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Asset bubbles in underdeveloped financial markets with influential DC pension funds: evidence from the Croatian financial market

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To my loved wife Ivana

Abstract

In the mid-1990s, the World Bank promoted a major reform of the pension systems in developing and transition economies; namely, the introduction of mandatory defined contribution pension schemes. Yet this was not accompanied by thorough analysis of the potentially speculative valuation side effects of influential institutional investors being introduced into underdeveloped financial markets.

In this Dissertation we developed a theoretical Overlapping Generations Model (OLG) with rational asset bubbles and influential institutional Defined Contribution (DC) pension funds. We report empirical evidence, based on data from the Croatian financial market, which confirms predictions of our theoretical model: namely, that when the financial market becomes dynamically inefficient, introduction of influential DC pension funds significantly increases the incidence and the intensity of the bubbly asset episodes. The empirical evidence also confirms that the shock of return to dynamic efficiency on the financial market, caused by the Global Financial Crisis, resulted in a sudden and swift collapse of the bubbly Croatian equity market.

We begin the Dissertation with a retrospect of the Efficient Market Hypothesis and the empirical evidence on asset price misevaluation episodes. The Efficient Market Hypothesis was contested with respect to its "joint hypothesis problem" contained in the distinction between the information efficiency of financial markets and the efficiency of financial market asset valuation. Major empirical evidence questioning asset valuation efficiency led to the development of four major theories of asset bubbles, which we treat in detail. We focus on the Rational Asset Bubbles theory as the most compelling and we use it in our own theoretical modelling. Building on a model developed by Jean Tirole (1985), we develop an original OLG Rational Asset Bubbles model with mandatory DC Pension funds as influential institutional investors. We inspect the dynamics of the modelled financial market using Phase Diagrams to derive the hypothesis that the introduction of a mandatory DC pension fund in a dynamically inefficient financial market leads to a higher state of the rational asset bubble. We also hypothesise that a sudden change in the dynamic efficiency of the financial market could lead to a swift crash of the rational asset bubble in such a market environment.

To test these hypotheses, we first develop novel approaches to identifying and measuring observable counterparts to each of the variables specified by the theoretical model. Next, we gather unique raw data from the Croatian financial market and extract time-series variables measuring: the relative value of the Croatian equity bubble; the dynamic efficiency of the Croatian financial market; and two different sets of DC pension fund investment data (one at annual and the other at monthly frequency), which arise from the introduction of DC pension funds on the Croatian financial market. Using this dataset, we estimate two empirical Vector Error Correction Models. In the first model we use the first set of two investment variables arising from the introduction of the DC pension funds: the direct equity investments in Croatian equities; and the indirect investments through the Croatian and SEE equity focused open-end mutual funds. While the empirical evidence showed that the direct equity investments behaved as productive investments, the indirect equity investments were identified as a bubbly (speculative) asset used by the Croatian DC pension funds. The model showed that the introduction of the DC pension fund led to a higher bubbly steady state of the equity market. This was confirmed by the second, simpler, VEC model, which was specified with only the indirect investment variable identified as bubbly in the first model. Specifying a model with only the indirect bubbly investment enabled higher frequency observations to better capture the speculative impact of DC pension funds.

In sum, we demonstrate the importance of the interaction between the introduction of DC pension funds and the dynamic efficiency of the financial market for the incidence and intensity of asset bubbles. In doing so, we draw to the attention of policy makers in countries introducing mandatory DC pension funds some hitherto unacknowledged consequences of this reform, once vigorously promoted by the World Bank.

Key words: rational asset bubbles, overlapping generations' model, financial market, mandatory DC pension funds, vector error correction model, pension reform, investments

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Introduction

In this dissertation we develop and present an Overlapping Generation Model with rational asset bubbles, which introduces Defined Contribution (DC) pension funds as influential institutional investors into the canonical model developed by Tirole (1985). We show through theoretical simulations derived from our model that the introduction of influential institutional investors, such as DC pension funds, onto the underdeveloped financial markets of the SEE transition economies created a fertile environment for rational asset bubbles. In our corresponding empirical analysis, we use data from the Croatian financial market, and its equity market in particular, because of the relative importance of this market in the SEE region and because of the availability of the data. Based on data from the Croatian financial market, we estimate two Vector Error Correction Models and find evidence supporting our theoretical hypothesis that the introduction of influential institutional investors onto an underdeveloped financial market could create a speculative market environment and increase the speculative market pricing equilibrium to the levels associated with asset bubbles.

The general motivation of this dissertation comes from the pension reforms in the SEE financial markets and the speculative equity market episodes on their small equity markets following the pension reforms. Global pension reforms in the last decade of the 20th and the first decade of the 21st century were marked by the highly influential research *Averting the Old Age Crisis* sponsored by the World Bank (World Bank, 1994). With this research, the World Bank promoted mandatory fully funded pension funds, the so called "second pension pillar", as the basis of pension reforms suitable for developing and transition countries. Those mandatory fully funded pension funds were designed as country wide saving/investment schemes, predominantly taking the legal form of DC pension fund institutions. These collect part of each employee's gross salary on a mandatory basis and invest this amount on the financial market to provide pensions in the future. Such reform was meant to help those countries overcome the issue of population aging and the inadequacy of their current "pay-as-you-go", the "first pillar" of their pension insurance systems. This World Bank initiative, followed by a set of active advising programs, resulted in 23 countries undertaking the reforms and accepting the so called "second pillar" of the pension insurance represented by the mandatory fully-funded privately managed pension funds (later in the text DC pension funds). The reforms started among Latin American countries[1] where, after being pioneered in Chile in 1981, 12 more Latin American countries undertook the same path. The second wave followed among the Eastern European transition countries[2] where 10 countries undertook the reforms in a period of 10 years or so (1998-2008) (Lago, 2014). Romania was the last country on the list, which converted to the system in 2008. The main consequence of the reforms was the establishment of influential Defined Contribution (DC) pension fund institutional investors rapidly growing by their assets under management and soon becoming dominant institutions on the underdeveloped financial markets of the reforming countries.

Besides the main focus on the issue of aging population, inadequacy of the current pension system and the suggested reform process, with detailed prescriptions on the forms of application and transition to the mandatory fully-funded saving schemes, there was very little analysis and information in the World Bank research about the potential negative impacts or challenges of such reform on the underdeveloped financial markets of those countries. Mostly, this was because before 1994, when this research project was published, only Chile had a (relatively short) track record of experience with the introduction of privately managed mandatory DC pension funds. The World Bank (WB) assumed in its research that the impact of the introduction of mandatory fully-funded DC pension funds on the financial markets of the developing and transition countries would be predominantly positive, stimulating financial innovation and market deepening. Such an assumption was based on the experience of US financial markets, where occupational pension schemes started dominating the financial market already in the 1980s and early 1990s (Allen, 2001).

> *In the United States, pension funds and life insurance companies became the main forces behind financial innovations after the Employee Retirement Income Security Act (ERISA) of 1974, which imposed minimum funding requirements and sharply increased the demand for hedging instruments. New instruments have been tailored to the needs of pension funds (such as zero coupon bonds, collateralized mortgage obligations, mortgage-backed securities, indexed futures*

[1] Latin America: Chile 1981, Peru 1993, Argentina and Colombia 1994, Uruguay 1996, Bolivia and Mexico 1997, El Salvador 1998, Nicaragua 2000, Costa Rica and Ecuador 2001, Dominican Republic 2003 and Panama 2008.

[2] Eastern Europe: Hungary 1998, Poland 1999, Latvia 2001, Bulgaria, Croatia and Estonia 2002, Lithuania 2004, Slovakia 2005, Macedonia 2006 and Romania 2008.

> *and options, and guaranteed income contracts). These financial instruments have transformed illiquid loans into highly liquid and tradable securities and enabled new forms of risk sharing, facilitating both business investment and housing finance. UK pension funds make active use of financial instruments in their international investment strategies, increasing liquidity and lowering transaction costs. So, one reason for encouraging private funded pension plans in middle income developing and transitional countries is they might become an instrument of financial innovation and capital market deepening*.
>
> (World Bank, 1994, p. 177)

However, even in developed financial markets such as the US market, those welcomed innovative financial instruments produced to satisfy the investment requirements of the growing occupational pension industry, stressed as positive in the WB research in the 1990s, were later criticized as being highly speculative and responsible for the real-estate bubble which triggered the Global Financial Crisis with its collapse in 2006/2007.

The potential issue of the over-accumulation of productive investments caused by the strong shift in the savings attitude of the developing and transition economies, as a corollary of the introduction of mandatory fully-funded pension schemes, was ignored as a possible problem for the financial markets of those developing and transition countries. The potential negative effects were disqualified without a thorough analysis but simply with the argument that reasonably high rates of productive capital returns existed in those economies (World Bank, 1994, p. 208; Vittas & Michelitsch, 1995, p. 24).

This lack of a critical attention toward the potential negative impact on financial market dynamics when influential institutional investors – pension funds – are introduced onto a small and underdeveloped financial market, in the World Bank (1994) research on the suggested reforms, resulted eventually in negative performance and an expensive reverse trend in the reforming process of the pension systems in many of those developing and transition economies after the Global Financial Crisis. Hungary fully reversed the reforms by nationalizing the "second pillar" DC pension funds, while many other countries such as Argentina and Bolivia in Latin America, and Poland, Estonia, Latvia, Lithuania and Romania in Eastern Europe applied transitional measures reversing the reforms, mainly by reducing the mandatory contributions to the fully-funded pension funds (Heinz, 2012). One of the main reasons was the significant financial loss and the disappointing returns achieved by pension fund investments (Rofman, 2011). The inadequate analysis of potentially negative effects of the introduction of fully-funded DC pension funds on the financial markets of the transition and developing economies, producing fertile ground for development of speculative investment episodes, was one of the main reasons for the failure of the reforms and for the costly reversing actions. The main challenge of this dissertation is to add to the understanding of the consequences of introducing influential institutional investors onto the underdeveloped financial markets of the developing and transition economies, in particular with respect to the probability of the incidence and the level of bubbly (speculative) market outcomes.

In order to understand the occurrence and the character of asset bubbles, in Chapter 1 of the dissertation we present a detailed introduction to the theoretical treatment of market speculation and asset bubbles. We begin our discussion with the still dominating financial markets paradigm of the Efficient Capital Market Hypothesis (ECMH) and we introduce some of its main issues such as the "joint hypothesis problem". A number of empirical studies testing the ECMH show that although financial markets generally provide confirming evidence of information efficiency, they often imply asset pricing mechanisms that significantly differ from the fundamental values of the underlying assets (Shiller, 2000). Those two elements, the information and pricing efficiency of the financial market mechanism are hard to separate and test individually, which describes the joint hypothesis problem. However, the impressive amount of empirical evidence focused on departures of financial asset prices from their fundamental values initiated new theoretical ideas describing the market pricing mechanisms, aside from the principles of the ECMH or the beliefs of market "fundamentalists". In the first chapter we make a complete account of the four major theories of asset bubbles, which present an alternative understanding on the operation of financial market pricing mechanisms. Those schools of asset bubbles or asset bubble theories are the Rational Asset Bubble Theory, Asymmetric Information Theory, the Agency Problem Theory and the Irrational Investors Theory. We find the most compelling to be the Rational Asset Bubbles theory, which with its Overlapping Generations modelling (OLG) structure was introduced in the canonical work of Jean Tirole in 1985. This presents the most complete platform for modelling financial markets with bubbly assets, connecting the macro dynamics with the micro specifics defined in the model (Tirole, 1985).

Next, we continue with Chapter 2, where we present our original extension to Tirole's Rational Asset Bubbles OLG model, augmenting it by introducing mandatory institutional

DC pension funds as influential market participants. Our specific theoretical modelling, built on the platform of Tirole's Rational Asset Bubbles model, allows us to analyse the dynamics of the speculative asset pricing mechanisms under certain conditions: namely, the introduction and the impact of the DC pension funds; the dynamic (in) efficiency of the financial market; and the impact of significant shifts in inter-temporal consumer preferences caused by events such as the Global Financial Crisis. Phase diagram simulations arising from our theoretical model are used to simulate certain scenarios and inform our main hypothesis about the effects of the introduction of DC pension funds. We hypothesise that the introduction of the DC pension funds is expected to increase the incidence and the intensity of rational asset bubbles, especially in dynamically inefficient financial market environments. We also hypothesise that strong shifts in the dynamic efficiency of the financial market, caused by either sudden fear among consumers/investors triggered by such events as the Global Financial Crisis or by the termination of the DC pension scheme, could produce swift increase of market volatility and a collapse of the bubbly assets.

We continue by developing an empirical strategy for testing our hypotheses in Chapter 3. There we first determine the most important variables that have to be measured, and we identify unique measuring methodologies used to extract the corresponding data from the Croatian financial market. Our main endogenous variables are the relative value of the Croatian equity bubble, the dynamic efficiency of the Croatian financial market and two separate sets of DC pension fund investment variables differentiated by the method of derivation and their genuine frequency. Within the DC pension fund investment variables, besides the direct equity market investments, we introduce investments in the Croatian open-end investment funds. These funds are financial intermediates that invest on the Croatian and SEE equity markets based on a pre-specified investment policy. The Croatian Open-end investment funds focused on Croatian and SEE equity markets boomed with their assets under management during the period of the Croatian equity bubble and were intensively used as an investment asset by the DC pension funds. Their extension of the principle-agent problem and information asymmetry makes them a suitable candidate for the "bubbly asset" in the context of the Croatian financial market. The open-end investment funds were the actual financial market innovation, in the spirit of the World Bank, which genuinely developed in order to satisfy the need of the DC pension funds for a bubbly asset in the bubbly financial market environment. In this chapter we also estimate the relative value of the asset bubble on the US market, as an exogenous variable to control for the effect of global financial markets. Finally, we construct set of dummy variables representing the introduction of certain legal acts affecting the operation and the investments of the Croatian DC pension funds. We make a complete descriptive analysis of our dataset in order to address some important issues such as stationarity and co-integration among non-stationary variables. We further use this data set in Chapters 4 and 5, where we estimate two empirical time-series models to test our main hypotheses.

In the first empirical model presented in Chapter 4, we estimate a Vector Error Correction Model (VECM) with lag order of four and with two long-term co-integrating relationships. This model is estimated with a set of two investment variables of the DC pension funds describing their introduction to the Croatian financial market: direct DC pension fund investments in Croatian equity; and indirect DC pension fund investments in Croatian equity through the Croatian and SEE equity focused open-end mutual funds. The data for both variables is directly extracted from the audited balance sheets of the four Croatian DC pension funds and interpolated from their yearly frequency to monthly frequency using the cubic fitting function. This first estimated empirical model has several drawbacks. First, it consumes a high number of degrees of freedom due to the high optimal lag in the model compared to the length of the data set. Second, its two DC pension fund investment variables are interpolated from their genuine yearly to a monthly frequency, which causes them to be highly cross correlated with each other. And, finally, the coefficients of the VECM show weak results when testing their stability. We also introduce a structural break within the co-integrating vectors, signifying the effect of the Global Financial Crisis, which suggests that the GFC had a dominant effect on the collapse of the equity bubble and on the change of the dynamic efficiency of the Croatian financial market. Our first theoretical model, confirms that the introduction of the DC pension funds, measured by the pension fund investments variables, have a positive effect on the relative value of the Croatian equity bubble. One of the unique benefits from the first empirical model with two investment variables is that it identifies the bubbly asset as the indirect DC pension fund investments via the Croatian and SEE equity focused open-end mutual funds, while the direct equity investments are identified as productive investments. This distinction is confirmed by the contrasting impacts of each investment variable on the dynamic efficiency of the Croatian financial market.

In order to overcome the problems identified with our first empirical model estimated and presented in Chapter 4, we estimate the second empirical model presented in Chapter 5, in which we introduce DC pension funds by a single investment variable. This is a smaller VECM with a lag order of zero and with two co-integrating vectors, and in which the investment variable of the DC pension fund is represented by an index of the assets under management of the Croatian and SEE equity focused open-end mutual funds weighted by their participation in the DC pension fund portfolios. The genuine monthly frequency of this index variable brings the benefit of much richer information about the DC pension fund speculative investments but at the cost of shorter period of availability. Using this variable we estimate a much simpler VEC Model, losing many fewer degrees of freedom. This second model is free from residual cross-correlation among the equations of the VECM, which was an issue with the empirical model presented in Chapter 4. The coefficient stability tests show also much more robust results for this second empirical model. However, when we test our hypothesis using our model in Chapter 5, we find similar results to the ones supported by our first empirical model.

Finally, we use our two empirical models to draw conclusions about the long-term and short-term interactions of our variables governing our modelled financial market dynamics and to test our theoretical hypotheses about the impact of the introduction of DC pension funds on the incidence of bubbly assets on domestic financial markets. We find evidence confirming the positive effect of the introduction of DC pension funds on the level of speculative asset bubbles in the financial market. We also find that the dynamic inefficiency of the financial market, representing a misbalance between investment opportunities and savings/investment demand, creates a fertile environment for the occurrence and for the growth of the rational asset bubbles on the local financial market. Namely, an environment with strong savings demand and low investment opportunities characterized by low interest rates, called a dynamically inefficient market environment, is the necessary condition for the occurrence of rational asset bubbles. The opposite, a dynamically efficient financial market environment, represents an environment in which rational bubbles collapse. In this structural environment of the financial markets, and especially in the case of the dynamically inefficient financial market, the introduction of influential DC pension funds, which soon gain significant market importance, stimulate the incidence and levels of asset bubbles. Such asset bubbles, increased to a higher level due to the introduction of the DC pension funds, come under high pressure in times of sudden return of dynamic efficiency of the market caused by some event triggering market distress. Such times are common in periods similar to the global financial crisis, which caused a structural shift to the financial market steady states and set the speculative asset valuation dynamics towards collapse.

Finally, we draw the attention of current and future policymakers to the importance of dynamic efficiency on the financial market and to the influence of institutional investors on the creation of speculative market episodes. Policies stimulating dynamic efficiency must follow the growth of the market dominance of DC pension funds. Otherwise the market environment can become a fertile platform for ultimately harmful speculative market episodes. We believe that this research opens up a new set of issues that should be further analysed in order to improve not only the process of pension reforms in the developing and transition countries, but also the process of pension fund influence on the global financial market.

Part I: Theoretical Modeling

Chapter 1: Theory of Capital Market Asset Bubbles

Introduction

Analysing the main factors explaining the occurrence of asset bubbles on small and illiquid equity markets, such as are characteristic for the South East European (SEE) equity markets, presuppose a wide understanding of the theory of finance, focused on the efficiency of capital markets and equilibrium asset pricing models. This knowledge sets the critical basis for understanding the potential reasons for asset price miss-valuation and for the occurrence of asset bubbles in different market environments. For this reason, we begin this dissertation focused on the effect of the pension reform and introduction of Defined Contribution pension funds in the Croatian financial market on the formation of equity asset bubbles, by reviewing the main theoretical foundations on asset pricing theory, market efficiency and the theory of asset bubbles.

We begin by giving summary ideas on the mainstream theory on market efficiency and equilibrium asset pricing models, where the Efficient Capital Markets Hypothesis (ECHM) dominates the present mainstream theory of finance. We try to evaluate some weaknesses and fundamental questions of ECMH, such as the joint hypothesis problem, which will lead us to the current disagreement among the most influential economists in the field, about the issues of market efficiency and asset price miss-valuation. This disagreement, prominent especially during the last two decades, and playing out against the background of many observed market miss-valuations of financial assets, led to the development of a new set of theories focusing on the arguments and models explaining the occurrence of asset bubbles.

The main idea of this theoretical chapter is to provide the platform for theoretical and then empirical investigation: in particular, theoretical tools, such as the Overlapping Generations Model, presenting a general equilibrium framework; and the main micro–structural features important for explaining asset bubbles. This is a condition sine qua non for specifying an empirical model capable of explaining the occurrence of asset bubbles in illiquid financial markets, such as the one present in the Republic of Croatia, affected by the introduction of influential institutional investors, namely the defined contribution pension funds.

This theoretical chapter will be structured in the following order. Subchapter 1.1 which follows, will focus on explaining the historical development of the main ideas leading to the establishment of the Efficient Capital Market Hypothesis(ECMH), with exposition of its main strengths and weaknesses, and especially focused on the distinction between the informational efficiency hypothesis of the capital markets and the hypothesis on efficiency of the equilibrium asset pricing. This distinction is known as the joint hypothesis problem.

Then follows subchapter 1.2 focused on the theories of asset bubbles, defined as a persistent pricing disparity over an assets fundamental value and inspired by the vast empirical evidence showing the persistence of capital asset market miss-valuations. Here, we will begin with the dominating general equilibrium model, the Overlapping Generations Model, which will be explained in detail since it represents the main macro modelling tool for theoretically modelling and analysing asset bubbles in different theoretical contexts with different micro structures. Then, further, this subchapter will be divided into four parts, each one covering one of the four major schools on asset bubbles focused on a specific micro structural feature of the market economy which leads to the occurrence of asset bubbles.

Finally, the conclusions will synthesize all the major ideas relevant for the incidence of asset bubbles on the Croatian financial market. This will help to specify an original model of asset bubbles, by means of which we will analyse the effect of introducing institutional investors.

1.1. Evolution of the Efficient Capital Markets Hypothesis and Equilibrium Capital Markets Pricing Models

The origin of ideas about efficient capital markets, as summarized in the work of LeRoy (LeRoy, 1989), begins in the early 1930s. The Random walk theory of price dynamics was the focus of the pioneering economists analysing stock price dynamics such as Holbrook Working (Working, 1934). Another rising group of authors, known as "fundamentalists",(Williams, 1938) and (Graham & Dodd, 1934), claimed that the best investment strategy is investment decision-making based on the value of the investment assets defined by their discounted values of future expected cash-flows (DCF). This claim about valuation based on the intrinsic value of assets, from the very beginning of its theoretical treatment within asset pricing theory, was confronted with increasing empirical evidence showing that there was not much pay-off from investment decisions based on fundamentalist prescriptions. This was an observation made by several academics (Cowles, 1933), who analysed the results of fundamentalist recommendations given by 45 professional agencies at that time, finding that their advice was on average useless. This opposition to the claims of "fundamentalists" even from the early stage of the foundation of asset pricing theory was becoming a widely accepted opinion among the market practitioners as well. The theoretical alternative for explaining stock market prices and the process by which they are set on the market, was to observe the market price dynamics as a pure "random walk", which made the rational investment strategy similar to that of a pure gambler. There was obviously a need for a new set of better founded ideas bringing together the observed stochastic stock price dynamics and the ideas of the fundamentalists for explaining how market prices are set.

New radical ideas came from the work of Samuelson who introduced the "martingales stochastic distribution", which has weaker constraints compared to the "random walk" distribution (Samuelson, 1965). The difference between the two is that a "random walk" is a sum of random numbers in our case representing the absolute change of the value between two periods, where each consecutive number is an independently and identically distributed (i.i.d.) random number or change. The formal statement of the random walks is the following:

$$S_t = X_0 + X_1 + .. + X_t \qquad (1.1)$$

where X_i is an i.i.d random variable with "i" indexes represent periods from 1 to t

On the other side, a "martingale sequence" is a sequence of numbers, where the expected value of the martingale variable reflects the information about all its previous period values. A completely natural consequence of this statement is that if prices represent a martingale process, then the expected value of the current period price will equal the value of the last observed price end so on. The formal definition of the martingale price dynamics could be described in the following way:

$$E(P_t | P_{t-1}, P_{t-2}, ..P_0) = P_{t-1} \qquad (1.2)$$

This in turn, for the purpose of its use in the theory of finance, all historical and actual information about the asset whose price dynamics is analysed and established by the market are included in the definition of the actual expected price. One important difference of martingales compared with the random walk, is based on the weaker predictability constraints of the martingales coming from the fact that the higher moments of the martingale variable could behave with inter-temporal correlation, while such a property is not present in the random walk stochastic variable where higher moments cannot be predicted, which means that no variance clustering could be observed. This also means that the expected variance of the price described by a martingale process is not necessarily equal to the last observed price variance or to its historical mean, which is a case with the random walk distribution variance. That implies that in the case of the random walk, variance or the second and every other higher moment could not be modelled using historical prices. On the other side, price variance and every higher moment could be modelled based on historical price values in the case of martingales. This is crucially important for application explaining stock price dynamics, because it implies that with martingales, trading agents could predict future variance clustering or could predict changes in asset price variation using the historical price variation.

This leads to a strong conclusion about the character of agents in efficient capital markets as defined in the work of Samuelson (Samuelson, 1965). Predictability of price variance with historical prices implies that this variation of asset price, or of the higher moments, does not enter traders' decision functions. Otherwise, if it is assumed that markets are efficient, traders will make the predictability of variation disappear by applying the prediction model to make arbitrage profit on variation based trading strategies. This makes one significant characteristic of martingales as applied by Samuelson, something that according to the interpretation of LeRoy, Samuelson might not have been unaware of (LeRoy, 1989). The implication is that agents in Samuelson's martingale model are defined as being risk neutral. This means that agents are indifferent about the risk defined as a higher moment of stock price dynamics, which could be arguably discussed as confronting some of the already established portfolio theoretical conclusions in portfolio theory(Markowitz, 1952), the classical CAPM model as well as the later observed "equity premium puzzle" (Mehra & Prescott, 1985). This also implied that the non-arbitrage condition, required at the equilibrium of efficient markets, will lead all financial assets to have the same expected rate of return in equilibrium, which equals the expected risk free rate of return. Finally, besides the drawbacks, Samuelson - describing the price dynamics of financial assets as a "fair game" martingale process -made a connection to the fundamental valuation of assets based on discounting the value of expected future cash flows where all available information is included in the estimation process. The stochastic feature of price dynamics was explained as a result of the uncertainty connected to the future cash flows. Later this approach was the basis for major criticism of martingales by empirical studies that applied volatility tests of asset prices in comparison with the volatility of dividends (Shiller R. , 1979; 1981; LeRoy & Porter, 1981).

Theoretical interest in the efficiency of capital markets in the late 1960s, culminated in the formal adumbration of the Efficient Capital Markets Hypothesis (ECMH) by Fama who defined efficient capital market equilibrium prices as ones which "fully reflect" all available information about the underlying financial asset(Fama, 1970). This definition of the efficient capital market, building on the work previously done by Samuelson, can be found even in today's finance textbooks mainly due to its simplicity in explaining the concept of market efficiency. Fama claims, similar to Samuelson, that if prices "fully reflect" all available information about the underlying asset, then price dynamics must represent martingale stochastic dynamics, which means that the actual prices are the best estimate of the next period prices(Fama, 1970).

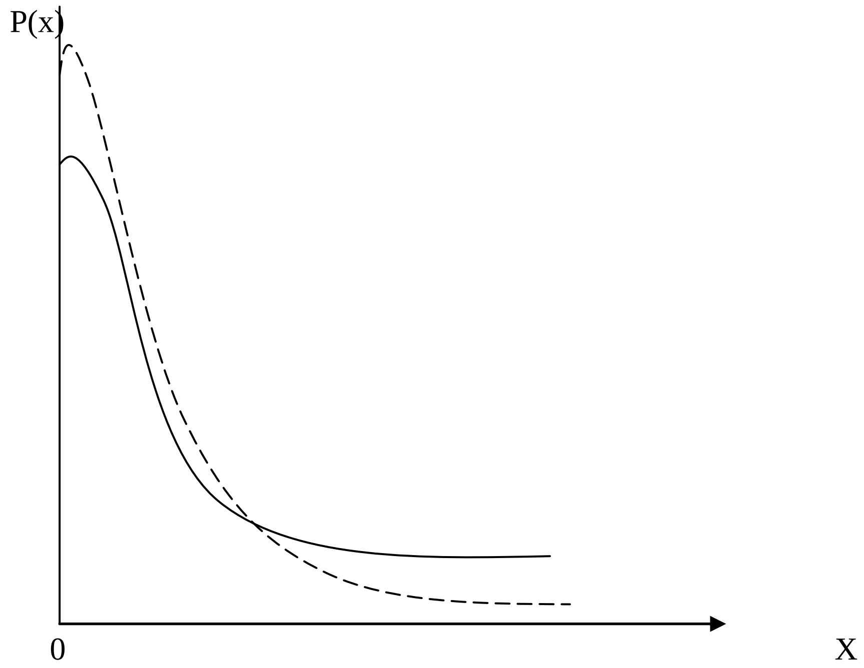
Fama goes even one step further, claiming that financial asset price dynamics represent a semi-martingale stochastic process (Fama, 1970). A semi martingale, compared to a pure martingale, contains one additional condition; namely, that the expected value of the current price is greater than or equal to its last price. This implies that there is a possible "non negative drift" in the stochastic process of price dynamics, which he used in his claim that such a case within martingales makes the "buy-and-hold" investment strategy dominate any other trading strategy based on all available information. The formal definition of the semi-martingale distribution with a non-negative drift is defined by the following inequality (Fama, 1970):

$$E(P_t | P_{t-1}, P_{t-2}, ..P_0) \geq P_{t-1} \tag{1.3}$$

In this respect, Fama's previous work (Fama, 1965a; 1965b) found that asset prices on such efficient capital markets tend to represent the intrinsic fundamental value of the financial asset, a crucial belief that he weakened in his later work in response to an increasing weight of counter-arguments coming from the empirical research of other authors. He pointed out that if the financial markets are efficient, information derived from technical analysis prescriptions, such as "the Dow Theory", based on finding patterns in the historical dynamics of asset prices, has no added value for the investment strategy of traders. Fama made extensive empirical research (Fama, 1965a), showing asset prices, although randomly distributed around their expected value, follow Mandelbrot's "fat tail"[3] infinite variance stable Paretian distribution with a specific coefficient a<2 (Figure 1.1.).

[3] A "fat tail" probability distribution is one which accumulates probability at extreme events in its tails. In this group belongs the T-student distribution which is leptokurtic especially at <10 degrees of freedom, and is often used by the industry simulating this "fat tail" character of asset price distribution.

Figure 1.1.: Heuristic presentation of the difference between the tails of normal and fat tail distributions



Source: Authors own interpretation of the difference between the right-hand tail of the Gaussian Normal distribution (broken line) and of fat tailed distributions (illustrated by the solid line)

Other authors also confirm this type of asset price distribution, also pointing to the importance of previous work by the French mathematician Louis Bachelier, who with his doctoral dissertation on the *Theory of Speculation* was claimed to be the first to model the stochastic process now called Brownian motion (Bachelier, 1900).

Fama initially realized that not all markets are completely efficient and that information efficiency plays a crucial role in determining the extent of market efficiency. Accordingly, when defining market efficiency, based on the empirical evidence about the levels of inclusion of information in the price of assets, he established three levels of capital market efficiency in his efficient capital market hypothesis (Fama, 1970):

1) "weak form" where the actual asset prices reflect all information of the previous historic price series;

2) “semi-strong form” where, besides the information on historical prices, actual prices reflect all publicly known fundamental information of significance to the underlying asset; and

3) “strong form”, where all information about the underlying asset is reflected in the actual market price of an asset, including insider information.

Even today, this is the most used distinction of different levels of capital market efficiency, based on the three different levels of fundamental information reflection in the prices. Yet, Fama soon changed his definition of efficient capital markets to one describing them such that they (Fama, 1976):

A) do not neglect any information relevant to the determination of security prices; and

B) Investors’ have rational expectations.

Introduction of the rational expectations investors was the biggest novelty in his newer definition, which received a better reception (LeRoy, 1989).

> *The assumption of rational expectation means that agents use all their available information to make those inferences about future events that are justified by objective correlations between the information variables and the future events, and only those inferences.*
>
> (LeRoy, 1989)

This could simplify to the conclusion that, with rational expectations assumed, agents are theoretically modelled in a way that they optimize knowing the structure of the market and the parameters of the model describing the market, which makes them rational, but also that they know the model and parameters describing the underlying asset of the investment, which makes them endowed with rational expectations. This definition certainly does not exclude the possibility of rational departures from the “objectively” defined fundamental value of assets, which will later come to be known as rational asset bubbles.

This new definition in Fama's textbook of 1976 could be the first sign that he realized the “joint-hypothesis” problem within the empirical test of efficient capital market based on martingales theory, which will be treated in more detail further on, especially in his own summary critique of the Efficient Market Hypothesis made in 1991.

In his later review of his seminal paper on the ECMH, Fama summarizes that there remains a significant amount of empirical evidence showing that stock prices follow martingale dynamics in the short run, but mounting evidence against it for longer period price dynamics (Fama, 1991). He also stressed that there is little evidence that based on historical price information one could develop a profitable arbitrage short-run strategy when calculating in the trading costs. On the other side, Fama also admits that in the long run, in contrast, there is significant empirical evidence that price dynamics can be predicted based on historical price dynamics (Fama, 1991).

Testing the empirical connection of the fundamentalists' ideas with the market asset prices, the empirical research showed that the variance of asset prices is higher that the variance of dividends, which was defined as an upper bound for price variance if the prices change according to the martingale model and based on asset intrinsic value (Shiller R. , 1981; LeRoy & Porter, 1981). This is part of the strong evidence against market efficiency described as setting asset prices equal to their fundamental value. Other evidence is the empirical work finding the existence of mean reversion of prices in their long-run dynamics. Many authors observed the mean reversion character of stock price dynamics in the longer run. Shiller, based on this evidence formed a model of "fads" for describing the dynamics of asset prices. According to Shiller, low forecastability of future values of cash flows and, overall, of the fundamental value of assets, creates a large discrepancy between the market price of assets and their fundamental value (Shiller R. , 1981). He suggests that martingale models be combined with the mean-reversion models describing asset price equilibrium into models with "fads", so that the theoretical model corresponds better with the actual empirical observations. Those models should have near-zero short-run asset price inter-temporal correlations according to the assumptions in the ECMH literature, but also negative or mean-reverting autocorrelations for longer-run asset returns and they should permit breaches of the variance bounds of dividend variation by price variation as observed in his empirical tests(Campbell & Shiller, 1988).

Back to his starting work on ECMH, Fama presented empirical evidence in support of the "weak" and "semi-strong" forms of the ECMH, but found little evidence in support of the "strong" form of ECMH (Fama, 1970). In his own review twenty years after he published the ECMH, Fama realized that there is an important difference between the hypothesis of market efficiency as described by the martingale asset prices process, and the hypothesis of efficient equilibrium asset pricing model used by the market (Fama, 1991). This difference is recognized as a joint hypothesis problem. Capital markets may be efficient in the sense that they instantly rationally incorporate all available information in the prices depending on the market micro-structure, but this does not necessarily exclude the persistence of asymmetric information and other factors which may impact the efficient equilibrium asset pricing model dominating the market. This leads to an increasingly accepted opinion that empirical tests of efficient market reaction on new information, which were widely used to provide evidence for the claims of markets efficiency, do not necessarily also provide evidence that markets equilibrate prices at their objective intrinsic value such as the DCF value described in the work of the market "fundamentalists". This represents a departure of Fama from his own work (Fama, 1965a), where he claimed that asset prices on efficient markets, defined as ones reflecting all available information, are equal to the intrinsic value of the underlying assets. This departure gained importance especially because, even though there is significant evidence that markets react instantly to new information, which means they incorporate new information in the price of assets, there exists significant evidence that conventional equilibrium asset pricing models are regularly inconsistent with the observed asset prices through time. This evidence also corresponds to the claims of economists such as Grossman and Stiglitz, who made one of the best critiques of ECMH (Grossman & Stiglitz, 1980), as recognized by Fama himself (Fama, 1991), and who theoretically proved in a simple model of a market economy that markets inevitably function in an asymmetric information context. An implication of their conclusion is that, although markets efficiently react to new information, because of the persistent asymmetry of information they still could exhibit large price disparities from some formally or objectively set criterion for intrinsic asset valuation such as the one of fundamentalist prescriptions. LeRoy shows many examples of empirical price regularities which cumulate evidence against such efficient equilibrium pricing models (LeRoy, 1989). The "Monday effect", the "Weekend effect" and the "January effect" are just three of the many, empirically observed regularities.

The joint hypothesis problem slowly drew the focus of academic research away from empirical tests of the informational efficiency of capital markets, based on testing martingale price dynamics, toward the suitability of different equilibrium market asset pricing models. One of the most influential models was the CAPM (Capital Asset Pricing Model) (Sharpe, 1964; Linter, 1965; Black F. , 1972), widely used by practitioners but strongly criticized for its inconsistency. Due to its observed inconsistency with the empirical observations (Fama, 1991), showing the significance of other factors such as the size of the company, or the value of some company specific valuation coefficient in their expected return estimation model such as E/P or B/M[4], CAPM was subject to further improvements. Merton introduced the idea that the consumer/investor when making investment decisions tries to actually smooth their future consumption by investing in market instruments which provide the inter-temporal bridge for their budget constraint (Merton, 1973). He augmented the CAPM into the Inter-temporal Capital Asset Pricing Model. He claimed that efficient market investment decisions incorporate a third portfolio or asset besides the efficient market portfolio, which serves investors or consumers to hedge risks of events affecting the other two portfolios (such as short-run treasuries and an efficient market portfolio) used by the classical CAPM. He finds that long-term government bonds might be playing this third instrument role. He recognized the problem of the CAPM model in ignoring inter-temporal investment decision characteristics, and the fact that investors care not only for the static expected returns but also for the future set of expected investment returns. The group of augmented models, forming the Inter-temporal Capital Asset Pricing Models (ICAPM) class of models, recognized the importance of the Inter-temporal Marginal Rate of Substitution (IMRS) when explaining the variance of stock prices. Other authors, such as Holmström and Tirole, extended the research on the CAPM by including the ideas on hedging the liquidity risks by companies with their Liquidity Capital Asset Pricing Model. They showed that companies try to hedge liquidity risks by holding liquid assets, which affects the equilibrium pricing model such as the classical CAPM (Holmström & Tirole, 2001).

So in general, this increasing focus on the equilibrium capital asset pricing models created augmented versions of CAPM trying to find better theoretical explanations by introducing additional factors affecting equilibrium asset prices where the most interesting work was done on the consumption based factors which, as will see later on, play a significant role in the general equilibrium rational asset pricing models with asset bubbles as defined in the OLG framework (Tirole, 1985).

Models such as CAPM, multi-factor asset pricing models or consumer betas models also proved inconsistent and unreliable through time. In their paper, presenting a critique of

[4] E/P – Corporate Earnings to equity Price; B/P – Corporate Book value to equity Price coefficients

equilibrium asset pricing models, Fama and French, find that the Sharpe-Linter-Blacks' classical CAPM model shows significantly inaccurate power in predicting asset price returns (Fama & French, 2004). They find that "Jensen's alpha", which according to CAPM has to be equal to 0, is in practice significantly different than 0 even for diversified portfolios. They propose further improvements of CAPM by controlling some state variables such as earnings, labour income, spending as in Merton's Inter-temporal CAPM, and so on. They also find that the estimation accuracy of the CAPM model is significantly weak and that choosing the market benchmark for the asset return estimates may create a significant bias.

Most of the recent empirical testing for the different forms or levels of the ECMH is based on inspecting the inter-temporal correlation of the current period price dynamics with the price dynamics from the previous period and the reaction of the price to news about some fundamentally important information for the underlying asset. One of the elegant empirical tests applied is the test for the profitability of some market strategy based on historical price data. But simple immediate response of asset prices to some new information, or the non-profitability tests of historical price based trading strategies, does not imply that the market equilibrium asset prices correspond to the discounted cash flow (DCF), which is one of the most commonly used criteria for establishing the fundamental value of an asset. It does not also imply that the price reaction to new information is adequate, over or under optimistic in terms of the intrinsic value of the asset. It does not even exclude asset bubbles such as the ones modelled by Tirole's "myopic" investors (Tirole, 1982) where prices also have martingale stochastic properties.

Twenty years after his seminal paper on ECMH, Fama states that there are numerous empirical findings of other authors showing that expected returns have significant mean reverting cyclical patterns over the long term, which make possible significant departures of prices from the intrinsic value of assets (Fama, 1991). This is strong evidence against the pure "fundamentalist" equilibrium asset pricing models and in favour of the possibility of asset bubbles defined as a significant departure of market prices from the fundamental value of their underlying assets. Authors searching for answers in another direction, such as focusing on asset bubbles theoretical research, e.g. Tirole, show that although bubbles cannot exist in fully dynamic models with rational expectations and a finite number of trades(Tirole, 1982), they can only exist in models with infinite trades such as OLG models(Tirole, 1985)and in "myopic" or "biased" expectation equilibrium models where asset price dynamics consist of some fundamental price and a bubbly element with martingale properties.

As a result, the evolution of ideas about the efficiency of capital markets came to the joint hypothesis problem. In turn, this pointed the way to growth of asset bubbles theory as an alternative to the equilibrium asset pricing theory based on the intrinsic value of assets as defined by the early fundamentalists. This introductory text on the theory of efficient capital markets led us through the evolution of the set of ideas and the main problems confronting the present academic thinking on asset pricing theories. One of the greatest of these is the empirically observed disparity between the market asset prices and their intrinsic values, which simply proves that “fundamentalist” asset pricing models such as Gordon’s pricing model (Gordon, 1959) derive from invalid assumptions for the equilibrium asset pricing models on markets at different stages of efficiency. This thesis belongs to the set of research efforts trying to contribute to the equilibrium asset pricing models where asset bubbles defined as departures from asset intrinsic values are accepted as an inherent market outcome. In the following sections, theoretical schools on equilibrium pricing models with asset bubbles are reviewed in order to arrive at the point where the introduction of influential institutional investors onto the financial market can be modelled and the consequences for the pricing of bubbly asset investigated.

1.2. Theoretical foundations of speculative asset bubbles

Speculative Asset Bubbles are not a new market phenomenon arising as an outcome of asset miss-valuation on financial markets. Their occurrence is known from the early stages of capitalism and on different kinds of markets. The only difference to today's notion and importance of asset bubbles compared to their earlier occurrences arises due to the process of globalisation. Globalisation increased the importance of financial markets to the global economy and amplified the extent and implications of episodes of massive asset bubbles and crashes. Such evidence is seen through the last episode of the housing bubble rise and burst in the US in the first decade of the 21st century, leading to a collapse of the major financial markets and to a prolonged economic downturn in the global economy.

This chapter reviews theoretical developments explaining the phenomenon of financial market asset bubbles. The main goal of this chapter is to identify and explain the mechanisms creating asset bubbles, which will later on be used to inform empirical analysis of the impact of institutional investors on their occurrence. Theoretical reflection on market miss-valuation such as the occurrence of asset bubbles focuses not exclusively on the creation and evolution of asset bubbles, but also on their rationality, connection to market micro structure and their dynamic properties within the financial market.

In terms of the initial theoretical ideas and awareness, asset bubbles and their incidence was stressed as an important market phenomenon even in the important contributions of Keynes, who claimed that asset bubbles are an inherent characteristic of capital markets (Keynes, 1936). Keynes stressed most of the deterministic factors and mechanisms inducing the creation of asset bubbles. He also distinguished between the assets fundamental value and their market price. Keynes defined the fundamental value by the present value of the discounted future cash flows similar to Gordon’s later established and popularised intrinsic model of stock valuation (Gordon, 1959), recognizing the importance of expectations or the probability of future events causing different cash flow outcomes. Keynes recognized the psychological profile of an “animal spirit” present among market participants, which drives prices away from fair value and stressed some important behavioural characteristics of investors. Although Keynes did not present a rounded theory of asset bubbles, he surely pointed to the essential ideas such as moral hazard and risk-shifting issues among investors, which informed developments in later years of theories of asset bubbles.

Asset bubbles draw significant attention of prominent economists because of their wide occurrence and impact. In this respect there is an amazing amount of literature trying to explain the occurrences of speculative asset pricing. Reinhart and Rogoff present a well-structured overview of the earlier episodes of speculative asset bubble occurrence (Reinhart & Rogoff, 2009), joined by many authors such as Paul Krugman who is one of the latest authors trying to raise the importance of understanding the occurrence and implications of financial market asset bubbles (Krugman, 2008).

Aside from the insightful ideas on asset bubbles set out by Keynes, and subsequent informal literature on speculative asset bubbles, there was a wide theoretical interest in explaining the occurrence and functioning of asset bubbles with higher rigour, which especially developed with the rise of the empirical evidence against the ECMH in the 1970s and 1980s. A general division of the main ideas, or schools of thought on the issue of asset bubbles was suggested by Allen and Rogoff, and we will keep this main structure on the main theoretical ideas throughout our work. These are the main schools in asset bubbles theory (Allen & Rogoff, 2010):

- Rational asset bubbles school
- Agency problem school
- Asymmetric information school
- Irrational investors school

In the following part of this theoretical chapter, we will focus on the characteristics of these four theoretical branches representing specific approaches to explaining and analysing asset bubbles. Every school tries to stress some important feature of the micro structure of the financial market, leading to the occurrence of miss-valuations and consequently the possibility of speculative asset bubbles. It is important to stress that most of the theories apply a specific micro-structural feature to some general equilibrium model where those specific micro structural elements lead to specific static and dynamic conclusions. The prevailing macroeconomic general equilibrium framework used by the authors from those four branches, is Samuelson's Overlapping Generations Model framework (OLG) (Samuelson, 1958), which is a very important tool for the theoretical modelling used in asset bubbles theory. Consequently, we devote significant attention to explaining and examining the OLG framework, first used by the Rational Asset bubbles school, which we later use to develop our own theoretical contribution. As we advance through the understanding of the OLG model, its enrichment with micro-founded insights will lead to more realistic outcomes and then, with the introduction of intermediary financial institutions embedded with an agency problem, to our own model in the following chapter. As most of the authors do, it is very important to be aware of the limitations of the theoretical modelling and to make a balance between the complication of the micro-structural features and the insightfulness of the model. Most of the good models are sufficiently simple to bring the most important conclusion to the forefront.

The main goal of the theoretical approach presented here will be to discuss the rationality and the factors leading to the occurrence of asset bubbles, especially among small non-liquid open financial markets. One such environmental characteristic impacting asset bubbles is the introduction of a mandatory defined contribution pension fund, with specific investment rules, investment incentives and other important qualities as an institutional investor. We first consider the Rational Asset Bubbles School, where the OLG framework will be explained.

1.2.1. Overlapping Generations Model and Rational Asset Bubbles School

The Overlapping Generations Model (OLG) was first introduced as a theoretical framework by Paul Samuelson, with claims present among economists that the actual author of the model was the French Nobel winning economist Allais Maurice. The OLG model represents a General Equilibrium multiple agent generations model. Samuelson's effort in his famous paper introducing the simple OLG model could be interpreted as a successful reaction to the critics of the Keynesian school concerning its weak connection of general equilibrium models to micro economic foundations. Samuelson succeeded in presenting a very simple, albeit well-rounded theoretical framework deriving its specific macro dynamics by allowing different micro-structural characteristics of the economy (Samuelson, 1958). In this way he made it possible to logically explain that there exists only a “special case” of the micro-specific environment in which the outcome of the market economy brings the general equilibrium to the socially optimal one. This “special case” was described as the “classical economy” outcome.

Samuelson, through the OLG framework, showed that there is a dominating set of common micro-structures for the market economy, where market mechanisms are inefficient in bringing the economy to the socially optimal outcome. Due to its simplicity and its ability to capture different important phenomenon, the OLG model was later on increasingly used and extended by many prominent economists. The OLG model serves to explain most of the principal economic phenomena, including financial market inefficiencies, and to help identify solutions, especially in modern monetary economics, by its dynamic macro system based on the complete use of neoclassical micro foundations allowing for different assumptions concerning the rationality of agents and the use of rational expectations.

The OLG model has one very important characteristic as opposed to the infinite living agent model (ILA) (Ramsey, 1928).It introduces an infinite existing economy but consisting of finite living agents instead of infinite living agents. This slightly complicates the inter-temporal consumer’s decision, but brings the savings problem and the long term perspective much closer to reality (Truman, 2007).

Although Samuelson originally set out a three-stage overlapping generation model with each individual agent living in three stages during their finite life, later in his paper he focused on a two-stage model without losing any generality (Samuelson, 1958). The main reason comes from it being more simple but sufficiently insightful and robust in explaining a series of economic and market phenomena. Economists, who followed his work, mainly accepted the general framework of the two-stage overlapping generation model. We will also focus on the two-stage OLG model in our later modelling.

The OLG model is based on an economy consisting of a finite number of homogenous agents, called households or the population, who live finite lives in two stages. In the first stage of its life, the agent is characterized as “young”; and in the second as “old”. Hence, in the overall infinitely lasting economy, there are two generations that constantly overlap their existence through the dimension of time. This means that persons who were young in the previous period (t-1) are old at the present period t, and they overlap or simultaneously coexist with persons who are young in the present period. Then, in the following period (t+1), the young born in period (t-1) no longer exist, they disappear (or simply die), and the young generation from period (t) become old and overlap with a new born generation of young. In this way the system continues to exist forever. This gives one important characteristic of constant preference heterogeneity within the market economy at any time of its evolution.

At the first of the indefinitely many discrete periods (t=0), called the initial or “the big bang” period, there are a number of Initial old agents. The initial old agents are numbered by N_0 and the first generation of young which is brought to existence by the initial old in the first period are numbered by N_1. Samuelson introduces population growth denoted by the rate n, which connects the number of young in the next period (t=2) with the young in the previous period (t=1) by the population growth first order difference equation $N_t=(1+n)*N_{t-1}$. This is the simplest way of modelling population growth in the OLG structure, and it could be defined by a more complicated dynamic equation depending on the needs of the analysis.

Population growth plays a significant role in the OLG economy, since it determines its growth and dynamics which, when introducing production, as within the model of Diamond, creates a more realistic model of the economy (Diamond, 1965). Another crucially important feature that OLG introduces, are overlapping economic agents aware of their finite lives and different wealth, productivity and consumption characteristics at each period. Looking at the infinity of the system it is an asymmetric system and this asymmetry arises because of “the big bang moment” and the fact that the system then lasts forever. This characteristic affects the welfare outcomes when finding the optimality solutions or the social planer’s solution of the model.

Here is how the development of the system can be explained graphically (Table1.2.1). Here we stick to Samuelson’s model determined with a constant growth of the population, equal to n+1 which in models with production determines the growth of the economy.

Table1.2.1: Population Evolution in the 2-Generation OLG model

t=1	t=2	t=3	t=4	t=5…
N0 (initial old) N1(young)	N1(old)			
	N2(young)	N2(old)		
		N3(young)	N3(old)	
			N4(young)	N4(old) N5(young) …

Source: Authors interpretation of the Samuelson (1958) OLG framework inspired by the lecture notes from Professor Paula Verme - Hernandez at Texas A&M University, US

Further, the OLG model is also based on the neoclassical micro economic foundations set by the consumer preference theory, but provides more realistic longer term solutions of the general economy and inter-temporal decisions compared to the models based on infinitely lived representative agents (Ramsey, 1928).

In the OLG model there is only one type of good which is not transferable between the two periods, which can be consumed in any period and which brings certain utility to the consumer following the consumer's homogenous utility function:

$$U(C_{1t};C_{2t+1}) \qquad (1.4)$$

Here, $C_{1,t}$ represents the consumption of the agent in the period (t) when young (1), and $C_{2,t+1}$ represents consumption of the agent in the period (t+1) when agents are old (2). Every rational agent, considering its budget constraint will try to maximize its overall life time utility function depending on its two period consumption decision. This process defines each agent's inter-temporal consumer preferences.

There is a very important characteristic of the economy arising from the inter-temporal consumer preferences of homogenous consumers constituting the economy. In general, there could be consumers who derive higher utility from their consumption of the good when young, who are called "impatient" consumers; and, on the other side, the implicit utility function can describe another type of homogenous consumers in the model who derive more utility from consuming when old, who are called “patient” consumers. Logically, considering their consumption preferences, those agents who are characterized as “impatient” consumers have a higher liquidity preferences budget wise, to be able to achieve their optimizing temporal consumption structure. On the other side of the spectrum, more “patient” consumers will be more exposed to long-term investments and will structure their budget toward more illiquid assets. In the extreme, there are agents deriving utility from consumption only when young (“extremely impatient”) and those who derive it only from consumption only when old (“extremely patient”). The OLG model can use only one type of homogenous consumer preferences, but the type used determines a very important characteristic of the economy. This distinction between “patient” and “impatient” consumers could further be modelled as being stochastic, which can explain the stochastic properties of financial markets where on extreme occasions markets become dominated by liquidity demanding “scared” agents or long-term investment demanding “optimistic” agents and not the mere intrinsic characteristics of the investment assets. This stochasticity of agents’ marginal rate of substitution (MRS) could further play a significant role in explaining the volatility of financial markets.

Going further into the micro structure of the OLG model, agents make rational decisions about their inter-temporal consumption structure and other use of available wealth throughout their lives. Samuelson gave only a certain one dimensional endowment to consumers in the periods when young and old, but their budget was later expanded by other authors using other sources of income and investment vehicles. Another budgetary source of income or investment instruments were labour and capital as used in the production OLG models (Diamond, 1965). This finally led to the distinction among productive investment assets and bubbly assets later introduced in Tirole’s model (Tirole, 1985).

Each consumer tries to maximize their homogeneous utility function by their decision on the distribution of consumption constrained by the available lifetime budget:

$$\max U(C_{1t};C_{2t+1}) \tag{1.5}$$

Later on, we could specify the utility function explicitly. As an example, we may use a log function of present and future consumption, or some other form like the Constant Relative

Risk Aversion (CRRA)[5](Mehra & Prescott, 1985) and we can manipulate the inter-temporal preferences switching from the states of having more patient to the states of having more impatient consumers. In order to keep the focus of our analysis on explaining the broad incidence of bubbles, we will stick to the implicit form of the utility function similar to the simple model described by Samuelson.

Samuelson tried to focus mainly on the inter-temporal problem of transferring consumption through generations. He simplified the economy defining non-dynamic, or the simplest possible budget constraint for each consumer, giving them a certain amount of endowment (an amount of the consumption good) in the period when young (w_{1t}) and in the period when old (w_{2t+1}).

Samuelson also introduced a savings function, as an investment instrument that creates a bridge between the budget constraint in the period when young with the budget constraint in the period when old. This savings function, implies some level of achieved real return called the interest rate, which represents a reward for sacrificing today's consumption for consumption in the next period. By the use of this savings function, which transfers consumption between the two periods, the notion of a life-time budget constraint for the consumer is created.

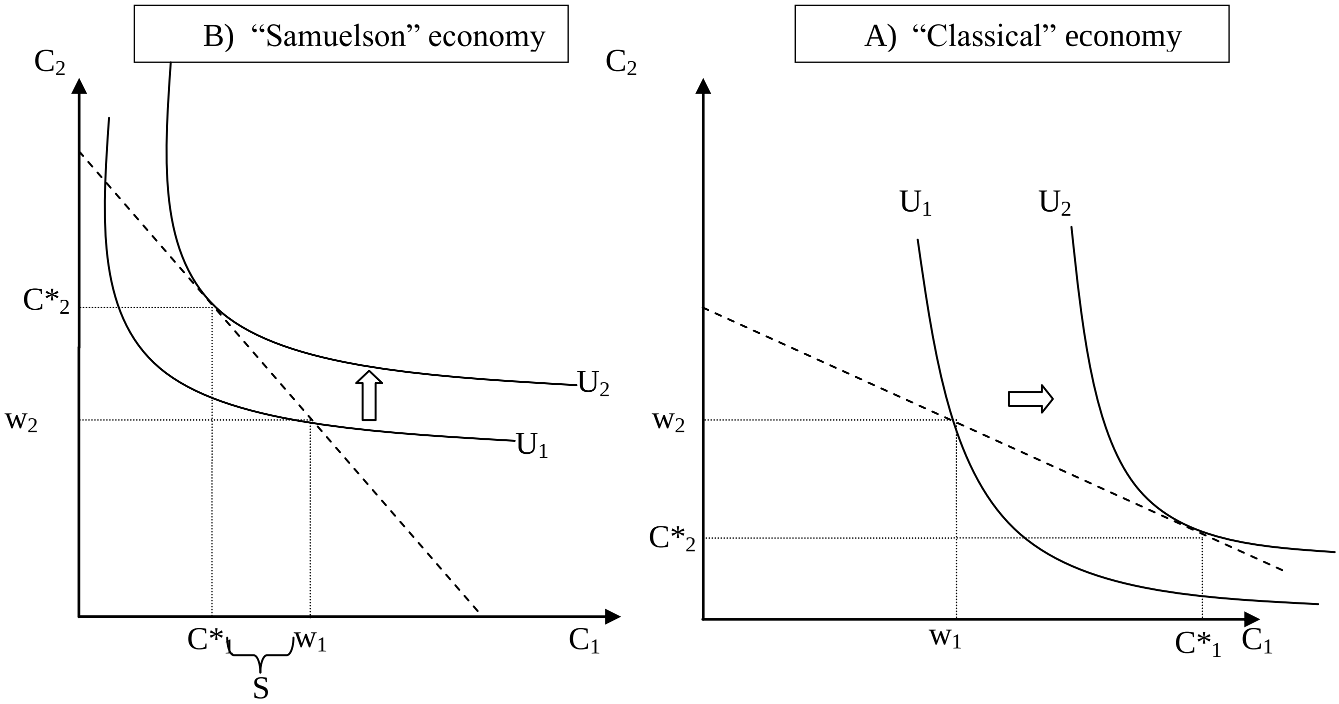
Technical solutions of the consumers' problem assumes plugging in this budget constraint into the utility function through consumption variables, or otherwise forming a Lagrangian with the application of the budget constraint defined by the lifetime budget constraint. Consumers solve their utility maximization problem, knowing the wealth given in the first and second period, and the return on savings (the interest rate, assuming a perfectly competitive savings market). This is a very important point for the use of the OLG model in asset price modelling, where different micro specifics could be added to the model, such as irrational expectation about the expected return from the investment asset used to transfer wealth between the two periods. Authors such as Delong, Schleifer, Summers and Waldman introduce a misperception as a random bias to the expected return, the interest, for that part of the agents described as "noise-traders" (DeLong, Shleifer, Summers, & Waldmann, 1990a).

[5] $U(c,\alpha) = \frac{C^{1-\alpha}}{1-\alpha}, 0 < \alpha < \infty$ - Where alpha measures the curvature of the utility function. With $\alpha = 1$, this function becomes logarithmic

Agents in the simple OLG model defined by Samuelson, solve their utility maximization problem by deciding only about their savings variable. Otherwise, if they cannot decide on their savings, they would have to consume all their wealth given in each period, which complicates their situation especially if they have wealth only in the first period, which without the availability of a transferring instrument, will cause them to starve and die in the period when they lack endowment or when they are old. This means that without the means of saving, or alternatively some social planner transfer, agents will not reach the desired Pareto optimal state of the economy, except in the case of the economy dominated by “extremely impatient” consumers. This corresponds to the issues of pension saving plans and the present debates about their systemic importance of smoothing the consumption of the population through their lives.

In making a distinction between “patient” and “impatient” consumers, we arrive at the important distinction between the two types of financial market economies, determined by the different consumer inter-temporal preferences of their respective representative agents. The first type, where representative agents have preferences characteristic of the “patient” consumers, is called the “Samuelson economy” (Weil, 2008). The other type, where consumers have preferences characteristic of “impatient” consumers is called the “Classical economy”. Figure 1.2.1 depicts the distinctive features of the two types of financial market economies.

Figure 1.2.1.: Distinction between the "Samuelson" and the "Classical" OLG economy



This figure represents the utility of each generation cohort derived from its consumption when young C_1 and consumption when old C_2. The two cases are presented where agents have different utility functions, one describing "patient" (left graph) and the other "impatient" (right graph) agents. The dotted line, describes the budget line under the assumption that agents can exchange their endowment (w1 and w2, for some reward based on their inter-temporal marginal rate of substitution). The slope of the budget line determines their intertemporal rate of substitution.A problem arises especially with the case of the market dominated by the impatient consumers, because it is not possible to transfer endowment from the future period to the current period, while the opposite is possible.

Symbols in the graph are as follows:

U_1, U_2- Levels of the life-time utility function of agents, where $U_2>U_1$
w_1, w_2- wealth endowment of the agents when they are young and old respectively
C^*_1, C^*_2 –Agents' optimal consumption structure when young and old, provided by the budget line whena saving/lending instrument is available (dotted budget line)
S – is the savings demand, determined as a difference by the wealth in the period when young and the optimal level of consumption when young C^*_1

Source: Authors interpretation of the distinction between the "Samuelson" and the "Classical" OLG economy inspired by the lecture notes from Professor Paula Verme Hernandez at Texas A&M University, US

These two graphs depict the two utility optimisation solutions of the two separate types of agent, "patient" and "impatient", in the Samuelson's OLG economy with endowment and without production (Samuelson, 1958). The different indifference curves describe their different inter-temporal consumption preferences and define the two separate types of "Samuelson" and "Classical" economies. Indifference curves are defined in the space consumption when young C_1 and consumption when old C_2, constrained by the given wealth when young and when old, respectively W_1 and W_2. Indifference curves U_1 and U_2 represent lower and higher utility of consumers in the economy respectively. Their utility is determined by their consumption when young defined by C_1 and their consumption when old defined by C_2. Their optimal lifetime consumption and utility depends on their lifetime budget constraint and the available means for saving or for transferring, if preferred, a part of their wealth from the period when agents are young to the period when they are old, in order to reach inter-temporal optimality. Agents who are "impatient" consumers would prefer to consume in excess of their available endowment when young, while agents who are "patient" consumers have a preference for saving and additional consumption in the period when they become old.

We can see that in both types of economies, if there are no means to save or transfer part of the wealth from the period when young to the period when old, consumers will consume their whole given wealth in the period when their wealth is endowed.

If we are in the economy consisting of "patient" consumers, as depicted by the graph on the left side (Figure 1.2.1), we could increase the utility of all agents if we could exchange part of the wealth from the young agents to the old agents, and keep this transferring rule forever so that the current generation of young agents when they become old receives part of the endowment of the next generation of young agents. This will lead to a higher level utility indifference curve or to a Pareto improving state of the economy. Such improvement could be done by an introduced rule of the social planner (social transfer), or by some market traded asset used for saving and for transferring wealth among generations in the decentralized market economy. This is called the "Samuelson case" economy, where there exists a meaningful use of money and some savings asset in the system as a socially agreed contract for wealth transfer among generations. If there is no such kind of asset or social contract introducing an investment vehicle for transferring wealth from young agents to the old, the economy will function in a Pareto suboptimal way.

In the other case, depicted on the right hand side of Figure 1.2.1, representative agents have consumer preferences and corresponding indifference curves characteristic of "impatient" consumers. In this economy, consumers derive most of their utility– and correspondingly want to consume more than their endowment– when young. They would prefer to transfer some of their endowment from the period when old to the period when young to maximize their lifetime utility. In this case, a social planner or a market mechanism cannot improve the utility of agents by transferring part of the endowment from the old to the young, simply because it is not possible to transfer something from the future to the present. This arises because of the asymmetry of the OLG economy, which means that at the moment of the introduction of any social planers solution, the initial old generation has an incentive to consume at minimum its whole endowment. So the social planer's solution and, potentially, the market outcome – in this case called a "Classical type" economy – is Pareto optimal, and there is no meaningful use of money or any savings asset in this case. In other words, there is no way to improve the economy's welfare by introducing a savings asset.

The crucial difference between the two types of economies is that in the case of the "Samuelson" economy young consumers have incentives to save and consume more when old, while in the "Classical" economy young consumers would like to spend more than they have and they don't want to save. Later on, when we introduce production and financial markets, we will see that the "Classical" economy is characterized by low levels of productive capital accumulation and high returns of productive capital, while the "Samuelson" Economy is characterized by low levels of returns of capital and productive capital over accumulation. In turn, this will raise the issue of the dynamic efficiency or inefficiency of the financial market, represented by the balance between the savings demand and the availability of savings vehicles. Such an issue, as later demonstrated through the example of the Croatian financial market, will lead us to the occurrence of rational asset bubbles.

We can imagine a social planner who can, without any cost, transfer the required amount of endowment between the two generations by a rule in order to maximize the utility of the society. Let's define the objective function of the social welfare function W:

$$W(C_1,C_2) = \alpha U(C_1,C_2) + (1-\alpha)U(0,C_2) \tag{1.6}$$

The social planner has to maximize the utility of the initial old generation, described by the second additive part of the social welfare function $(1-\alpha)U(0,C_2)$, together with every following generation cohort living and consuming when young and old as described by the first part of the social welfare equation $\alpha U(C_1,C_2)$ as depicted in the Table 1.2.1. In this social welfare equation, the "α" coefficient represents the weight that the social planner puts on the importance of the utility of all generations other than the initial old, when maximizing the utility of the society. If for example social planner sets $\alpha=1$, this means that the social planner does not care for the utility of the initial old when maximizing the utility function of the society. This decision stresses the previously mentioned importance of the asymmetry in the model arising by the "big bang" moment of inception.

The social planner wants to maximize the utility of all generations born at or after the "big bang" or period 1 of the economy, by setting their consumption when young C_1 and when old C_2. So, at each period, the social planner has a certain total amount of wealth of the agents available for division, which represents the social planner's budget constraint and is manipulated in his social utility maximization process. We will describe this available wealth in per young agent terms, by dividing its amount by the number of young agents, because this standardisation of wealth will help us to better explain the social planner's optimal solution for all present and future cohorts. We should also mention that wealth endowment used at this point, is only a simplifying assumption, where each agent is endowed by certain amount of the numeraire good, because the focus at this point is on the savings incentive. But further, in models with production, we could substitute wealth endowment by labour endowment, which agents could use to participate in the production process and earn wages.

The total wealth defined as the available endowment to the social planer, in per young agent terms, is equal to w_1 which is the amount of endowment of the young in per young agent terms, plus the endowment of the old in per young agent terms in the same period which equals $w_2/(1+n)$, because there are $1/(1+n)$ smaller number of old than young in every period. This total wealth available to the social planner defines its budget constraint and, at the tangency solution, will have to equal the consumption of the young per young agent C_1 plus the consumption of the old per young agent $C_2/\ (1+n)$, again due to the fact that the old are of a smaller number compared to the young by $1/\ (1+n)$. Formally, this maximization problem of the social planner previously described can be represented:

$$\begin{aligned} &\max u(c_1;c_2) \\ &s.t. \left\{ \begin{array}{l} u(c_2) - given \\ \\ c_1 + \dfrac{c_2}{1+n} = w_1 + \dfrac{w_2}{1+n} \end{array} \right\} \end{aligned} \tag{1.7}$$

The social planner problem could be solved by setting some Gama function (Γ) where the budget constraint is defined by the Lagrangian condition:

$$\max \Gamma(c_1, c_2, \lambda) = U(c_1; c_2) + \lambda\left(w_1 + \frac{w_2}{1+n} - c_1 - \frac{c_2}{1+n}\right) \quad (1.8)$$

Taking the first order derivatives with the respect of the control variable of consumption of young and old agents, and the Lagrangian, we get the First Order Conditions (FOCs) for the optimal solution, which gives us the rule by which the social planner will maximize the utility of the society. This rule represents the Marginal Rate of Substitution (MRS) optimality requirement and is formally represented in the following way:

$$\frac{U_1'(C_1; C_2)}{U_2'(C_1; C_2)} = 1 + n \quad (1.9)$$

Here $U'_1(.)$ and $U'_2(.)$ represent the partial marginal utilities of the cohort agents determined by increasing their consumption by one numeraire unit of the consumption good when young and when old respectively. Their ratio, determines the marginal rate of substitution, and should equal the required reward for sacrificing one unit of consumption in period when young by an amount of units of consumption in the period when old.

The intuition behind the social planner's solution is very simple, it says that the utility of the society will be maximized if and only if the marginal sacrifice of utility of the young when transferring one of their consumption units to the old equals the marginal benefit of the consumption of the old multiplied by one plus the rate of the growth of the population. This is solely determined by the fact that in each period there are 1+n more of the young than the old agents. This is similar to saying that the required reward for sacrificing one unit of consumption when young for one unit of consumption when old is equal to the rate of population growth. In other words, the reward for sacrificing consumption, known as the optimal interest rate from the savings market is equal to the rate of population growth rate in the socially optimal state of the economy. The rate of the population growth is the implicit interest rate in the case of the Social planers solution. The distribution of wealth satisfying this condition is known as the "Golden Rule" solution of the economy which is Pareto Optimal, and corresponds to the dynamic efficiency of the economy.

This means that the socially optimal rate at which the social planner should exchange wealth among the young and the old generations of consumers is the rate equal to the growth of the population. This is a very important rule and it is a benchmark for determining dynamic efficiency and Pareto optimality when analysing cases of decentralized market solutions in the economy without the presence of the social planner.

We saw when analysing the two types of economies, that there is no need for savings in the “classical type” economy and so the Pareto optimal solution will lead to young agents spending their whole endowment when young, and old agents spending their whole endowment when old. So solving for market participants in the decentralized model, in the “classical” economy, there is no meaningful need for money as a mean to transfer wealth from one period to another, and there is no inter-temporal trade allocation which is Pareto improving. Conversely, in the “Samuelson” economy, allocation where young and old agents consume their endowment when young and when old without intergenerational transfer is not Pareto optimal and there exists a rational need for a savings instrument such as money balances, productive or non-productive saving vehicles, to provide the market execution of this inter-generational trade by transferring part of the endowment of the young to the period when they become old.

This implies that when consumers have preferences describing them as “patient”, we are in the “Samuelson” economy and a pure market solution without savings instruments can’t lead to a Pareto optimal solution. In this case, we need to introduce some social contract, such as money or some other savings asset to transfer part of the wealth and provide trade among generations that will lead the economy to the socially optimal outcome.

By his work, Samuelson introduced and popularized one great model showing there can be market imperfection in other than the special case market economy (Samuelson, 1958).Other authors later developed Samuelson’s model further. Diamond introduced the government debt function and used production in the model, both of which introduced savings vehicles for lending to government and/or entrepreneurs and, thereby, allowed solutions of the dynamic inefficiency problem previously discussed (Diamond, 1965). This way the OLG model was brought closer to the real functioning of the financial market economy and further increased its analytic popularity among economists. This gave the opportunity to analyse optimality and market outcomes when consumer agents are endowed with a unit of labour in the first period when they are young, or a labour endowment which they use to engage in employment, earn wages and optimize their lifetime budget constraint by saving in different types of saving vehicles. So besides the market for goods, the modelled economy was enriched by a market for factors of production such as labour and capital and a market for government debt, a complication bringing the model closer to the functioning of the real world economy.

In Diamond's model economy, young consumers are deciding on how much of their earned income to save to be able to provide spending in the second period when they don't have the ability to work and earn wages. Besides holding real money balances, in this extension to the OLG model, agents can save by lending to entrepreneurs. So they can transform part of their wage into capital, or invest, which in the second period will provide them with an amount of dividend. This capital is managed by entrepreneurs who, besides capital, also demand labour from the young generation at the market wage rate equalling its marginal rate of production.

Finally, from the perspective of our interest in asset bubbles, the biggest progress of the OLG model in terms of enriching its micro-structure by introducing investment alternatives was the Tirole (1985) model.

It was just a few years before the US stock market crash in 1987 that Tirole published his 1985 *Econometrica* paper on rational asset bubbles. He made an innovative step further from the Diamond OLG model with production and productive saving (investing in capital) by introducing so called non-productive saving, which made his model a seminal work in analysing asset bubbles. In his model investors have an option to pass forward their wealth by the use of capital investments, or so called "productive saving", and/or by investing in some fixed supply of a non-productive fundamentally worthless asset, called "the bubbly asset". At this point, when introducing the idea of the "bubbly asset", it is important to mention Tirole's definition of the value of risky assets from his earlier paper on the non-existence of asset bubbles in a finite trades model (Tirole, 1982). Tirole thought that the market value of every risky asset consists of two parts: first is its "fundamental value"; and the second is its "bubbly value". In the extreme, an asset may have zero fundamental value and all of its value might consist of the "bubbly" value. It is the "pure bubbly asset" which is used in Tirole's OLG model (Tirole, 1985). But, there can be a whole spectrum of assets between the fully fundamental or fully bubbly type of asset. Tirole used only the two extremes in his Rational Asset Bubble's OLG micro-structure to analyse the pure effect on market dynamics of the overall value of the "bubbly" value of assets in the economy which, as we will see, gives very interesting insights.

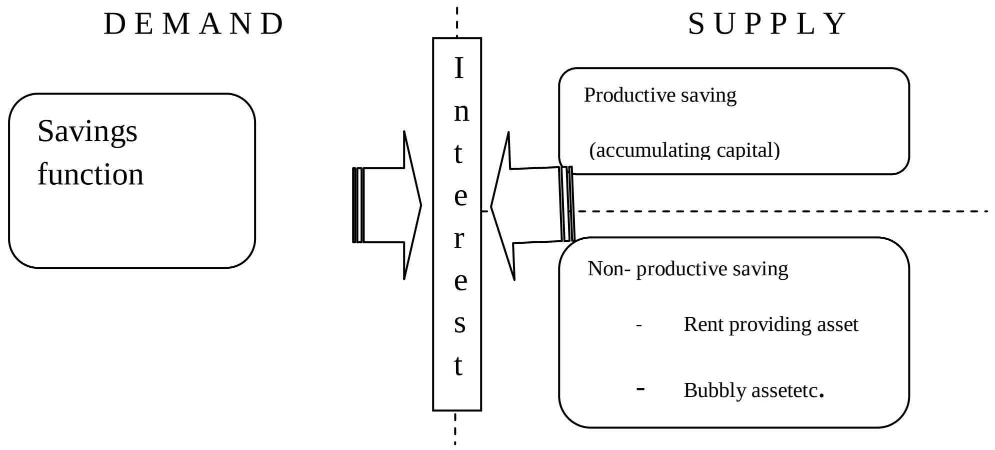
At the essence of the systemic solution in Tirole's OLG model is the "no-arbitrage condition" for the "interior solution", which connects the two - or later, if extended, more than two - assets used for saving, assuming perfect foresight of investors who are as if risk neutral. We note that risk-neutrality was an implied feature of Samuelson's theory on "martingales" explaining asset price dynamics, which was widely accepted as a great contribution to the efficient capital markets theory (Samuelson, 1965).

In the later work of Tirole and Farhi, the authors extend the model using a three stage OLG model and introducing several saving assets (Farhi & Tirole, 2011). In this and many extensions, risk neutrality and the non-arbitrage condition plays the same significant role.

At this point it is important to understand the intuition behind the "non-arbitrage" condition and the "interior solution". The "Non-arbitrage" condition, as depicted below, means that with the perfect foresight assumption within risk neutral agents, we need to have the return on savings equal among all assets used for this purpose. Otherwise, there will be an arbitrage opportunity, which will be instantly cleared on the assumed perfectly competitive market.

The other understanding of the "internal solution", which follows the "non-arbitrage" condition, is in fact a solution where all introduced assets in the model are used for savings. Otherwise, in the "corner solution" we could have a situation where only productive capital is used but the bubbly asset is not. We will see, in solving the dynamics of the model, that this might happen if and only if the pure bubbly asset is not initially introduced at any point of time during the dynamics of the market economy, which is hard to imagine having significant empirical evidence showing the existence of asset bubbles.

Figure1.2.2.: Structure of the savings market within the OLG modelled economy



Source: Author's own interpretation of the financial equilibrium in Tirole's OLG economy

Whenever we introduce a market transfer of wealth through periods, by the use of real money balances, per capita capital or a per capita non-productive investment asset such as the pure bubbly asset, we have as a result one or more not necessarily linear first order difference equations governing the market economy. They describe the dynamics of the evolution of the per capita savings instruments and the dynamics of whole market economy through time. Each of those difference equations is called an Equilibrium Law of Motion (ELM) and is formally specified for each separate investment instrument. So if there are n-multiple investment instruments, the market economy simultaneously evolves in n-different dimensions. In its evolution through time, the decentralized market mechanism is equilibrated at all times(satisfies the market clearing condition at any time), and the interaction of the rationality of agents (in terms of continuous optimisation of their own objective functions) and the market clearing determines the dynamics of the economy at any moment in time.

At this point, we must stress that Tirole's model is deterministic, because there is no stochastic factor introducing uncertainty. Further, this assumption could be eased by introducing stochasticity into the inter-temporal marginal rate of substitution (MRS) among agents, which affects interest rates and asset prices, or we could also introduce stochasticity in the model through some other variable such as monetary policy etc. But for the moment, we will keep the deterministic character of the model.

There are states of the dynamic variables, the variables describing the evolution of the market economy, at which dynamic equations mutually became stable. Those points are called the Steady State (SS) equilibrium points and, most of the time, the dynamic system of equations (ELMs) will lead the economy to some of its SS equilibrium points, which are called “sink points”, but occasionally the system will tend to go away from these points, which are then called “source points”.

So to analyse this system and its dynamics, we need to first find the locus of steady state points of each dynamic equation describing each ELM of the savings instrument governing the market economy, and then analyse the behaviour of the economy in the close neighbourhood of the steady state of all dynamic ELM equations simultaneously, which is called Dynamic Stability Analysis. Formally, this is done by inspecting the gradient matrix of partial derivative values (or difference values, since time is discrete in the model) in the close neighbourhood to the global steady state points. If the value of the partial derivative is $0<dx<1$, we have a stable monotonic dynamic to the steady state. If $-1<dx<0$, we have a fluctuating dynamic but also leading to the steady state. In other cases, we have a dynamic exploding away from the steady state, which then defines a “source”.

When we have more than one dynamic equation describing the system dynamics, we could use a phase diagram, inspecting the dynamics of the system in the plane around the steady state points or, alternatively, by applying matrix algebra and inspecting the first order partial derivatives of the system of linear or linear approximation difference equations at the SS points. Finding Eigen values and vectors and their determinant and trace values, helps to determine the overall system dynamics at those points.

Back to Tirole's work (Tirole, 1985), the revolutionary point introduced in his paper was not that he used more means of saving, which complicates the dynamics of the system, but that he introduced non-productive saving, which can take the form of a non-productive rent-providing investment or an intrinsically worthless bubbly asset. When solving the model with more than one asset available for satisfying agents’ savings demand, such as money and capital or capital and a bubbly asset as in the case of Tirole, rational investors tend to solve their consumer and savings problem by simultaneously holding both assets (the interior solution) only when the non-arbitrage condition is satisfied. This is actually the solution of risk-neutral agents to choosing among saving alternatives the one which gives them higher benefit in the future, under the perfect return foresight assumption. This means that at the interior solution, both productive and non-productive "bubbly asset" will have the same expected rate of return.

At this point, an important feature to stress is the diminishing rate of return from investments in productive capital, which plays a significant role by creating a systemic incentive for the introduction of the bubbly instrument. Without extensive countervailing technological improvements, the diminishing rate of return to productive capital investments will create a lack of productive savings instruments providing satisfactory real returns from savings, which is a direct consequence of the over-capital-accumulation. This problem is identified as a dynamic inefficiency of the financial market (Abel et al., 1989), which creates a systemic incentive to crowd out this over-accumulation into investments in other instruments such as the bubbly instruments. This intuition is well recognized in Tirole's model (Tirole, 1985). This incentive of unsatisfactory return from productive capital investments, originating from the lack of technological progress and innovation, could create financial innovation leading to increased use of bubbly instruments.

Solving the model, Tirole's two first order difference equations for the capital per young agent (k_t) and the bubbly asset per young agent (b_t) were the following:

$$b_{t+1} = \frac{1+f'(k_{t+1})}{1+n} b_t$$

$$k_{t+1} = \frac{s(f(k_t) - k_t f'(k_t), f'(k_{t+1})) - b_t}{1+n}$$

"Equation 16'' from (Tirole, 1985)(1.10)

The first equation (Equation 16'', Tirole, 1985) derived by the non-arbitrage condition describes the ELM of the bubbly asset per young agent b_t. Here, the first derivative of the production function per young agent with respect to capital is described by $f'(k_{t+1})$ and represents the marginal product of capital at the level of its accumulation in per young agent terms at time t+1, while n stands for the rate of population growth. Consequently, satisfying the non-arbitrage condition, the bubbly asset will grow in absolute value in per young agent terms if and only if the productivity of productive capital investments is greater than the natural rate of population growth.

The second dynamic first order difference equation (Equation 16'' at (Tirole, 1985)) defines the ELM for per young agent productive capital accumulation, which as seen by the right hand side of the equation, is positively affected by the implicit savings function solution, and negatively affected by the crowding out effect of the use of the bubbly asset b_t, satisfying the non-arbitrage condition.

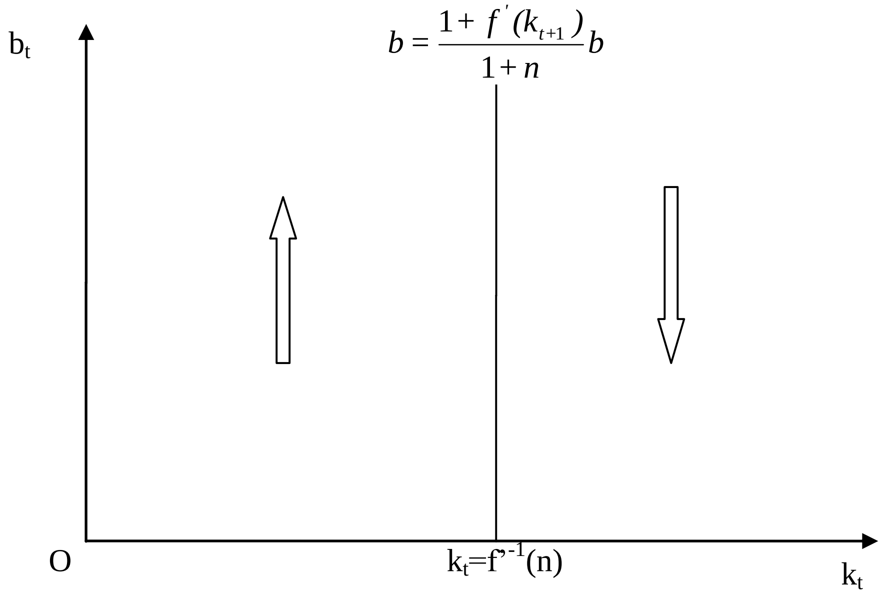
The savings function is assumed to be "normal", which implies that the savings increase as the endowment when young increases and it also increase with the rise of the reward for saving, the interest rate. The savings function being "normal" also implies that savings decrease with the increase of endowment in the period when agents are old.

Depicting the loci of the steady state points for the bubbly asset and for the productive capital following those two dynamic equations leads us to the phase diagram of the market economy. Having two savings instruments, productive and non-productive saving assets, which define two ELM functions governing the market economy, Tirole's phase diagram consists of two parts each describing the dynamics of the specific asset depicting the locus of its stability points (Tirole, 1985):

a) **Bubbly asset ELM** $b_{t+1} = \dfrac{1+f'(k_{t+1})}{1+n} b_t$

For the bubbly asset steady state solution we have one trivial solution, when there is no bubbly asset used for saving $b_{t+1} = b_t = 0$, and indefinitely many non-trivial solutions satisfying the condition $1+f'(k_{t+1}) = 1+n$, when the bubbly asset is used. This means that the locus of stability points for the bubbly asset is a vertical line starting at the point of capital accumulation at which the marginal rate of return of capital equals the rate of growth of the population (Figure 1.2.3).

Figure 1.2.3.: Dynamics of the bubbly asset Equilibrium Law of Motion and its locus of stability points



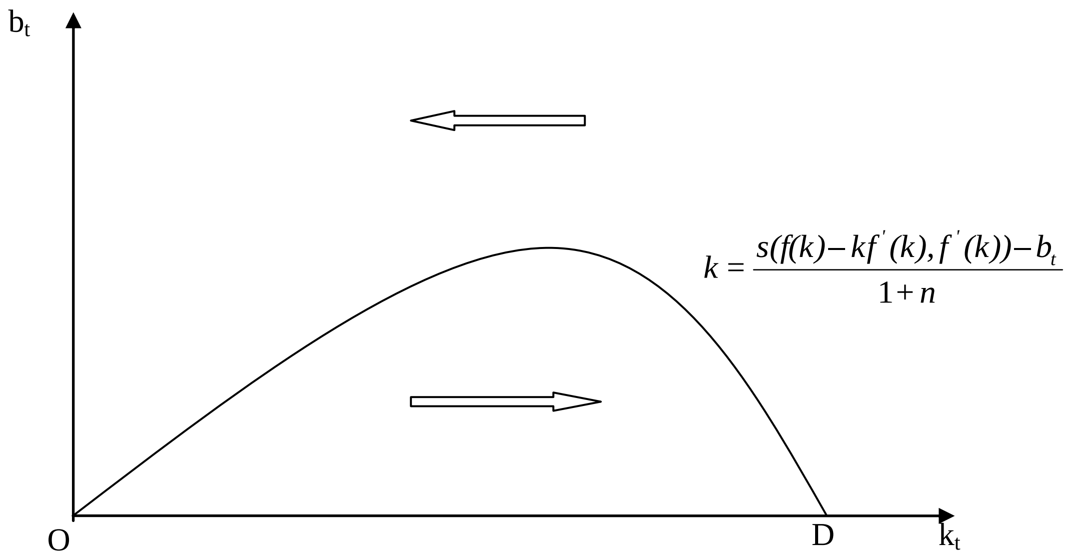
In this figure:
*In the stability loci, "b" is constant which solves the bubbly ELM to $k_t = f'^{-1}(n)$
K_t- represents the level of productive capital per young agent at time t in the economy
b_t-represents the level of bubbly asset per young agent at time t in the economy
Source: Authors own interpretation of Tirole's rational asset bubbles model phase diagram

Inspecting the dynamics in the near neighbourhood of the locus of stability points for the bubbly asset (Figure 1.2.3), depicted by the vertical line where the amount of the bubbly asset per young agent is stable $b_t=b_{t+1}=b$, we notice that if we are on the left side of the steady state locus, due to the fact that in this region we have less productive capital accumulation and return on capital is higher than the rate of growth of the population $f'(k_{t+1}) > n$, the bubbly asset ELM tends to increase the amount of bubbly asset per young agent through the non-arbitrage condition. The opposite happens on the right side of the locus and in this region the ELM of the bubbly assets drives the economy to a lower level of the bubbly assets value per young agent. The two arrows, on the left and on the right of the steady state locus line for the bubbly asset, describe the vectors of systemic forces that determine the dynamics of bubbly asset accumulation.

b) Productive capital ELM $k_{t+1} = \frac{s(f(k_t) - k_t f'(k_t), f'(k_{t+1})) - b_t}{1+n}$

Analysing the ELM for per-young agent productive capital accumulation (k_t), we also find one trivial and a continuum of non-trivial steady state points called the locus of stability points for productive capital per young agent. The trivial steady state (SS) is at the point where $k_{t+1} = k_t = 0$ and is located at the origin of the diagram. At this point no production and capital accumulation are observed. The other set of SS points follow a concave locus line of steady states for the productive capital in per young agent terms as described below (Figure 1.2.4). At each point along this locus of steady state points $k_{t+1}=k_t=k$, which means capital per young agent is stable. The shape of the locus is determined by the diminishing returns of productive capital, and describes how aggressive, due to the non-arbitrage condition, capital accumulation is at points of high rate of returns near the origin point, driving up the value of the bubbly asset accumulation toward higher levels in per young agent terms. As more productive capital is accumulated, the effect of the diminishing rate of return to productive capital causes decreased interest for saving, which turns the curvature of the steady state locus of the productive capital accumulation in per young agent back to the bubble-less equilibrium point D.

Figure 1.2.4: Dynamics of the productive asset Equilibrium Law of Motion and its locus of stability points

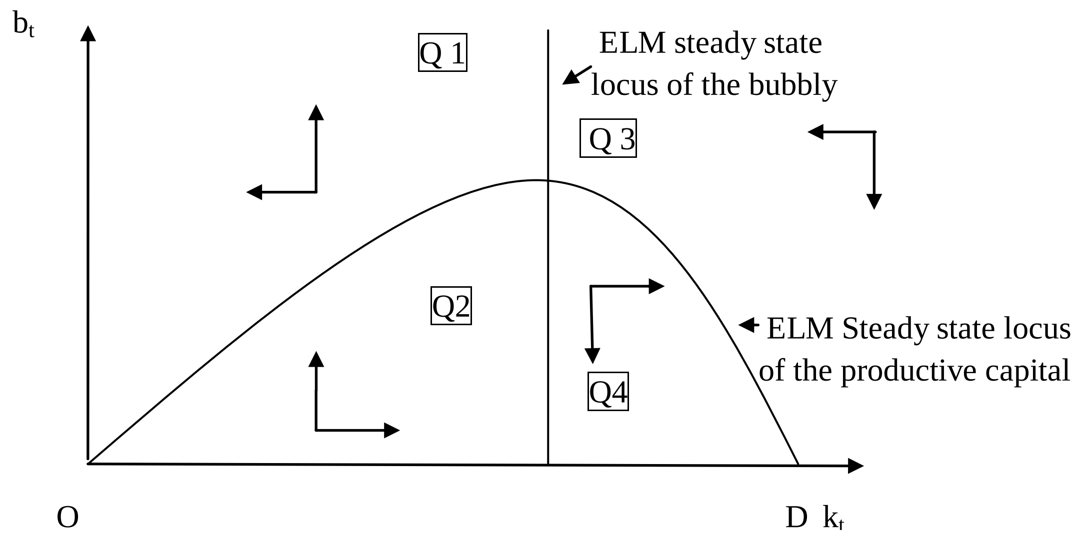


In this figure:
K_t- represents the level of productive capital per young agent at time t in the economy
B_t-represents the level of bubbly asset per young agent at time t in the economy
Source: Author's own interpretation of Tirole's rational asset bubbles model phase diagram

Again, to analyse the dynamics of this nonlinear first order difference equation in the near neighbourhood to the steady state locus for per-young agent productive capital accumulation, we need to determine the vectors describing the dynamics when we are below and above the locus line of stability points. When we are above the locus, the crowding out effect from the growth of the bubbly instrument dominates the savings market, crowds out productive savings and leads to decrease in productive capital accumulation in per young agent terms. The opposite happens when we are below the capital accumulation steady state locus line.

Finally, we can represent the two dynamic ELM equations together to represent and analyse the complete dynamic of the system. The phase diagram derived from the separate diagrams for the ELM of the bubbly asset (Figure 1.2.3) and for the productive capital (Figure 1.2.4) is characterized by the vectors of forces directing the system dynamics at any time t in its two dimensional space according to the level of the bubbly asset and of productive capital accumulated per young agent. This joint interaction of the two ELMs is depicted in Figure 1.2.5.

Figure 1.2.5: Vectors of forces driving the accumulation of productive asset and bubbly asset in the economy



Source: Author's own interpretation of Tirole's rational asset bubbles model phase diagram

The two ELMs mutually govern the dynamics of the economy toward its general equilibrium steady state point, if such a point exists.

If the financial market economy is characterized by higher returns of productive capital compared to the rate of growth of the wealth that has to be invested (equal to the rate of growth of the population), then – according to the non-arbitrage condition – given that the bubbly asset is present it rises in value. This region is represented by Quadrant 2 in Figure 1.2.5., where the vector of forces shows b_t and k_t rising together up to the golden-rule steady state here the two loci intersect.

If, on the other hand, the financial market economy finds itself in Quadrant 4 as depicted in Figure 1.2.5, this is the situation in which capital accumulation has proceeded beyond the point at which diminishing returns cause the rate of return on productive capital to be lower than the growth of the capital that has to be invested (population growth). In this case, the dynamics of the financial market lead to suboptimal productive capital over-accumulation but bubble-less steady state. In this state, productive capital over-accumulates to the point where the rate of return is zero, in which case – through the non-arbitrage condition – the value of the bubbly asset is also zero.

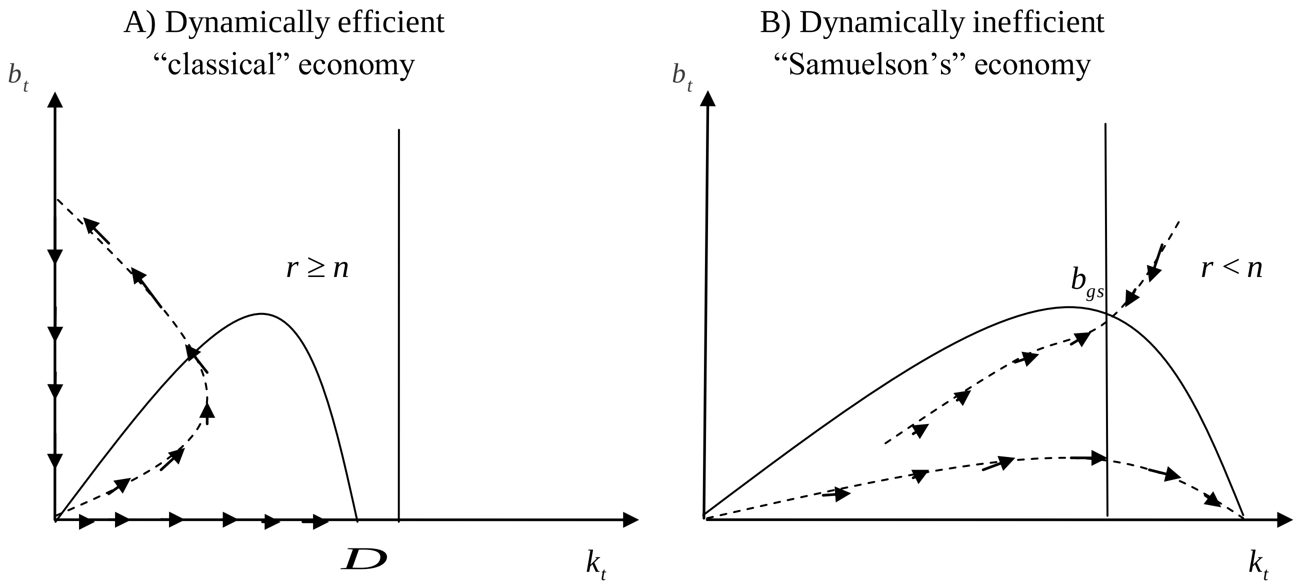
The other two quadrants, Quadrant 1 and especially Quadrant 3 are the area which is characterized by over-accumulation of the bubbly asset in the financial market. In this area – above the productive capital ELM – the bubbly asset is already highly present in the financial market. If the financial market economy finds itself in the Quadrant 1, the bubbly asset becomes a massive frenzy speculative investment episode. It completely takes over the financial market, increasingly crowding out productive capital accumulation, ending in a quick burst and bust cycle. Finally, in Quadrant 3, the financial market is characterised by low interest rates (because of productive capital over-accumulation) and bubbly asset over-accumulation, hence will be driven toward lower levels of both productive capital accumulation (higher interest rates) and lower levels of the bubbly asset.

But there is one crucial point that needs to be stressed, which as a conclusion was brought forward in the OLG model with endowment (Samuelson, 1958) and in the OLG model with production and productive savings(Diamond, 1965).

In the Samuelson's solution, the market leads to a solution which is dynamically efficient if and only if interest rates, without the presence of the bubbly asset, are higher than the rate of population growth. Later the population growth could be understood by the rate of growth of the economy. This is because the growth of the economy, described in Samuelson's case simply as the growth of the population, determines the growth of the overall wealth and imposes a continuous incentive for inter-temporal wealth transfer within the budget constraint of agents. This is an incredibly important savings demand driving force, which initiates introduction of different savings and investment vehicles, as well as other innovative or bubbly vehicles, when productive saving vehicles are not productive enough to satisfy the need for transferring wealth from one period to another. The financial innovation and the introduction of bubbly assets come naturally when the pace of growth of the wealth per young agent cannot be followed by the productivity of capital, which is diminishing in the absence of technological innovations. So this balance makes the difference between the dynamically efficient and dynamically inefficient market economies where bubbles become rational.

In Diamond's model (Diamond 1965), there is an efficient market solution, similar to Samuelson's, if the interest rates are higher than or equal to the population growth rate, which happens when consumers are impatient, as in the "classical" economy. Otherwise, the market solution with production and capital is not efficient when equilibrium interest rates are lower than the population growth leading to capital over-accumulation, and a rational need for introduction of the bubbly asset. So we actually have the following two alternative cases for the dynamic efficiency of the economy with productive and non-productive savings (Figure 1.2.6):

Figure1.2.6: Dynamic analysis of the "Classical" and the "Samuelson's" OLG market economy

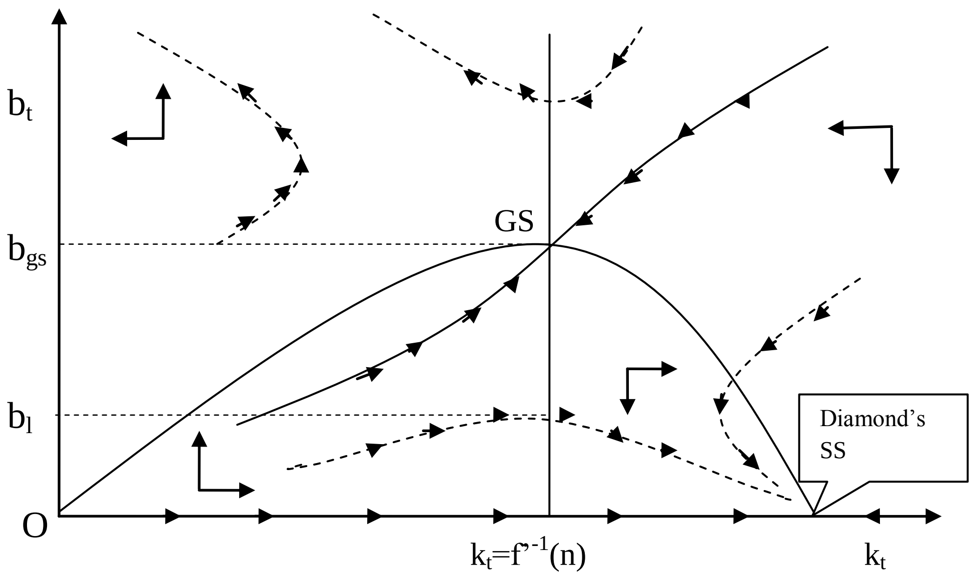


Source: Author's own interpretation of Tirole's rational asset bubbles model phase diagram

In the case when we are in the "classical" economy (Left hand side of the Figure 1.2.6), where the market interest rates (r) of the OLG model with productive capital accumulation and without bubbly asset are higher than the growth rate of population (n), because the economy is dominated by agents not willing to save (impatient agents), then there exists a bubble-less rational equilibrium depicted by the point D. There could be speculative stages along the path to this bubble-less equilibrium, when certain bubbly instruments are introduced, but they lead to short episodes of aggressive speculative inflation and collapse cycles of the bubbly instrument. Ultimately, the economy follows the dynamic of capital accumulation to the Diamond steady state depicted by D (Diamond, 1965), which is stable and dynamically efficient in the context of the "classical economy". Financial markets without bubbly instruments, as in this case, lead to the Social planer's Pareto optimal outcome.

If the other case dominates, we are in the “Samuelson” economy, dominated by agents who are patient consumers, want to smooth their life-time consumption and are willing to save for consuming more when they get old. In this case, there exists an efficient asymptotic bubbly path that leads the economy to the “Golden Rule” steady state equilibrium (the point “b_{gs}” in Figure 1.2.6.B), as defined by Phelps (Phelps, 1961).At this point, in Tirole’s deterministic OLG market economy, the value of the bubbly asset per young agent (b_t) is constant and its total value in the economy in absolute terms continues to linearly grow at the rate of growth of the population (in extensions this could be the rate of growth of the economy, or the rate of growth of wealth per young agent). Capital accumulation per young agent is constant as well at this point, and gives a return equal to the growth rate of the population. So at this state of the economy, bubbles as defined by Tirole, are a rational outcome leading the economy to its dynamically optimal Golden Rule steady state by preventing useless productive capital over-accumulation perceived as such by the agents (Tirole, 1985). This means that otherwise, if the system starts at a point of bubbly asset inception below the asymptotic saddle path leading to the Golden Rule steady state of the bubbly asset as depicted by level (b_{gs}) in Figure 1.2.6.B, there will be a lower level of asset bubble reached, level (b_l), below the golden rule steady state, and from this point the bubbly asset will subsequently decrease leading to the Diamond bubble-less suboptimal steady state. Diamond’s SS is inefficient in this economic context, because it represents a steady state characterized by an over-accumulated level of capital per young agent and leaves a lower level of consumption to be divided among young and old agents of the economy through its infinite existence. So rational asset bubbles, crowd out some of the productive savings and investments, decrease the level of capital per young agent, increase productivity and provide a higher level of consumption to be divided among agents. This corresponds to the social planner Pareto optimal solution and is favoured compared to the market solution, which leads to the Diamond SS. Asset bubbles in Tirole's economy play a similar role to government debt in the Diamond economy, crowding out part of the capital, thereby leading the inefficient “Samuelson” economy to its Pareto optimal steady state represented by the “Golden Rule”. In the following graphical representation, we see the mutual interaction of the two dynamic equations in the dynamically inefficient “Samuelson” market economy (Figure 1.2.7).

Figure1.2.7: Heuristic phase diagram describing "Samuelsson's" OLG market economy



Source: Author's own interpretation of Tirole(1985) -rational asset bubbles model phase diagram

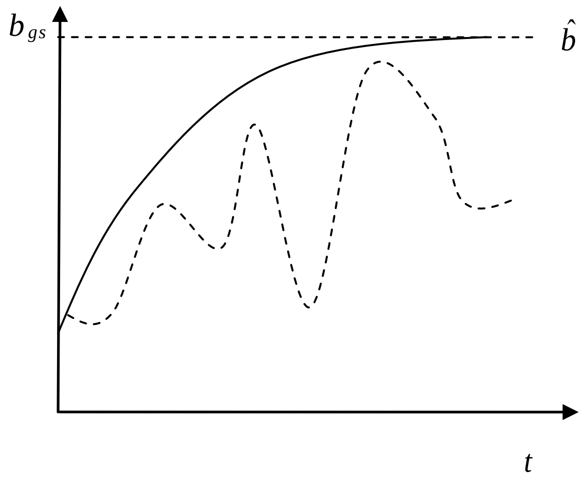
This shows that in an economic environment with both productive capital and a bubbly asset, where agents rationally interact on the markets, asset bubbles can be a rational outcome. The zero point (Point O in Figure 1.2.7), with no capital and no bubbly asset as a trivial solution is non-stable, which means it is a source. The economy will eventually develop away from this initial point. If worthless assets, defined as "bubbly" are introduced near the asymptotically-bubbly saddle path, the economy moves to the global stability point depicted at GS on the graph. This point is optimal for the economy, and it represents a "sink point", which means that the dynamics of the economy is attracted to it by its near environment or by the asymptotic path leading to it. When the economy reaches this point, ceteris paribus, it achieves its stable bubbly equilibrium.

If on the other hand, the economy accepts the bubbly instrument below the asymptotically-bubbly path, the bubbly asset reach some maximum level (heuristically described by the point b_l on the Figure 1.2.7), but then disappears attracted by the second "sink" point in the economy, the inefficient Diamond steady state. At this point, there is no bubbly asset in the economy, interest rates are very low and there is over-accumulation of productive capital which brings lifetime consumption and the utility of agents to a sub-optimally lower level.

The third possible dynamics are achieved if, euphorically, the bubbly asset is introduced way above the asymptotically-bubbly saddle path. From this point, the bubble rapidly explodes, crowds out all of the productive capital and subsequently rapidly collapses itself to the origin (see Figure 1.2.7.A, graph on the left hand side). This cycle represents an inefficient speculation, which is a non-sustainable but possible outcome for the market economy. The logic of the existence of this phenomenon is described by many prominent economists explaining bubbly cycles on the stock market (Shiller R. , 2000).

Tirole claimed that there can be infinitely many bubbly assets introduced in the inefficient “bubbly” market economy (Tirole, 1985).He claimed that there can be more than one bubbly asset, or even fundamentally worthy rent providing assets that belong to the category of non-productive assets. He also claimed that assets could be a mix of fundamental and bubbly assets (Tirole, 1982), which explains the bubble present on the stock market. In such a circumstance, only the total amount of all the bubbly and other non-productive assets could be observed growing to the SS depicted at the point GS. But the individual value dynamics of different bubbly assets, among and within the sum of the bubbly assets can wildly differ and fluctuate. Among the different bubbly assets, there is no possibility to determine which one will inflate and which will deflate and when; the only thing that can be expected to be observed is the dynamics of the sum of the bubbly assets toward their joint steady state (Figure 1.2.8).Tirole compares this process of fluctuation of the value among different bubbly assets with the occurrence of sunspots (Tirole, 1985). The only thing that can be predicted is the overall sum of the value of the bubbly instruments. Similar to the logic of Tobin's portfolio (Tobin, 1969), or “the overshooting mechanism” of interest rates introduced by Dorbusch (Dorbusch, 1976), there could be a large volatility between different groups of bubbly assets. Some groups will have negative correlation but the total of bubbly assets will be led to the steady state of the bubbly asset described by the point GS, which can be affected by some changes in the market microstructure. This outcome together with the result that the sum of the bubbly assets will tend to reach its steady state level, leads to the conclusion that in general there will be a negative correlation among groups of bubbly assets or, when one falls, the sum of the others will have to rise offsetting the fall in value of the first one. This is similar to the empirically evidenced correlation between equity and bonds, both having characteristics of bubbly instruments, but also giving insights into the equity premium puzzle (Mehra & Prescott, 1985).

Figure 1.2.8.: The interaction within different "bubbly assets" as presented by (Tirole, 1985)



This Figure represents the evolution of the value of total of bubbly assets toward their steady state equilibrium level b_{gs}, represented by the solid line. The dashed line heuristically describes the result of the fluctuation of value of one bubbly asset among different types of bubbly assets in the group.

Source: Author's own interpretation of (Tirole, 1985) rational asset bubbles model phase diagram

Discussing the preconditions that trigger one intrinsically worthless asset to be used as a bubbly asset, Tirole specifies several factors. Those factors are durability, scarcity and common beliefs. Some of those factors, especially the common beliefs, will be extensively treated by the other schools on asset bubbles described further in this chapter. One asset needs to have a critical amount of those qualities, to arise and serve as a rational bubbly asset in the economy.

Tirole indeed presented very compelling insights. However, the relationship between interest rates, growth of the economy and asset bubbles proved to be more complicated in reality than in Tirole's model. This complexity was seen during the last decade of the twentieth century. At a time of strong growth of the US economy during the 1990s, asset bubbles on the equity market were developing followed by high productive investment and high interest rates as opposed to the claims present in Tirole's result. It seemed that bubbles are possible even in the other type of economy, even in the classical dynamically efficient economy. But this could also be a result of insufficient understanding of the dynamic inefficiency criteria imposed within the OLG model. In our empirical strategy, we consider whether it might be not the growth of the population, or growth of the economy which best define the common dynamic efficiency criterion but, instead, the growth of the investable wealth per young agent vis-a-vis the real return of the productive assets which determines the dynamic inefficiency of the economy and which brings the need for asset bubbles onto the stage.

Farhi, Caballero and Hammour, three scholars from the new generation of leading economists, compared Tirole's results with the evidence of the equity market bubble in the late 90sin the US economy (Caballero, Farhi, & Hammour, 2006). Motivated by the findings of authors who, using the comparison between the growth rate of the economy and the return on equity investments (calculated by comparing the sum of dividend and stock repurchase to the net market value of equity), has found that the US and OECD economies were dynamically efficient (Abel et al., 1989), Farhi, Caballero and Hammour, searched for more explanation about the occurrence of asset bubbles in the US economy. In their model, based on the foundations set by Tirole, within the resulting savings function they introduced an extension, named “the growth-funding feedback”, which relaxes Diamond's SS result for the dynamically efficient economy.

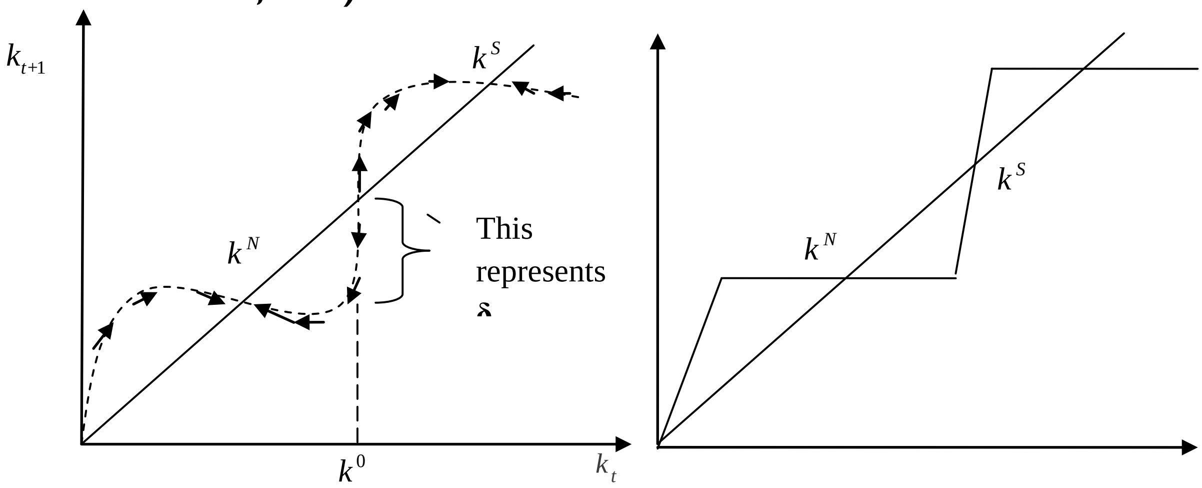
Compared to the simple non-linear savings function in Diamond's model, this extended “funding function” is not only determined by the wage of the young agents and by the interest rate or marginal productivity of capital, but also by other factors positively correlated to the level of capital accumulation in the system. This is atypical to the standard rational asset bubbles model result of Tirole, and allows for multiple equilibrium and occurrence of asset bubbles even in the classical dynamically efficient economy. Farhi, Caballero and Hammour accomplished this by introducing a “step” into the “funding function”, as described in the following equation from their paper (Caballero, Farhi, & Hammour, 2006):

$$s(k_t, r_t) = \begin{Bmatrix} s_0 + s_k k_t + s_r r_t & k < k^0 \\ s_0 + \delta + s_k k_t + s_r r_t & k \geq k^0 \end{Bmatrix} \quad (1.11)$$

In their step-funding function, when capital reaches some threshold value defined by some k^0 critical capital accumulation level, then the savings function initiates a “growth-funding feedback” induced by the effect of some exogenous factor δ. This initiates a jump of the market economy to the new saddle path leading to a higher capital accumulation steady state called the speculative state.

This allows the system to have multiple steady state equilibriums and to be able to jump from one to another, provoked by some exogenous factor, such as a technological development, fiscal policy etc. The first steady state, the so called “normal” steady state, which corresponds to Diamond’s result; and the other “speculative” steady state, which relaxes Diamond’s result and can be achieved as a result of this growth-funding feedback. The resulting steady states are denoted k^N and k^S, respectively in Figure 1.2.9.

Figure 1.2.9.: Representation of the growth-funding feedback (Caballero, Farhi, & Hammour, 2006)



Source: (Caballero, Farhi, & Hammour, 2006)

On the left hand side in Figure1.2.9, there is a graph of the multiple equilibrium capital accumulation of the economy. There are two stable SS (sink) equilibriums for the capital k^N and k^S. As capital reaches the critical level of capital accumulation described by the point k^0 , funding initiates the jump above the 45 degree line and drives the economy to a new ELM for capital accumulation leading the economy to the new sink called “speculative” SS represented by k^S. The dynamics of the new equilibrium SS are characterized by increased investments, a lower interest rate and an environment “friendly” for the development of rational asset bubbles. Farhi, Caballero and Hammour used a linear representation of the funding function for simplicity, so they used the graph represented on the right hand side of Figure 1.2.9 as a good approximation for the ELM of capital accumulation.

This so called “speculative” SS equilibrium is characterized by over investment into productive assets and low interest rates as depicted by the point k^s on the graph. Farhi, Caballero and Hammour claimed that this point corresponds to the empirical data on the US economy in the 90s. Higher capital accumulation SS, provides more capital funding itself by triggering speculative productive capital accumulation, which more than offsets the decrease of funding due to the lower interest rates.

Another important phenomenon described in Farhi, Caballero and Hammour's work was the stock market boom-bust cycle, which accompanies this “speculative-growth episode”. They introduced capital gain, in the fashion of Tobin, denoting it by “q”, which is the driving ingredient in this jump to the new speculative SS equilibrium. This way, they introduced Tirole’s description of asset prices being a sum of the intrinsic value of the underlying productive capital and its bubbly value (Tirole, 1982). They addressed the problem of having the system at which higher capital accumulation, which presumes lower marginal productivity of capital, drives higher investment interest in productive capital after the speculative jump of the economy, which was not an outcome of Tirole’s model (Tirole, 1985)but which was observed in the US economy in the 1990s. To solve this problem, they introduced “adjustment costs” of capital, being costs of moving away from the level of investment needed to maintain the effective capital stock, which create a wedge between the price of the installed capital and the marginal product of capital. This way, they assumed capital appreciation, which allowed for continual productive capital accumulation even though the marginal product of capital was decreasing. This capital appreciation was the actual capital gain. In this way, Farhi, Caballero and Hammour introduced the potential bubble within the market pricing of the productive capital (Caballero, Farhi, & Hammour, 2006), in line with the Tirole’s logic (Tirole, 1982).

Finally, the authors derive the two equations governing the system dynamics. Different from Tirole (1985), where the dynamics of the pure bubbly instrument (b_t) and the accumulation of productive capital (k_t) were the two dynamic equations determining the dynamics of the economy, here the authors use productive capital (k_t) and capital gain described by (q_t) as the two defining dynamic equations. They describe the bubble in a more artificial way by using capital gain-i.e. as a bubble within the pricing of a fundamentally worthy instrument such as equity and not necessarily as a pure bubbly asset such as in Tirole’s model.

Farhi, Caballero and Hammour, define certain triggers which can jump-start this speculative growth path of the economy. Among the triggers are Technological Advance, Fiscal Surpluses, External Capital Flows and Financial Deregulation.

Finally, focusing on Emerging Markets, Caballero and Krishnamurthy (2005) develop a stochastic OLG model with rational asset bubbles focusing on Emerging Markets. They show that although the return on productive assets can be much higher in Emerging Markets, their representative companies fail to capitalize their expected future returns due to poor equity ownership rights and protection. Such an issue increases the problem of the dynamic inefficiency, because of the lower effective returns on capital as stressed by Abel (Abel et al., 1989). Caballero and Krishnamurthy focus on an open market economy, where agents can also use foreign financial market investments as saving vehicles. The combination of low returns abroad and on the domestic financial market together with the higher rates of growth of the wealth of the domestic agents, creates a fertile environment for the development of rational asset bubbles. Although Caballero and Krishnamurthy stress the benefits of the rational asset bubbles developing on the domestic market, which tend to prevent capital outflows, they also warn of the negative implications due to the reverse cash flows when the bubble crashes in the stochastic environment.

Caballero and Krishnamurthy focus on EM financial markets with significant openness and impact from foreign financial markets. Accordingly, for the investigation of this thesis their model has the disadvantage of displacing the focus of the analysis away from the role of domestic institutional investors such as the DC pension funds. Moreover, relatively modest portfolio capital flows to the Croatian equity market compared to the assets under management of the Croatian DC pension funds together with low integration of the Croatian equity market with global financial markets reinforce the case for using the Tirole's closed market OLG model with rational asset bubbles. No model in the OLG tradition attempts to include all possible characteristics of actual market economies, because the models would be intractable and so incapable of yielding useful insights. For these reasons– our focus on dominant domestic institutions in the context of low openness – we judge Tirole's model to be the most appropriate platform for addressing the impact of DC pension funds.

To support our view in this thesis that during the period under consideration the Croatian financial market was characterised by low openness, we note that during the boom of the Croatian equity bubble in 2005-2007 the total AUM of the Croatian pension funds was between 1.5 and 3 billion EUR, while the AUM of the Croatian Open End Mutual funds investing in Croatian Equity markets was 2 billion EUR. In comparison, the impact of the foreign portfolio equity inflows of less than of 200 million EUR per year on average can be considered as marginal. Table 1.2.2 provides an overview of the relevant flow data.

Table 1.2.2 Dynamics of the foreign portfolio investments on the Croatian equity market

Values in mio. EUR

year	2004	2005	2006	2007	2008	2009	2010
estimated new foreign portfolio investments	-5.1	178.9	-121.6	491.7	274.4	33.2	400.9

Source: Calulated using data from the Croatian National Bank and Croatian stock exchange

Moreover, current trends of increased global capital market correlations suggest that the degree of closedness/openness of the financial market could be achieved by controlling for a price related foreign capital markets variable in the model. This is the strategy we adopt in our empirical analysis, which is reported in Chapters 4 and 5.

In accord with these arguments, we continue our theoretical modelling focusing on Tirole's OLG closed financial market model with rational asset bubbles.

We won't lose generality in analysing the consequences of the introduction of institutional investors such as pension funds using Tirole's model. In other words, the same results applicable to pure bubbles would hold if bubbles were to be represented as a part of the price of the productive assets via some "capital appreciation" mechanism as defined in the Farhi, Caballero and Hammour model.

We have analysed the OLG model of rational asset bubbles and some of the proposed extensions trying to give more complete and more insightful conclusions. Finally, we conclude that Tirole's contribution was seminal in understanding some of the most important characteristics of capital market asset bubbles (Tirole, 1985). They arise as a rational consequence driven by the dynamic inefficiency of the capital market, defined as a significant disparity between the accumulated wealth that needs to be continuously transferred from one period to another smoothing consumption between generations, and the systemic availability of investment instruments to productively achieve such inter-period transfer of wealth with satisfactory returns. In this fashion, bubbles become a rational outcome for the system and for participating agents who accept them as an inevitable part of their portfolios. Although very abstract, providing significant depth of ideas, Tirole's model proves simple, insightful and capable of extensions in its microstructure which, in turn, will enable us to investigate the reaction of the overall market economy to specific changes such as the introduction of the mandatory pension funds that affect the investment behaviour of agents and the overall savings dynamics in the financial market economy. This is the reason why we will model our theoretical capital market economy, and derive our hypothesis on the asset bubble effect of the inclusion of pension funds, based on the theoretical framework of the OLG general equilibrium model.

Having explained the main school of rational asset bubbles and the OLG general equilibrium theoretical framework, we move to critically evaluate other schools of thought on asset bubbles. These give additional, more micro-structure related specifics, which could explain and reinforce the process of systemic adoption of the bubbly financial instrument. The main ideas of these schools complement the analytical framework set by Tirole. We will provide a brief review of the ideas of other schools treating asset bubbles in the following sections, starting with the asset bubble theories based on the presence of asymmetric information in the financial markets. We will find that other schools as well, especially the noise-traders school, have widely accepted the OLG framework as their specific framework for financial market modelling.

1.2.2. Asymmetric information asset bubbles theory

1.2.2.1 Capital markets in the environment of asymmetric information – a recognized critique to ECMH

One of the strongest critiques to Fama's Efficient Capital Markets Hypothesis (ECMH) (Fama, 1970) came from the prominent authors, representatives of "Information economics", Grossman and Stiglitz. Their critique (Grossman & Stiglitz, 1980) followed naturally, because information efficiency was one of the major assumptions of Efficient Capital Markets Hypothesis and its implication was crucial to achieving efficient equilibrium market asset prices. In this respect, Grossman and Stiglitz, in their seminal work on the impossibility of the existence of perfectly information efficient capital markets, claim that market information, which is necessarily costly, in a situation when perfectly available to every agent participating in the capital market cannot lead to sustainable competitive market equilibrium (Grossman & Stiglitz, 1980). They prove their claim by formally stating a simple model where if information is costly even with a minimal cost, and if capital asset pricing reflects all information as one of the basic assumptions of the ECMH, then there won't be an incentive for the marginal trading agent to purchase any information, which will immediately hold for all market participating agents. In such a situation, no agent will have an incentive to be informed, getting the whole information for free from the observed asset prices and, consequently, there will be no equilibrium for the financial asset on the capital market (Grossman & Stiglitz, 1980). This simply means that a capital market on which asset prices perfectly reflect all available information about the asset is an impossible outcome.

The only market outcome, to be a rational outcome when information is costly, is the one where agents have different private information, so that price does not perfectly reflect all information about the financial asset. Consequently, there exists a permanent disparity between the private and common knowledge relevant for asset pricing, which is the measure of information asymmetry. This leads to the existence of asset miss-valuation and to the creation of capital market asset bubbles as a possible rational market outcome, besides the assumption that financial markets react efficiently to new relevant information. The presence of miss-valuations may not be inconsistent with the empirical evidence confirming compliance of the hypothesis of short-run martingales asset price dynamics.

Grossman and Stiglitz in their influential theoretical work claim that trade on the financial markets is driven by heterogeneous beliefs of trading agents, and as we will later see, inspecting the consequences of the presence of asymmetric information, the biggest informational asymmetry affecting equilibrium market prices comes from the disparity between the private and common beliefs. The nonexistence of competitive market equilibrium in the world of perfect information, as shown by Grossman and Stiglitz, is also in compliance with the "no-trade theorem" (Milgrom & Stokey, 1982). The same nonexistence of market equilibrium happens in the other extreme situation when agents are wholly uninformed. Grossman and Stiglitz, claim that the capital market always operates in an imperfect, asymmetric information context between those two extremes, and this leads to equilibrium pricing of the capital assets different compared to their intrinsic value (Grossman & Stiglitz, 1980). This persistent difference means that the bubbly value as a component of the price of the financial instrument, similar to the previous exposition by Tirole (1982) will always have a non-zero value, hence the continuous possibility of asset price bubbles.

When this is compared to the evidence stated in the ECMH, it is clear that the empirical evidence in favour of the short-run martingale properties of asset price dynamics is not necessarily a proof that equilibrium market asset prices are reflecting the intrinsic value of the underlying asset. Moreover, if prices permanently reflect a bubbly component, which has martingale properties as in the case theoretically described by Tirole's "myopic bubble" (Tirole, 1982), besides their intrinsic component, then martingales could not be a proof of efficient equilibrium asset pricing. This corresponds to the recognized "joint hypothesis problem" stressed by Fama and French (Fama, 1991) (Fama & French, 2004).

The Grosmann and Stiglitz critique was later widely accepted by Fama, especially the possibility of prices being away from the intrinsic value of the underlying assets (Fama, 1991). This gives weight to the important role of asymmetric information arguments when defining the rational expectations models of equilibrium market asset pricing. The strength of the argument of persistent asymmetric information defined a new focus in the theory of finance, creating a rising group of authors belonging to the asymmetric information school describing reasons for the occurrence of asset bubbles. In the following section we will focus on some of the most important arguments and specific ideas introduced by those authors and on the insights of this school of thought towards explaining asset bubbles. We will also present the arguments of this school on the roles of large institutional investors in the context of asymmetric information.

For this purpose, we begin with the very roots of the "information economics". Authors such as Stiglitz, Akelrof and Spence, who won the Nobel Price for their "Information Economics" theoretical contribution, demonstrating that in the most commonly present information imperfect market environments, Adam Smith's "free hand" doesn't perfectly work when allocating scarce resources (Grossman & Stiglitz, 1976; Spence, 1973; Akelrof, 1970). This means that the price mechanism tends to function imperfectly when market participants decide based on asymmetric information or, in other words, price as an allocation device doesn't reflect the objective intrinsic value of assets. Akelrof shows in his "market for lemons" that information asymmetry between buyers and sellers could lead to "missing markets" for fundamentally sound assets and could drive out good in favour of bad assets when such information is hidden or unknown before the trade occurs (Akelrof, 1970). This set of ideas corresponds to Gresham's Law observed on the money market[6]. Its logic also leads to the outcome known as "the winners curse" or a Pyrrhic victory, which translates into a possession of a traded asset with smaller fundamental value by the less-informed auction "winner".

This analogy of simplistic relations developing in the environment of asymmetric information, leads to a financial market where, based on knowledge about the presence of imperfect information, an agent could decide to buy an asset at a certain price over its intrinsic value, believing that there will be another agent who, not knowing the true intrinsic value of the asset, will further bid its price to a higher level. Similar to this logic, agents in Tirole's rational asset bubbles model (Tirole, 1985) accept a bubbly asset into their savings portfolio.

The analogy of the simple but extremely important ideas from the "market for lemons" could also be found on financial markets, where high numbers of traders interact in simultaneous auction games within the similar context of imperfect information, bidding up price away from its intrinsic value. The result could lead to persistent market price deviations from intrinsic asset values known as asset bubbles. Since information about common beliefs,

[6] Based on the observation by Sir Thomas Gresham in the 16th century who found out that bad money, or money with a worse mixture of metals in terms of the intrinsic value of the coins, was driving out the good money from the market.

private beliefs of the other market participants and about disparities among them are the most profitable types of information for the individual market decision maker in such a market environment, especially if the market can be described by a sequential game, market participant will try to discover ways to learn this valuable information. The importance of market signalling and finding other ways of solving this persistent information imperfection problem is already recognized and is part of the modern theory of finance.

Imperfect financial market pricing mechanism as a signalling or information dissemination tool that disseminates the information about the value of financial asset from informed to ignorant market participants is a consequence of the asymmetric information market environment explained by Grossman and Stiglitz. Such financial market environment was in the focus of many of the following representatives of this asymmetric information school on asset bubbles.

Jackson and Peck, stressed that in an asymmetric information environment, when the market game is simultaneous, actual price cannot enter the decision function of market traders a-priori because it is ex-post determined after the interaction of all trading decisions has been made (Jackson & Peck, 1999). This way the information about the current common beliefs, disseminated by the actual price, cannot affect a-priori the uninformed agents. Jackson and Peck further claim that the rational expectations capital market equilibrium model must account for this fact, which further causes (Jackson & Peck, 1999):

1) Excess volatility of prices vis-à-vis the volatility of the value of asset fundamentals;

2) "Uncoupling" of prices from the intrinsic value of assets, making the occurrence and persistence of asset bubbles possible;

3) Makes uninformed agents earn a lower premium from participation in the risky (bubbly) asset market; and

4) Makes market equilibrium price exhibit a "V" shape functional mapping related with the increase of costs of information.

Although known as being most efficient, it becomes obvious that financial market copes with plenty of inherent information problems and that the pricing relevant information is not inexpensive. Those problems are especially present in less developed financial markets, with underdeveloped regulation, setting weak or non-existent issuer's transparency requirements and having less educated market participants, which has impact on their rationality and on the formation of their beliefs and investment decisions. Such financial market environments are more often found in countries such as the Croatian financial market belonging to the SEE group of countries where financial markets are young and where the asymmetric information problem is vividly present. On such markets the assumed speculative market premium is also higher.

1.2.2.2. Asset bubbles, learning process and the role of "technical analysis" in financial markets with asymmetric information

There is a group of authors theoretically modelling financial markets operating in the environment of asymmetric private and common information. In their models the market context with asymmetric information is blamed as a major cause for the occurrence of asset bubbles. One of the seminal contributions within this branch of theories on asset bubbles is the work of Allen, Morris and Postlewaite (1993), who present a finite period general equilibrium model of an exchange economy with asymmetric information. In market conditions where:

a) Each agent is short sale constrained for his trades with the risky (bubbly) asset;

b) Agents' trades are not common knowledge; and

c) Agents use their private information when determining their fair value belief for the risky asset;

The authors model a rational market outcome for the risky asset where the market value reached is higher than the asset's intrinsic value (Allen, Morris, & Postlewaite, 1993).

They show that because of the presence of asymmetric information, even if all agents know the correct intrinsic value of the financial asset, because of the fact that they are not sure if the other agents have the same knowledge, or they are not sure that their private knowledge equals the common knowledge, they are prone to bid the price of the asset above its intrinsic value. Agents act this way believing that there is another agent who could help them achieve capital gain at or over their rational, information constrained assumption of the expected return by buying the asset at a higher price. The intuitive logic behind the asset bubbles occurrence in such an environment could be explained by the decisive importance of the private information about other traders' beliefs affecting individual agents trading decision. Previous period price growth observation could lead to a conclusion that other agent beliefs are "bullish", which could prevent the rational dominance of the decision to sell the asset for which the private intrinsic value calculation finds a much lower fundamental value than the present market value. This speculative decision is rational in the market game with asymmetric information as Allen, Morris and Postlewaite show. This is so because the assumed price dynamics pattern closely corresponds to the later observed one due to self-fulfilling prophecies. The authors show by presenting theoretical examples with constrained private beliefs that, due to the existence of asymmetric information, although all agents know that the financial asset is worthless, they are ready to speculate on a higher price of the asset as they speculate on the content of the common knowledge about the value of the worthless asset (Allen, Morris, & Postlewaite, 1993). This idea intersects with the ideas of Brunnermeier, who went one step further, defining such expectations, which lead to short-run felicity derived from optimism, resulting in overestimating the positive outcome probabilities of events. He named this "near rational attitude" as "optimal expectations" (Brunnermeier & Parker, 2005).

Authors call this extreme case a "strong bubble", when everyone knows the financial asset is overvalued but they still bid its price up toward higher values. It corresponds to the idea popular with market practitioners called "The greater fool theory". It says that market traders in such a case believe that they have unique private knowledge for the financial asset being worthless but, because of their belief that the common knowledge values the asset inaccurately over its fundamental value, they believe and optimally apply to their decision a model of belief that there will be another trader to whom they could further sell the asset at a higher price and achieve the expected higher returns. This is considered as a rational decision in such an informational setting where this prophecy is self-fulfilling.

F. Allen, S. Morris and A. Postlewaite also question the objectivity of the criteria for the measurement of the intrinsic value, stating the heterogeneity of the different fundamental valuation approaches used (Allen, Morris, & Postlewaite, 1993). This issue is also recognized by other authors such as Tirole who questions the implication of the existence of different intrinsic valuation models applied by different market agents (Tirole, 1982).

Further in this respect, Morris (1996) explains why in the context of asymmetric information, IPOs find that their values usually reach their maximum at the moment when the public offering takes place. Morris explains the learning process as a process of diminishing the effects of information asymmetry which, as Grossman and Stiglitz showed, cannot be completely eliminated. Morris claims that although this learning process continuously lasts, even small differences between agents' private beliefs could initiate the process of "eternal switching" in the trading positions of agents, leading to a bubbly outcome on the market. This means that a small information asymmetry leading to different private beliefs could initiate the process whereby groups of agents sequentially bid up the price among themselves, believing that after each bidding sequence there will be another agent who will help them realize capital gains (Morris, 1996). Analogue to this is the famous Cambridge story about Isaac Newton, a good example of a significantly rational agent, who speculated on the South Sea Bubble, nearly doubling his 3500 pounds initial investment on his first exit from the market, but then, realizing that the price continued growing, invested a larger sum again and finally made a significant loss. He famously remarked: "I can calculate the movement of the stars, but not the madness of men". His attitude is a perfect example of the "switching process".

So even in an environment with a minimal information asymmetry among private beliefs on all possible contingencies for the fundamental asset, speculative premiums tend to be positive, driving the market to a bubbly price equilibrium (Morris, 1996).

In this respect, trying to model and understand private learning solutions for agents learning the beliefs of others and the common beliefs, as one of the most important tasks for them, even more important than establishing the fundamental value of the asset, leads to the importance of "technical analysis" and common opinion disseminators such as the media. It could be seen that today's media, answering to the high demand for this crucially important informational need of individual market decision makers, makes many innovations such as regular "expert opinion" interviews, "technical analysis" and "price trend analysis" reports, selling this valuable information to the capital market participants.

One of the proposed solutions of the “market for lemons” problem, making individual investors more confident when making their investment decisions, is investing through professional financial institutions such as actively managed mutual funds. This is partly because investors believe financial institutions have better information or better and more sophisticated solutions to the lack of information about the common beliefs and beliefs of others for the intrinsic value of the asset. This is similar to the logic a buyer of a used car applies when he decides to go to a dealer believing that way he will make a better informed decision about the quality of the used car he is buying.

Another way to solve the “common knowledge learning problem” is the use of some innovative tools of learning or figuring out the common beliefs of the mass of investors. This set of tools is known as “technical analysis” tools and trend following techniques, which have gained significant popularity among investors. The popularity of books on technical analysis is amazing in recent times and corresponds to the massive demand of trading agents to develop tools for discovering changes in common beliefs before others do. This confirms the presence of the asymmetry of information and the importance of knowing such beliefs, being even more important than the knowledge of the fundamental values of assets.

Consequently, although authors such as Fama found evidence against the benefits of the technical analysis (Fama, 1965a), later empirical work weakened his claims showing the effectiveness of technical analysis. Matthew Jackson tried to explain market outcomes when agents use a way to solve their problem of ignorance on the private beliefs of other market participants by utilising “technical analysis” methods, in that way trying to gain better pay-off through their market participation. He presents a model with “technical analysis” where market participants follow historical price dynamics by sketching price trends and reacting to sudden price breakthroughs from the trend, representing their private signal about the change of beliefs of other market participants (Jackson, 1993). He adds in the model this feature of “technical analysis” as a learning solution, by consequently affecting private decisions each time an agent observes two consecutive price moves in the opposite direction. Jackson names this technical analysis information as “speculative information” and tries to model its role and discover whether it gives additional value to market participants. He discovers and theoretically shows in his model that agents using this “technical analysis” information learning strategy are better off, since it helps them better predict the equilibrium bids of other agents and the consequent market returns (Jackson, 1993). This result represents an opposing result to the claims implied by the ECMH where, due to the martingale properties of capital market asset prices, there is no market strategy based on historical prices that can make some traders better off compared to those who accept the current price as the best predictor for the price dynamics and the expected investment return in the next period (Fama, 1970). This theoretical result corresponds to the mounting empirical evidence on the long-run price regularities such as the mean-reversion process found by Shiller, LeRoy and many other authors (Shiller R. , Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?, 1981; LeRoy, 1989).

There are also other authors, who show that "technical analysis" gives valuable information to the trader in an asymmetric information context (Treynor & Ferguson, 1985; Brown & Jennings, 1989). "Technical analysis" helps agents with superior fundamental information to better predict the time when the market learns about their already known information, which is crucial for them to realize their pay-off, as shown in the Treynor and Ferguson model (Treynor & Ferguson, 1985).

"Technical Information" gives value, especially to the myopic agent in the Brown and Jennings model (Brown & Jennings, 1989), while Bennet and Sias show that the "money flow" indicator, which is defined as a difference between up-tick and down-tick dollar trading volume has significant asset return predicting value (Bennett & Sias, 2001). Even Fama in his earlier work admits himself that, in the short run there is some empirical evidence against perfect inter-temporal independence of prices, which at that time he explains by the time requirement of markets to absorb and evaluate the new fundamental information (Fama, 1970). So even in a "weak form" of the ECMH, he admits that there could be some market imperfection potentially based on the asymmetry of beliefs, and then later on he applies this to the joint-hypothesis problem where the problem lies in the inability to separate the empirical testing of the hypothesis about the efficient equilibrium asset pricing models from the hypothesis about market information efficiency.

The expected return of the risky asset in this market environment with agents using speculative information is uncorrelated with the fundamental value of the traded financial assets. Jackson also finds that the use of speculative information increases the variance of financial market asset prices above the variation of their intrinsic values creating excess market volatility (Jackson, 1993), a phenomenon realized by many other economists conducting empirical research (Shiller R. , 1981).

Jackson's work, belonging to the asymmetric information branch of theoretical asset bubbles, uses an OLG general equilibrium framework in his model, describing an infinite living market economy with agents living in two periods. He stresses the importance of defining the price formation mechanism for which he uses a Vickery auction model where single agents cannot price manipulate the market with individual extreme bids. Agents in such a context bid up prices using their optimal Nash equilibrium price responses, which means they use their private beliefs about the fundamental value of the asset together with the beliefs about the beliefs of others and common beliefs inferred from technical analysis when they make investment decision. The asymmetry of information about the beliefs of the beliefs of others, and the common beliefs, plays a crucially important role in the rational market decision making process. This was a factor much devalued by authors belonging to Fama's camp, where inference or learning tools such as "technical analysis" played an insignificant role. The final outcome of this model produces departures from the intrinsic value of assets denoted as asset bubbles having risky assets priced at higher than their fundamental values (Jackson, 1993). Asset prices are prone to significant price crashes with low probabilities in this model, which corresponds to the permanent Paretian distribution defined by Mandelbrot and described by Fama (Fama, 1965a).

1.2.2.3. Signalling, Leadership and the Role of Large Institutional investors in the context of asymmetric information

Other very important work in this branch on asset bubble theories comes from the economists Corsetti, Dasgupta, Morris and Shin. They are trying to explain the role of large influential market participants on currency markets with asymmetric information (Corsetti, Dasgupta, Morris, & Shin, 2004). Claims that speculative attacks on Asian currencies in 1997 were initiated by some large hedge funds such as the one owned by George Soros, making the currencies of those countries collapse, presented an incentive for this prominent group of economists to theoretically model and explain this particular situation. What they produced presents a significant improvement in the theory of asymmetric information on the capital markets, especially on the role of the market microstructure, size and disparity among agents in equilibrium asset pricing.

It explained the role of the large market participants and their signalling role in particular, in this process of information discovery about the beliefs of other market participants and critically important discovery of common market beliefs. In their model, large market participants play a significant signalling role for the mass of small market players. Corsetti, Dasgupta, Morris and Shin model and assess the importance of the following four characteristics of agents in the context of financial markets with asymmetric information (Corsetti, Dasgupta, Morris, & Shin, 2004):

1) The importance of the relative size of the market participants;

2) The importance of market participants' private information accuracy about the intrinsic value of the risky asset;

3) The importance of the extent of the differences of private information, which defines the level of asymmetry of information; and

4) The importance of the visibility of market actions by the large market participant to the rest of the market agents, which affects their signalling role.

Corsetti, Dasgupta, Morris and Shin, modelled two different types of market games: first, a simultaneous game; and second, a sequential game. They tested different outcomes based on predefined market environmental characteristics in each game. The conclusions confirm some previous conclusions and refine the understanding of the issues important for explaining asset bubbles based on asymmetric information (Corsetti, Dasgupta, Morris, & Shin, 2004). Their findings are as follows.

In the context of asymmetric information, the higher the asymmetry measured by the difference in fundamental valuation precision among large and small participants the larger is the price variation produced by the entrance of a large trader at the market. The threshold for speculative attack, or the trading decision threshold, in terms of the fundamental strength required for a speculative attack on a currency is higher when large market players are present in the market. The sole presence of the large trader makes small players more aggressive market participants by the fact that its presence and its signals "clarify" small market participants' beliefs about the common beliefs, which is important decision determining information in their Nash optimal response function (Corsetti, Dasgupta, Morris, & Shin, 2004).

Large traders in the environment with asymmetric information have a larger market power affecting the market equilibrium if their relative market size is larger or if they have better information precision about the fundamental value of the traded asset vis-à-vis the information precision of the rest of the relatively small agents. The second line of the argument actually confirms that the greater is the asymmetry of information the greater is the market power of a better informed large trader. A large trader increases its market influence if it can perform more effective signalling in the market environment, which may be enforced by its presence in the media (Corsetti, Dasgupta, Morris, & Shin, 2004).

Although not treating asset bubbles in particular, Corsetti, Dasgupta, Morris and Shin discover a significant influence of the large market player in the context of markets with asymmetric information.

In conclusion, according to this branch of theoretical work treating asset bubbles in the environment of asymmetric information, the presence of higher asymmetry of information measured by the imprecision and heterogeneity of private information about the fundamental values of financial assets and about the difference of the private beliefs, beliefs about the private beliefs of others and beliefs about the common beliefs, could be an important factor leading to the occurrence of asset bubbles. This informational asymmetry and its relative extent could be empirically measured by measuring the difference in price variability against the variability of information fundamentally important for asset valuation. Such difference in variability does not affect the efficiency of markets as defined by their timely reaction to new information. This reinforces the importance of the "joint hypothesis problem" of the ECMH as stressed by Fama, which points that prices may fully reflect all available information but still significantly depart from the intrinsic value of the corresponding assets based on the extent of information asymmetry. Increased price variability which defines investment risks is one of the consequences of higher information asymmetry. In this respect, large traders such as DC Pension Funds and other institutional players, especially if their market actions are highly visible to the "herd" consisting of small market followers, could lead the market coordination game toward equilibrium with prices reaching values higher than the fundamental value of financial assets with increased market volatility. The agency problem inherent to large institutional investors, connected to the potential moral hazard when investing, could even add to the arguments for asymmetric information market environment when explaining the creation of asset bubbles. This visibility of the large institutional investors' market actions is also stressed by other authors such as Robert Shiller who in its famous "Irrational Exuberance", categorizes the informational distribution of media about the moves of well-established and large market players among its 12 factors driving markets to the speculative bubbly asset equilibrium (Shiller R. , 2000).

1.2.3. Agency problem school on asset bubbles

1.2.3.1. Asymmetric information within the organisational unit of the institutional investors

The problem of the presence of asymmetric information has two dimensions relevant to financial markets, which are both important for explaining the occurrence of asset bubbles. The first dimension was covered in the previous text; namely, that is the asymmetry of information inevitably occurring between the trading agents participating in capital markets (Grossman & Stiglitz, 1980). The first dimension of informational asymmetry is concerning the inter-agents information asymmetry about private beliefs, private beliefs of others and the common beliefs. We have seen that this information asymmetry plays an important role when forming the expected asset returns among traders placing bids and offers to the market. Evidence from the growing literature on equilibrium asset pricing models contains significant empirical and theoretical arguments showing that asset bubbles could arise because of the specific interaction among traders developing in the context of asymmetric information.

The second dimension of the asymmetric information problem related to the occurrence of asset bubbles is the one focused on the information problem within the market participating agents, being institutions who are dominant market participants or traders. This information asymmetry affects the relationship and interaction between principals and agents within the organisational unit of the institutional investment industry. This second dimension of the asymmetric information problem is partly the focus of the Agency Theory and it brings to the forefront the principal-agent problem present in the context of financial institutions. Its potential effect on the equilibrium asset pricing models could lead to financial asset bubbles, positive or negative (Allen & Gale, 2007). Before we explain the specifics of the Agency theory applied to financial institutions, affecting the equilibrium capital asset pricing models and their outcomes, such as the asset bubbles, we will first define the main subject of interest to the Agency theory, its main problems and its main solution ideas.

1.2.3.2. The Agency theory and The Principal-Agent problem

The main subject of interest of agency theory is agency itself. The agency represents a contracted relationship between the provider of the means (principal) and the provider of the service (agent), where the product of their mutual incentives based on the division of the benefits and liabilities in their joint cooperative effort arises and defines the essence of the agency problem. There are two main parties in the agency relationship treated by the theory, and those are the "principal" and the "agent". The "principals" are most commonly the providers and owners of the capital engaged in the organisational unit, whose main interest is the preservation and the return on the invested amount of the capital. On the other side, the "agents" are the individuals who are hired to manage the given means or the capital, toward achievement of the goals of the principals compensated by a certain reward for the service. The relationship between the two parties is called the "principal-agent" relationship.

The most commonly treated relationship by academic researchers in relation to the "principal-agent" relationship is the one between the representatives of the owners of capital on the one side and the representatives of the management of the firm on the other side. But the principal-agent relationship could be found in many other instances among different economic relationships besides the stockowner-CEO relationship and its main principles could be used to explain relationships beyond the pure field of corporate governance.

In finance, and in particular in the equilibrium asset pricing theory, which will be the focus of agency theory explaining asset bubbles, the principal-agent problem will treat the relationship between the owners of capital or the individual investors and the institutional investors who are providing asset management and advising services to the investors, the principals. The problem of conflicting actions of agents against the interests of the principals, mainly based on the asymmetry of information within the organisation, is generally called "the principal-agent problem".

The agency theory developed itself as a part of organisational theory, but it intersects with contractual theory and covers issues arising both in micro and macro contexts. This is because one of the most important tasks of the design of the contract, representing a primary focus of the contractual theory is finding the solution to the principal-agent problem.

One broad division of the theoretical work in the field of the general agency theory is made by recognizing two major streams of academic work in the field: 1) positivist agency theory research and 2) principal-agent research (Eisenhardt, 1989).

The first group, of the agency theory positivists, tends to recognize situations where principals and agents have differing goals and, without much mathematical rigour, tries to give prescriptions for solving such situations of differing incentives. Economists belonging to this first group are mostly focused on the relationship between the owners of the capital and the representatives of the capital managers. Their main propositions are focused on the types of contracts and the information systems used for the control of the agent. They generally claim that when more information about the outcome is known by the principal, or when information systems providing for such information are in place and are efficient, then the adequate principle-agent contract that aligns mutual interests is the "behaviour based" contract. In this case agents behave following the interests of the principles. On the other hand, when little information is available to the principal about the behaviour of the agent, the other type of so called "outcome based" contract could partly solve the information problem, the disparity of interests and the principal-agent problem (Eisenhardt, 1989).

The second group, the principal-agent theorists, tends to use more mathematical modelling and rigorous analysis for explaining the principal-agent problem in different, broader and more general sets of environments than those covered by organisational theory and the agency positivists. They compare and analyse the outcome and the effectiveness of the "outcome based" versus "behaviour based" contract incentives in different environments. The common environments under inspection are characterized by different levels of uncertainty about the final outcome, different quality and effectiveness of the informational systems, different levels of task programmability of agents operations and responsibilities, different levels of risk aversion of the principal and the agent, different outcome measurability of the joint effort, different duration of the principal-agent relationship etc. Because their mathematical modelling includes a greater number of factors, principal-agent theorists manage to cover a wider range of agency problem issues, yielding more tailored prescriptions and insights.

1.2.3.3. Asymmetric information, Moral Hazard and Adverse Selection

One of the most important factors of interest in agency theory, from both the perspective of agency positivists and principal-agent researchers, is the effect of the level of asymmetry of information between the principal and the agent in the principal-agent problem. There are two different cases of interest dependent on the extent of information asymmetry. The first one is when there is perfect and complete information within the principal-agent relationship, or when the principal perfectly knows what his/her agent is doing, what are his/her interests, his/her abilities and the outcomes of their cooperation. In this case there is no agency problem arising solely because of the fact that the perfect information relevant to the relationship is mutually known and because of the contractibility of all observable conflicting situations to the cooperative undertaking. This situation is extremely rare in practice.

The other most commonly present situation is when there exist an asymmetric information between the two parties, when the principal does not perfectly know what his agent is doing on his behalf and what are the outcomes and implications of his work, given the agent's self-interest, and when the agent may or may not behave following the interests of its principal. In this second case, the agency problem arise causing outcome anomalies, which are subject for research by Agency Theory. The agency problem arises because the principal and the agent have different goals, and the principal cannot determine if the agent has behaved appropriately following the principal's interests.

There are two aspects of the agency problem arising from the information asymmetry, which are commonly cited in the agency theory literature. The first is the problem of the "Moral hazard", or a problem arising when the agent doesn't devote the contracted and expected effort and care for the benefit of the principal. This is also called "shirking". The second problem is the problem of "Adverse selection" which refers to the misrepresentation of the personal abilities by the agent, which causes adverse selection or a non-identification of the correct critical values of the agent by the principal when the second is deciding on hiring the agent (Eisenhardt, 1989).

The problem of "Moral Hazard" is explained as a direct consequence of the presence of asymmetric information in the relationships between the principal and the agent who engage in "risk-sharing" relationship, where agents' privately taken actions can affect the probability distribution of the outcome important to the principal. This definition of "Moral Hazard" with its especial importance for the insurance industry, carries even higher importance when such an agency problem and moral hazard arises within the institutions affecting the equilibrium financial asset pricing models (Holmstrom, 1979).

Although the pay-off or the outcome is observable, moral hazard could arise placing the informational system at the relationship between principal-agent at the highest level of importance. Inadequate "risk-sharing" among the principal and the agent combined with imperfect information about agents' behaviour, abilities and the outcome of their effort, produce sub-optimal Pareto equilibrium outcomes. Investing in monitoring and in instruments enforcing adequate "risk-sharing", as proposed by Holmstrom may improve the equilibrium outcome (Holmstrom, 1979).

1.2.3.4. The agency problem within financial institutions, equilibrium asset pricing models and asset bubbles

The focus and treatment of the "Agency problem" and, in particular, the "principal-agent" problem by researchers was on the particular relationship between the share-owners and the management of the firm rather than on its potential impact on equilibrium asset pricing models (Allen, 2001). Yet, the observation of an exponentially growing number and market participation of institutional investors in global financial markets, made by Franklin

Allen in his presidential address at the American Finance Association in 2001, motivated a rising group of economic researchers to begin inspecting the impact of the inherent principal-agent problem within financial institutional investors on equilibrium asset pricing models.

Allen observes significant absolute and relative growth of assets invested by institutional investors. According to the Federal Reserve Board statistics that Allen cites in his exposition to the AFA, the total value of holdings of corporate equities in the US grew from 142.7 billion USD in 1950 to 19047.1 billion US dollars in 2000, or by more than 133 times (Allen, 2001). This significant growth of corporate equity holdings was followed by a significant shift in the ownership structure in the last 50 years. In 1950, households dominated with 90,2% participation in total equity holdings, which means all of their investments they directly invested on the equity market. Later on, this participation was constantly falling, reaching 39.1% by the year 2000. On the other side, the participation rate of institutional investors was rapidly growing. Public and Private pension funds and insurance companies, referred to as "opaque"[7] (or least transparent) institutional investors where the potential principal-agent problem predominates (Ross, 1989), increased their participation from 4.1% in 1950 to a significant 29.7% in 2000. Among this group the rise of assets under management is especially significant for the pension fund institutional investors who increased their participation rate from 0.8% in 1950 to 23.2% in 2000 or from insignificant amounts close to zero, to almost a quarter of corporate sector equity holdings in 2000 (Allen, 2001).

Allen stressed the fact that even in the group of household equity holdings, there was an incredible rise in assets invested according to the investment advice of institutional investors, investments which could also be considered as affected by the "principal-agent" problem, which might even increase the overall market participation or impact of institutional investors in today's financial world. Allen further asks, having in mind the fact that institutional investors are the most important investors in the present capital markets, and knowing the general common occurrence of the principal-agent problem among productive companies, why then there is a lack of interest in questioning the potential principal-agent problem among financial institutions and its impact on equilibrium market pricing models

[7] Ross classifies institutional investors as: 1) opaque or the least transparent; 2) translucent or mid transparent and 3) transparent. Mutual funds such as ETF's belong to the 3th group while the other extreme is represented by the pension funds, insurance companies and hedge funds (Ross, 1989).

(Allen, 2001). Allen notes that Mehra-Prescott's "equity premium puzzle" (Mehra & Prescott, 1985), or the extraordinary high average stock premium over the risk free rate of return observed within equities, and the other anomalies observed by the increasing amount of empirical research, such as the "size effect" on the applications of CAPM, the "value effect", the "momentum effect" and other price anomalies and especially the occurrence of asset bubbles, all of them should inspire analysis and the rise of new ideas for different equilibrium asset pricing models (Allen, 2001).

The new approach different from the classical asset pricing models based on the neoclassical Arrow-Debreu framework should stress the growing importance of institutional investors and the inherently present agency problem among them (Allen, 2001). The problem arising within the institutional investors, which could cause the occurrence of asset bubbles, is related to the previously mentioned "risk-sharing" problem in the insurance industry. Within the institutional investors this problem is called the "risk-shifting" problem and is similar to the agency problem between debt-owners and stock-owners in a company observed and explained by Jensen and Meckling. In their work (Jensen & Meckling, 1976), they find that debt financed firms could accept even projects with a negative net present value (NPV), because their owners, the stock-investors, shift a significant part of projects risk to the debt owners. This is the basic example of a "risk-shifting" problem or improper "risk sharing" among the two parties. The main reason simply arises from the fact that shareholders obtain most of the upside potential gain from the outcome of the risky projects but disproportionally bear the downside risk because of the limited liability of their participation in the joint contractual effort (Allen, 2001).

Allen and Gorton, being representatives of the earliest academic researchers focused on the agency problem within the financial institutions, examined the impact of the agency problem on the equilibrium asset pricing model (Allen & Gorton, 1993). They found that even knowing the price of the asset exceeds the maximum expected value of its cash flows, agents bearing only limited liability on the trusted funds for investment, could rationally bid up the price to higher levels because of the fact that they are not bearing the cost of a potential loss. Since agents take part only in the potential gain from their speculative action but bear no loss in case of the negative return, their expected return from the speculative investment becomes strictly non-negative even when the probability that the asset reaches a higher price is extremely low. This argument has the highest importance for the further development of the theory of financial institutions' agency problem (Allen & Gorton, 1993).

Allen and Gorton compare the fund management contract of portfolio managers with a put-option, where the agents' only potential loss is the eventual loss of their jobs. With this type of investment incentive on the side of the institutional investors and their portfolio managers, if they start dominating the market as observed by the rising amount of their market participation (Allen, 2001), this may induce a shift in the probability distribution of the price of a bubbly asset toward the realisation of higher prices. The agency problem within institutional investors, dominated by the risk-shifting problem, makes portfolio managers increasingly risk-loving with respect to their attitude toward investments in speculative (bubble) assets (Allen & Gorton, 1993). This is explained by a simple model, where portfolio managers optimal response when their contract reward is being "outcome based" representing some percentage "α" of the achieved positive return on investments, makes them overinvest in the riskiest asset. This is called "risk-shifting", which literally means that the risk about the negative performance of the risky investments is shifted from the portfolio manager to the investor principal, disproportionally in comparison to the shift in the expected return.

The limited liability of the portfolio manager, combined with the "outcome based" reward inducing such "risk-shifting", makes their personal expected return on the investment positively skewed within the range between the opportunity cost of losing their employment and positive infinity, which is different than the distribution of the actual returns of the risky asset. Even with "behaviour based" contract rewards for the portfolio manager, where the portfolio manager is rewarded with a predefined fixed amount of salary, "reputation incentives" which are especially present among inexperienced first-comer portfolio managers, could create a significant positive skew to their limited liability reward distribution from risky fund investments.

Dass, Massa and Patgiri, in their empirical work observed that portfolio managers with lower salaries are more prone to taking risky investments and "ride on the bubbles" motivated by the "reputation" incentives or by a utility derived from making a reputation as good performers (Dass, Massa, & Patgiri, 2008). The whole point of the "risk-shifting" problem present within institutional investors points to the importance of certain contractual and personal characteristics that should be taken care of when employing a portfolio manager. This especially can have a significant impact on equilibrium asset pricing and the occurrence of asset bubbles. If the downside risk or liability from eventual loss on investment for the portfolio manager could be induced to be higher, which might come from the height of his salary raising the opportunity cost of losing his job, professional certification programs with high ethical dimension such as the CFA Institute program, age in employment in the investment firm, or some contracted incentive or bonus based on longer-run performance, then the principal-agent problem that tends to induce asset bubbles could be decreased.

1.2.3.5. Central Bank Liquidity Policy and Asset Bubbles

Allen also stressed the importance of liquidity to asset pricing anomalies and the "cash-in-the-market pricing" in situations of sudden liquidity shortages (Allen, 2001). In their book on Financial Crisis, Allen and Gale, stress the importance of credit and liquidity inducing the principal-agent problem and equilibrium asset prices to their asset bubble equilibrium level (Allen & Gale, 2007). There is significant empirical evidence of the coincidence of loose monetary policy and the inflation of asset bubbles. Availability of credit and the low borrowing costs seem to coincide with the exponential rise in asset prices, as observed by Allen and Gale. On the other side, the authors explain the collapse of the bubble by the interaction of the change in liquidity preferences by consumers/investors, and the decision to tighten monetary policy by the central bank (Allen & Gale, 2007). Such an explanation could closely describe a crash of the bubble in an OLG model, such as the rational asset bubble model (Tirole, 1985).

Allen and Gale develop a model, based on the asymmetry of information within the institutional investors and the presence of the principal-agent problem, where the random variable representing the aggregate amount of credit available to the financial institutions-banks in the system - denoted "B", which is partly controlled by the central bank, enters the derived asset pricing function having a positive effect on asset prices (P). The result from their simple example is given below (Allen & Gale, 2007, p. 244).

$$p = 8(-1 + \sqrt{1 + 0.25B}) \qquad (1.14)$$

The importance of this equation, derived from the example as stated in Allen and Gale's text, in our context comes from the fact that the price is a function of the amount of available credit, or of the monetary policy of the central bank. Here, p denotes the asset price, while B denotes the credit availability.

The outcome of their simple example as stated above, is different compared to the standard asset pricing model based on discounted expected pay-offs. When liquidity provided by the central bank is vast, the price path of the assets departs increasingly from their fundamentally established intrinsic value.

> *Changes in aggregate credit can cause relatively large changes in asset prices when there is an agency problem*
>
> (Allen & Gale, 2007)

The author's also claim that in a situation when there is a liquidity shortage, which could be affected by the reserve requirement or the interest rate policy of the central bank, equilibrium asset prices could fall even below their intrinsic value measured by discounting assets future cash flows. This means that the bubbly part of the price of financial assets could take not only positive but also negative values. Allen and Gale call this "Cash-In-the-market" asset pricing, which could be even worsened by the sudden systemic need for liquidity resulting from shifts in the consumer inter-temporal preferences or the deterioration in the quality of banks assets (Allen & Gale, 2007).This situation is connected to the financial innovation of financial institutions for the purpose of risk rebalancing and liquidity needs (Ross, 1989). Allen and Gale explain the potential of the "cash-in the-market" asset pricing based on sudden liquidity scarcity as a basis for development of "negative" asset bubbles, which also represent market inefficiencies. Their occurrence could partly be solved by loosening monetary and credit policies by the central bank, a policy response widely observed as a response to the last crash of the real estate market bubble in the United States in 2007/2008.

The authors also find the health of the financial system and the balance sheets of the financial institutions to be directly connected to the occurrence of liquidity runs, which further affect capital market asset prices. Allen reminds us that institutional interconnection within economies and between national markets stresses the importance of further research into financial fragility and contagion (Allen, 2001).

To conclude, the asymmetry of information is widely present not only between the market participants but also within the organisational structures of the institutional investors. This intra institutional asymmetry of information increases the importance of the “risk-shifting” problem where, because of the limited liability for potential losses, asset managers turn into extreme risk lovers and tend to prefer assets with high investment risk where the risk shifting is the highest for their own benefit but at the cost of the final owners or lenders of the entrusted capital. In such a situation, young and inexperienced portfolio managers and those with low base salaries who are dominated by the “reputation” incentive, together with those whose contracted reward is based on the percentage of the achieved return over the return of the risk free asset, have the highest incentive for risk-shifting by bidding up the price of bubbly assets. Availability of credit and liquidity in such environment plays a crucial role, placing the central bank in a situation where it can stimulate the occurrence of the asset bubble by providing massive liquidity in the system, or could induce even a “negative” bubble to occur when it decides to tighten monetary policy in a situation of systemic increased liquidity preference, a situation called “cash-in-the-market” equilibrium asset pricing.

1.2.4. Irrational investors and asset bubbles

1.2.4.1. Rational agents, rational expectations and opposing ideas

One of the bases of modern economic theory and one the main assumptions of the theory of finance and its main efficient capital markets hypothesis is the rational expectations assumption characteristic for the principal rational agents in the capital market. The rational agent assumption is based on the idea that economies consist of only rational economic agents acting within the economy's micro-structure. This theoretical micro economic basis defines the concept of the "economic being" or the "economic man" as a primary economic agent representing the unit particle of the system. His actions represent a rational response to the relevant available knowledge and information about environmental changes and are aimed toward achieving individual maximum beneficial outcome established by his individual objective function. But there is an obvious difference between the notion of "rational agents" and "rational expectations" which must be distinguished. Simply explained, if an agent expects that path A will bring him home in half the time than path B, a rational agent would ceteris paribus choose path A, which makes the agent rational. But this doesn't mean that the expectation that path A gets him faster home that path B is rational, if the outcome has probabilistic character. He might mistakenly have calculated the expectation of path A being time wise shorter than path B which, although he made rational decision being a rational agent, would lead him to an irrational outcome, because he used an irrational expectation as an input in his decision making process.

The "Rational expectations" theory imposes an additional requirement on the rational agent namely the presence of complete structural knowledge about the economy (Heaton & Brav, 2002). Often, irrational agents are confused with the rational agents who base their action on irrational expectations who are commonly named "noise traders". So we must distinguish between noise traders and irrational agents, because the first are not the second.

The notion of "Rational expectations" is based on the idea that within the process of forming outcome expectation or outcome forecasts by rational individual agents, being crucially important inputs in their decision making process, logical factor relationships are used by economic agents whether consumers, employees, entrepreneurs or investors. This idea implies that the real world laws of causality, relationships and interdependences between observed events and expected or projected outcomes, which define the structural knowledge of the economy, as used by the agents, have or tend in their limit to have the identical probability distribution to the actual distribution of outcomes in the real world. This definition of rational expectation, which is the basis of the mainstream economic theory, and of the mainstream theory of finance, is based on a very strong set of assumptions, some of them strongly opposed by numerous observations of authors closely related to the science of psychology, which analyse and observe many common anomalies with respect to the assumption of rational human behaviour.

One of the first economists, who openly confronted the ideas of the "economic man" as a rational decision maker, asking for "a fairly drastic revision" of the microeconomic foundations of economic theory, was the Nobel Prize winner Herbert Simon (Herbert, 1955). But even before him, Keynes stressed the importance of the "animal's spirit" within human behaviour, pointing to human beings' common departures from behaviour based on the rational expectations, especially in the context of the functioning of financial markets (Keynes, 1936).

Besides Keynes and Simon's earlier work, it was not until recently that Behavioural Economics and Behavioural Finance opened a new chapter of strong reconsideration of the basic assumptions of rationality of the actions of market participating agents. Nobel prize winning psychologist Daniel Kahneman, and his close collaborating colleague Amos Tversky, initiated a new significant wave of thinking on the economic principal agent's rationality in the decision making process with their "prospect theory" (Kahneman & Tversky, 1979). Their ideas were further popularized by the considerations of economists such as Richard Thaler and Fisher Black (1986) who, in his American Finance Association presidential address (Black F. , 1986), introduced the idea of "Noise traders" being agents who make investment decisions based on irrational expectations, stressing their significant importance for the analysis of the functioning of financial markets.

Kahneman and Tversky showed that subjects consistently tend to violate the assumptions of preference theory (Kahneman & Tversky, 1979), which are the basis of the classical principles of utility theory (Mas-Colell, Whinston, & Green, 1995). Kahneman and Tversky in their "prospect theory" claimed that agents form their preferences under uncertainty based on some reference point, not on expected absolute changes, and decide distinctly in situations faced with prospects of loss in comparison to decisions considering prospects of gains, which contradicts the principles of classical preference theory (Kahneman & Tversky, 1979).

Further on, in his critique of Daniel Bernoulli's Utility Theory (Bernoulli, 1954), formed on axiomatic ideas about preference theory (von Neumann & Morgenstern, 1944), Tversky shows the existence of significant problems in the descriptive powers of utility theory based on economic agents' rationality (Tversky, 1975). Using experiments based on decision problems under uncertainty, mainly gamble choices among agents, Tversky determined the existence of significant common inconsistencies against the basic axioms of the preference theory such as transitivity. Later on, together with his colleague Kahneman, they showed that based on decision making experiments designed and conducted on university scholars and students, significant violations of utility theory were again confirmed. They criticized utility theory for significant departures from the empirical evidence, such that it should not be used for either descriptive or for normative purposes.

Those two authors opened a new growing theoretical field of ideas, founded on significant evidence derived from experiments describing the psychological decision-making profiles of agents when deciding in the context of uncertainty. Hirshleifer (2001) documents the incredible track record of the main contributions of Kahneman and Tversky, as well as of other authors in the development of behavioural finance theory and towards finding explanations for the behavioural anomalies of decision makers observed through the work of these Nobel Prize winning authors. We will briefly review some of the main characteristics of the growing research of the behavioural finance school of thought describing the unit agent's irrationality of expectations and their judgemental biases in the following section based on Hirshleifer (2001).

1.2.4.2. Behavioural Finance ideas on agent's irrationality

The rising empirical evidence following the work of Kahneman and Tversky (1974) against the correctness of the von Neumann Morgenstern Utility function theory and its main assumptions, motivated the interests of psychology scholars in treating the issues of rationality of choice and rationality of expectations of agents in the economy. Psychologists stressed a significant number of biases to human rationality and behaviour, which significantly impacts the ideas of rational agents and rational expectations and opened a new field of theory of behavioural economics and behavioural finance. David Hirshleifer (2001), in his account of the progress made in the field of behavioural finance, synthesized the main findings on human behaviour, which present significant evidence against its rationality. In his exposition, Hirshleifer makes a wide survey of the theory and evidence regarding investors' psychology suggesting biased rationality as a determinant of market equilibrium asset prices. Focusing on the main contribution of Behavioural Finance, Hirshleifer states that the question that has to be primarily addressed and answered by this new theoretical branch, being essential for equilibrium asset pricing models, is how risk and investor's misperception affect the expected returns of investors and translate into pricing anomalies. On the increasing importance of psychology for the theory of finance Hirshleifer notes:

> *Over time, I believe that the purely rational paradigm will be subsumed by a broader psychological paradigm that includes full rationality as a significant special case*.
>
> (Hirshleifer, 2001)

The dynamic psychology-based asset pricing theory, which is in its infancy, could potentially provide explanation and solutions of the anomalies associated with most of the mainstream asset pricing models based on the existence of only rational agents applying rational expectations. To arrive at the models including agents who base their actions on irrational expectations, in the following lines, we will first review some of the main psychological characteristics typical of human beings that bound their rationality of expectations.

1.2.4.3. Judgemental biases bounding agent's rationality of expectations

Agents face two important subjective limitations not accounted for by the rational expectations theory and the ECMH, which currently dominate the theory of finance. Those two limitations are based on the information processing time and on the available cognitive resources of human beings. Facing those limitations on the one side and forced by the need to efficiently make decisions on the other, the natural selection process has made human minds implement "rules-of-thumb" or heuristic simplifications as decision making short-cuts (Herbert, 1956). But this approach of applying simplifying algorithms and developing heuristics as an answer to resource limitations of the human brain creates misjudgement and decision biases, which significantly affect the outcomes observed on the markets vis-a-vis the theoretically modelled outcomes assuming complete expectations rationality of agents.

It has often been argued by the rational expectations theory that valuation errors based on the mistakes made by cognitive resource limited agents cancels out (Friedman, 1953). However, psychologists and representative scholars of the behavioural finance theory have observed the presence of judgemental biases based on judgemental heuristics shared by most people, which level the potential expectations irrationality bias up to the systemic level making miss-valuation and misjudgement a common market-wide issue. Hirshleifer summarizes three broad groups of behavioural characteristics causing judgemental biases implying common miss-valuation and expectations irrationality among market participating agents. Those three main groups are the following (Hirshleifer, 2001).

A) Heuristic Simplification, which presents a short-cut cognitive method for making conclusions developed by human reasoning with its limited attention, memory and processing capacities. Heuristic simplification associates with a thinking process focused on a small subset of information and on the use of subconscious associations when making economic decisions. One of the products of this heuristic simplification, as stressed by Tversky and Kahneman, is the so called "availability heuristics" (Tversky & Kahneman, 1973), where common things that are easily noticed compared to more potentially substantial but difficult to notice characteristics, are the ones getting attention and consequently gaining misleading importance in the decision making process. Another implication of heuristic simplification is the tendency to develop "habits" that can be optimal mechanisms for human beings to make routine reactions, when addressing their problems of memory loss and limited cognitive abilities. Those "habits" create rules of self-regulated strategies, which could apply to investment decisions also leading to persistent misjudgements. In this respect, the Andreassen and Kraus experimental evidence, when conducting stock market experiments with educated agents, has shown that they tend to develop a "habit-like" response specific to a certain market situation. When they recognise a trend in the asset price dynamics, they tend to follow the trend, while when they recognize that prices are moving in a range, they adopt a different "habit-like" response buying at range lows and selling at range highs (Andreassen & Kraus, 1988).

Another related effect to heuristic simplification is the "halo effect", which associates all characteristics of one subject as being positive if and only if the judging person finds one simple salient main characteristic about the subject that he finds positive.

The "Illusion of control", or belief that the decision making agent is gifted with some ability of a "magical thinking" can make him use irrelevant cues leading to irrational judgements. This illusion can be found in the psychological profiles of many investors and it also relates to the heuristic simplification characteristic of human beings. This form of bias leads to the "illusion of control", whereby people become overconfident in their belief that they have a magical but still rational way of thinking when making certain choices. It explains the observation that, commonly, lottery players act completely irrationally by applying irrational expectation when refusing an offer of being paid a positive amount of money in addition to another same lottery ticket for the exchange of the ticket he already personally picked. He foolishly believes that the fact that he picked the ticket himself increases the winning probability of the ticket based on his "illusion of control" bias.

Further on, based on the idea of heuristic simplification, academics at the field of psychology stressed the importance of the use of "narrow framing" when making choices. It represents another biased characteristic of human cognitive behaviour related to financial and economic decisions. It is based on analysing problems in a too isolated fashion. An example of "narrow framing" is often stated by the common disparity of observed agents' decisions on the same problem presented differently. Agents tend to choose differently and inconsistently on a completely equal problem when the outcome is presented in terms of gains compared to an alternative presentation of the same outcome in terms of losses. One of the outcomes of the "narrow framing" of agents is the well-known "disposition effect" associated with investors' common habit of selling winning stocks and keeping losing stocks.

The representative heuristics could also explain misjudgements arising from the "gamblers' fallacy" or a belief of the presence of a "hot hand" among gamblers, consequently providing additional explanation for agents' attitudes of "chasing trends" and other misperceptions. Finally, there are a number of other examples of observed biases that prevent agents forming rational expectations, which are also explained by the heuristic simplification argument. However, their further explanation exceeds the aim of this text.

B) Self Deception is the second important psychological feature commonly tied to human beings as a decision making agents. The first in the group of biases caused by self-deception is the “overconfidence bias”, arising from the commonly present misleading belief of agents about their own knowledge being more accurate than it really is. This leads to systematic misjudgements of the predicted probabilities of an expected event, skewed toward realisations of extreme outcomes. Psychologist experiments observe that agents’ confidence intervals, when making projections about some economic outcomes, are systematically too narrow compared to the ones derived from the observed variation of outcome realisations. This “overconfidence bias” leads to the presence of “over-optimism”, which has vast influence on the irrationality of expectations and, thereby, on asset pricing on financial markets. Self-deception bias has also a feedback on the learning process creating Bayesian learning bias, which leads to the incidence of “rationalisation” and “confirmation bias”. Agents, when learning from their past experiences, commonly tend to understand and accept their own failures as being a consequence of a “bad luck”, in comparison to the understanding of successes, which are most commonly attributed to their own high personal abilities. This is also connected to the “false rationalisation” of agents, finding a biased ex-post explanation supporting the rationale of their previously made decisions. Finally, “confirmatory bias” and “belief conservativism”, where people try to find information confirming and enforcing their beliefs, shapes persistent rigidities among agents’ knowledge and cognitive abilities, making for strong effects on their persisting self-deception and on their miss-valuation of financial assets.

C) Emotion and self-control belong to the third group of psychological characteristics which bound and bias agents’ rationality of expectations. Distaste for ambiguity, distaste for fear, avoidance of unpleasant feelings such as loss, or realisation of being wrong, and the effect of mood on investment decisions are only few of the many observed characteristics from this group of psychological characteristics affecting agents’ investment behaviour.

“The weather effect”, is one example of a mood related effect, where the weather over the stock market has an impact on the attitude of investors. This is just an example of the expectation irrationality of agents affected by the impact of emotions. Empirical studies also show that disruption of sleep patterns caused by changes to and from daylight saving time, affecting the mood of investors, also can have an impact on stock returns (Kamstra, Kramer, & Levi, 2000). Time preferences and self-controls are also an important part of this group of behavioural characteristics leading to agents' irrationality of expectations.

Having briefly explained commonly summarised psychological characteristics of the human psychological profile bounding its rationality of expectations, we could go further in defining its impact on the asset pricing models getting the theory of finance close to the functioning of the real world markets. Based on the arguments about the psychological biases on agents' rationality of expectations, behavioural finance theorists assume that agents will misjudge not only the random nature of the residuals (mistakenly seeing patterns) but also the factors used in the causality models used for predicting the expected return of assets (Hirshleifer, 2001). With the increasing work of psychologists, showing that investors as human beings could act guided by irrational beliefs and expectations, a group of economic scholars introduced the idea and importance of modelling markets with investors deciding based on irrational expectations. The main such contribution is the "noise traders" model, or the DSSW models (DeLong, Shleifer, Summers, & Waldmann, 1989). This model use an OLG 2-stage framework similar to Tirole's rational asset bubbles model (Tirole, 1985), but with the inclusion of investors with irrational expectations acting side by side with the rational expectation investors. The main point in the DSSW model is that irrational expectation investors, or the "noise traders", miss-value the expected return of the risky asset by some random number ρ_t, which might have some modelled probability distribution. Further, one could model any of the irrational expectation biases of such "noise traders", as suggested by the behavioural economists, by modelling a predefined miss-valuation pattern. Irrational expectations investors, called "noise traders" do not know that they act irrationally; they are rational but guided by irrational beliefs, which again stress the important distinction between the rationality of agents and the rationality of expectations. On the other side, there are also rational investors endowed with rational expectations who are aware of the existence of "noise traders" who follow irrational beliefs, which promotes additional risk to those rational investor investments called the "miss-valuation risk".

This idea of including "noise traders" in the OLG model, gave important insights into asset valuation anomalies, their persistence, the mean reversion effects and short-run positive and long-run negative auto correlations of asset prices observed, as opposed to the theoretical assumptions of the ECMH. With the publishing of the series of DSSW models, the number of further models based on the inclusion of "noise traders" in equilibrium asset pricing models increased significantly. Many authors tried modelling different patterns of miss-valuation based on observed psychological biases by adding different features to their irrational expected return formation. Besides the "pure noise trading" models, which we will examine in more detail below, there was an increasing number of authors trying to introduce different observed biases to the rationality of expectations, based on the work on psychologists, when formulating the miss-valuation rule in the model similar to the pure noise traders DSSW model.

The "Positive Feedback Trading" effect, as one such characteristic bias, where irrational agents extrapolate trends, was an extension to the DSSW model made by De Long et al., who defined three types of agents: rational speculators; irrational speculators; and passive investors (DeLong, Shleifer, Summers, & Waldmann, 1990b). According to this study, rational speculators could increase market volatility only because they want to profit from the presence of foolish irrational expectation speculators who overreact, something that was explained to be an optimal investment strategy in such a context by George Soros, who claimed to use that strategy himself (Soros, 1987). Further on, in the Cutler et al. extension, adding "positive feedback trading" to the model, two types of noise traders were included, differentiated according to the speed of their reaction while trading with a positive feedback (Cutler, Poterba, & Summers, 1990).

Other authors, (Gervais & Odean, 2001) go to an even more complicated modelling of irrational expectation agents' miss-valuation, trying to accommodate solutions of the learning process under biased self-attribution, which is related to the miss-valuation caused by the presence of "overconfidence bias" belonging to the group of "self-deception" biases present among agents.

Generally, the DSSW model based on the OLG framework opened a wide range of theoretical investigation opportunities by introducing ideas of behavioural psychologists on the expectations irrationality of agents. Those ideas were applied to equilibrium asset pricing models in finance and the effects of noise traders were inspected. In the following subsection, we focus on the basic DSSW model with pure noise traders and its main implications, which carry most of the intuition important for explaining asset price anomalies and the occurrence of asset bubbles.

1.2.4.4. Asset bubble models with noise traders

Increasing evidence and theoretical knowledge on the expectation irrationality of agents, paralleled by the need to explain occurring equilibrium market asset pricing anomalies, put forward the idea of modelling financial markets with inclusion of agents with irrational expectations. Those agents, named "noise traders" (Black F. , 1986), are agents who base their investment decisions on irrational expectations called "noise" or on irrelevant information that they consider to be important. This causes them to be rational in the sense of optimizing their objective function, but to be guided by irrational expectations instead of rational expectations. This definition of "noise traders" is in line with the arguments of behavioural economists, bringing onto the stage the psychological irrationality biases previously mentioned in the text explaining the reasons for the irrational expectations and consequently irrational investment decisions.

Bradford De Long, Andrei Shleifer, Lawrence H Summers and Robert J Waldman created a new branch of theories of asset bubbles based on the idea of the presence of irrational expectation agents, who represent investors deciding based on biased rationality as observed by behavioural economists. Their noise traders' asset pricing model is also known as DSSW after the capital letters of the authors' last names (DeLong, Shleifer, Summers, & Waldmann, 1990a).

In their main paper (DeLong, Shleifer, Summers, & Waldmann, 1990a) from the series of DSSW contributions (DeLong, Shleifer, Summers, & Waldmann, 1989; 1990a; 1990b; 1991), this group of economists use the OLG framework as a macro modelling basis for their theoretical investigation. Their originality is coming from the inclusion of "noise traders" or agents with irrational expectations, acting side by side with the rational expectations agents as main components of the micro-structure in the model. In the model, "noise traders" establish their expected return from their investments in stocks (the bubbly instrument) based on a misjudgement factor represented by a normally distributed number with a non-zero mean, added to the rationally expected rate of return. This misjudgement factor “ρ”, as described by DSSW (DeLong, Shleifer, Summers, & Waldmann, 1990a, p. 708), has the following normal distribution, where the mean is some positive value:

$$\rho_t \approx N\left(\rho^*, \sigma_\rho^2\right), \tag{1.15}$$

Here, ρ_t represents the value of the misjudgement added to the rationally expected return as applied by the noise traders at time t. It is assumed to have a normal distribution with a positive mean $\rho^*$and a standard deviation σ_ρ.

On the other side, rational arbitrageurs (also called rational speculators), establish their expected rate of return based on their rational expectations. This imposes a logical necessity for the rational expectation investors to include the fact of the presence of “noise traders” when they define their investment demand decision function based on their rational expectations. Simply, the market impact of the “noise traders” becomes a part of the rationally expected return from the risky asset by adding uncertainty about the persistence of the noise traders’ irrational trading positions. This affects rational expectation agents’ demand for investment and influences the equilibrium asset pricing model by introducing an additional component, which DSSW called the “miss-valuation risk” or “noise trader risk”, to the pricing function. The presence of “miss-valuation risk” increases the overall risk of the financial instrument over its fundamental risks in the perception of the rational expectation agents and prevents them from fully arbitraging out the present mispricing from the rational value of the asset.

DSSW (DeLong, Shleifer, Summers, & Waldmann, 1989, p. 685), define the risk of such instrument with the following equation, now with the “noise traders” on the market, consisting of two independent elements:

$$\sigma_R^2 = \sigma_\pi^2 + \sigma_\varepsilon^2, \tag{1.16}$$

Here, besides the fundamental risk σ_ε^2 coming from the variability of fundamentals of the asset itself, the total risk of the asset includes the noise traders’ risk, σ_π^2 which is uncorrelated to the fundamental risk, making the total risk of the asset σ_R^2 greater or equal compared to its value in the environment without the presence of “noise traders” in the perception of the rational expectation investors.

As previously mentioned, this additional risk perceived by rational expectation investors, changes their rational investment market response. The corresponding additional premium required by rational expectation agents is added because of the presence of “noise traders” and, in turn, this affects the equilibrium pricing of the risky instrument by imposing a higher rate of return. Consequently, as a result of the overlapping generations model, the inclusion of the “noise traders” leads to a formal result of the equilibrium asset pricing equation such as the one derived by DSSW (DeLong, Shleifer, Summers, & Waldmann, 1990a, p. 711)(the authors slightly changed their notation in 1990a compared to their 1989 paper):

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\mu^2\sigma_\rho^2}{r(1+r)^2}, \qquad (1.17)$$

In this equation, p_t represents the equilibrium price of the risky asset at time t, which is determined: first by its fundamental (rational) value of 1, representing the first pricing component on the right hand side of the pricing equation; second, the pricing effect of the actual young generation of noise traders bullishness or bearishness based on theirmisjudgement about the expected return of the asset defined by the difference between the actual misjudgement variable and its mean $(\rho_t - \rho^*)$ multiplied by the relative number of young noise traders in the market defined by μ and discounted by the dividend return of the asset $1+r$; third, the effect on the price of the average bullishness or bearishness of the young noise traders positively depending on their relative number in the market μ and their mean misjudgement value ρ^* capitalized by the dividend rate r; and fourth and the last component which determines the risk premium of all market participants which is determined by their mutual risk aversion exogenously defined by some risk aversion factorγ, the squared presence of noise traders in the market μ^2 and the variance of their misjudgements σ_ρ^2.

The interplay between the persistence of noise traders’ irrationality, their average misjudgement bullishness or bearishness and their relative presence in the market on the one side, compared to the constrained longevity of the investment horizon of risk-averse rational arbitragers on the other, imposes potential bounds on aggressive arbitrage profit exploitation opportunities created by noise trader’s miss-valuations by the rational arbitrageurs (DeLong, Shleifer, Summers, & Waldmann, 1989). Rational arbitrageurs observe the presence of the “noise traders” and include their impact of prolonged asset price miss-valuation in their own investment decisions. The presence of the noise traders plays a crucial role in the establishment of the equilibrium asset prices. Rational arbitrageurs, who are risk averse, as well as the noise traders, have a short-term investment horizon and bear liquidity risk, which actually reflects their potential need to sell their investment ahead of the time required for the realisation of its fundamental value, in order to satisfy their consumption needs. In this respect, as could be seen from the last component of the pricing equation (1.17), the risk of the next period misjudgement of the group of noise traders, as observed by the rational speculators, prevents them from arbitraging out the mispricing of the risky asset. This risk is especially enforced when rational speculators are more risk averse, when noise traders have higher participation at the market and when their misjudgement has greater variance. Those are all characteristics common to young, illiquid and underdeveloped financial markets such as the financial markets in Croatia and the other SEE countries. In this case, mispricing of the risky asset could become persistent at the market and could create asset bubbles due to the presence of noise traders.

As opposed to the assumptions of Friedman and Fama (Friedman, 1953; Fama, 1965a), who claimed that rational arbitrageurs quickly wipe out present market mispricing profit opportunities created by the foolishness of others, and that irrational “noise” traders fail to survive, DSSW show that due to the presence of the “miss-valuation risk” or the “noise traders risk”, rational arbitrageurs might not attempt trading on the obvious arbitrage opportunity. This leads to noise traders taking more systemic risk and consequently gaining higher return from the premium they create by their own presence, leading to their ultimate market survival as a group (DeLong, Shleifer, Summers, & Waldmann, 1989; 1990a). Rational arbitrageurs include this risk from prolonged mispricing of the financial asset in their investment function, because of the present probability of a continued miss-valuation by noise traders, who have the power to lead the price to even more disparate values from its fundamental intrinsic value. This knowledge of the presence of irrational expectation noise traders, combined with the liquidity risk among rational arbitrageurs, could result in their avoidance of bidding against noise-traders and, finally, in prolonged asset price anomalies and in the creation of asset bubbles on financial markets.

Even if financial markets are small and illiquid, noise traders might gain market power, which additionally lowers the incentive of rational arbitrageurs for aggressively betting against noise traders, and makes small and illiquid markets even more dominated by noise traders’ behaviour (Palomino, 1996). Palomino, describes this situation as similar to a result of the “Cournot oligopoly”, where an irrational agent can cause more cost to rational agents then to himself. In the case of bounded competition such as oligopoly, irrational participants tend to produce more, in effect lowering the price, but causing more damage to the revenues of other rational participants than to their own revenue. This simple example explains the solution of Palominos extended DSSW model on non-competitive small markets leading to the dominance of asset price dynamics by the behaviour of noise traders, and to increased volatility and disparity from the fundamental value of assets. This also explains the observed departure of the price from the fundamentals of small company stocks closed-end mutual funds' net asset value, known as the closed-end funds puzzle (DeLong & Shleifer, 1991; Lee, Shleifer, & Thaler, 1991). Markets for closed-end funds and small stocks are similar to the non-competitive markets modelled by Palomino. Higher volatility of closed end funds discounts and premiums, as observed by De Long and Shleifer corresponds to this theoretical finding.

The importance and impact of noise traders on prices, perceived by the rational expectation investors and professional arbitrageurs, makes them spend much more resources on examining and predicting the signals of the group of noise traders, than spending resources on following and predicting the value of fundamentals of the asset. These "technical analysis" signals include sentiment indices, volume and price patterns and the forecasts of Wall Street gurus and analysts. Professional arbitrageurs find this information highly valuable and find it worthwhile to invest significant amounts in learning the mispricing attitude and mood of the noise traders. In their extended work on the impact of noise traders, modelling irrational expectation investors' miss-valuation as based on following a "positive feedback strategy" (DeLong, Shleifer, Summers, & Waldmann, 1990b), the authors find that rational arbitrageurs, knowing that noise traders would follow this pattern of miss-valuation, decide to speculate the price to a higher level over the fundamentally established asset price level on the occurrence of news at the market, or to overshoot the target price, in order to better exploit the opportunities provided by noise traders' response by further extending the price increase. This explains the "price overreaction hypothesis" and excess volatility as observed by Shiller, DeBondt and Thaler (Shiller R. , 1981; DeBondt & Thaler, 1985). In the models with noise traders, their presence adds on the volatility in the market. Further, Barsky and De Long, in their account for the major bull and bear markets of the 20thcentury, conclude that markets tend to over-react at the moments of highest optimism and highest pessimism, showing examples of respected scholars' statements that proved wrong about market perspectives[8] (Barsky & De Long, 1990).

Returning to the pure DSSW noise traders' model, noise traders could be significantly compensated for bearing the risk they partly create by their own presence and actions based

[8]They quote Irvin Fisher's statement in 1929, that markets have reached a "permanently high plateau". Fisher is not an outlier among academics commenting on the market.

on mispricing, simply because they over expose to the risky asset compared to the rational investors who restrain from investing because of the noise traders risk. Thereby noise traders on average earn higher risk premiums than the rational arbitrageurs. This could explain some observed market anomalies, especially the price/earnings mean reversion and the Mehra-Prescott equity premium puzzle (Mehra & Prescott, 1985). One very important conclusion linked to the presence of market miss-valuation, and excess volatility, is the importance of the investment horizon of rational arbitrageurs. The longer the investment horizon of rational arbitrageurs, the shorter and milder is the impact of possible miss-valuation or “noise trader” risk on their aggressiveness in using the profit opportunities present from the departure of price from the fundamental values of financial instruments.

In their earlier work inspecting the size and the incidence of losses from noise trading (DeLong, Shleifer, Summers, & Waldmann, 1989), DSSW find that a higher proportion of noise traders in terms of their wealth participation imposes a higher cost on rational investors by preventing their productive investments. Except in the case when noise traders are on average bullish, the economy is hurt by the presence of noise traders. In this special case, “noise traders” provide for lower interest rates, which increase the amount of capital, thereby outweighing the loss from rational arbitrageur’s non-investment. DSSW prescribe solutions, associated with Keynes’s ideas for taxing speculative capital and providing regulatory measures to stimulate a long-run investment perspective in order to decrease the cost from noise trader’s speculations on the economy (DeLong, Shleifer, Summers, & Waldmann, 1989).

At another occasion, DSSW analyse the survival of noise traders using an infinite lived agent model, where authors analyse the dynamics of wealth distribution among the group of rational arbitrageurs and the group of noise trader investors (DeLong, Shleifer, Summers, & Waldmann, 1991). In their model, noise traders are assumed not to have any impact on market prices individually, which corresponds to a practical situation with a relatively small individual wealth of each single noise trader, which is opposed to Palomino's case where noise traders can individually affect prices (Palomino, 1996). In such a situation, although the idiosyncratic probability of each individual noise trader for losing all its wealth is close to certainty, as a group, they tend to dominate the wealth distribution because of the higher expected return and the cancellation of the individually present idiosyncratic risk within the noise traders as a group. DSSW assume, based on the previously mentioned psychological characteristics of individuals, similar to the gamblers "hot hand" bias observed by psychologists, that noise traders' miss-valuations will have a positive correlation; in other words, they might act similarly to a herd and their actions will not cancel out (DeLong, Shleifer, Summers, & Waldmann, 1991). This finally leads to the increasing relative wealth participation of noise traders as a group and to the definite survival of noise traders in the market.

Within this group of noise traders, there will be a significant number of losers and a small number of incredibly lucky winners. But the group will always survive in the market (DeLong, Shleifer, Summers, & Waldmann, 1991). The unknown is the result of the effects derived from their model, beyond the point of "invasion" of noise traders caused by the growth of their wealth participation in the market. In this case, the group of noise traders becomes so wealthy that they dominate the market and start impacting the price, which is against the assumption of the DSSW model (DeLong, Shleifer, Summers, & Waldmann, 1991). So there is no conclusion on how the accumulation of wealth will continue to develop beyond that point of "invasion", when the group of noise traders start influencing market prices, or what will be the further impact on the distribution of wealth among noise traders and rational arbitrageurs.

Another interesting point is the presence of noise traders among institutional investors. Authors such as Dow and Gorton (1997) analyse the possibility that asset managers, when not having rational incentives or abilities to choose an investment asset, based on their contracted incentives, will act as noise traders which might have significant impact on the market due to the relative size of the assets under their management. Those ideas intersect with the principal-agent problem and stress the importance of the profiles and incentives imposed on the asset managers within institutional investors.

Further research departing from the rational expectations assumption is the modification to utility theory made by Brunnermeier and Parker, introducing the idea of "Optimal Expectations" (Brunnermeier & Parker, 2005). The authors try to explain and introduce behavioural biases by defining and adding the benefit of optimism in agent's utility function. Their idea is connected to the agent's benefit from being optimistic and from having optimistic beliefs. This leads to a biased investment decision overestimating the returns from investment. Even a small optimistic bias could create first-order gains within investors in terms of creating positive initial utility from having optimistic beliefs, while future inevitable disappointments from overoptimistic investments will create unexpected financial loss and disutility. Authors such as Alpert, Raiffa and Buehler show the presence of overestimation bias in expected returns among trading agents (Alpert & Raiffa, 1982; Buehler, Griffin, & Ross, 1994). This is the basis of the "Optimal Expectation" theory, where agents tend to be optimistic and to overestimate expected returns causing market miss-valuations.

To conclude, we have seen that psychologists confronted the basic principles of preference theory, based on arguments from observational evidence about human behaviour under uncertainty. Humans as unit economic agents display many biases from the theoretically assumed rational expectations behaviour, which further questions the normative usability of the classical utility theory when modelling rational expectations. Authors such as DSSW (De Long, Schleifer, Summers and Waldmann) opened a new page in analysing financial markets by introducing the impact of the presence of noise traders on the market equilibrium outcome. The "noise traders" are the ones who depart from deciding and investing based on rational expectations, but their decisions are affected by multiple behavioural biases. DSSW show that "noise traders" will not disappear as previously believed, and- on the contrary - that they will survive to accumulate wealth on the market. Their actions will cause persistent asset price miss-valuations based on their interaction with the rational arbitrageurs. This is a simple consequence of the added "noise trader" miss-valuation risk on top of the "fundamental risk" from investing in risky instruments. This can explain the occurrence and persistence of asset bubbles. On the other hand, crowding out of the rational arbitrageurs by the noise traders is increased in small and illiquid markets, because they gain market power. This explains the close-end funds premium/discount and the miss-valuations observed within small company stocks. Further research analyses the potential noise trading behaviour of institutional investors, affected by poor asset manager selection and by the wrong contracting incentives with the asset managers. Those issues intersect with the issues stressed in the agency theory explanation of asset bubbles, and are of significant importance for analysing the impact of pension funds on the induction and dynamics of asset bubbles in small illiquid markets.

Conclusion

Reviewing the equilibrium asset pricing theory, we could conclude that, mainly due to the repeated observational evidence against the efficient equilibrium asset pricing hypothesis, the theory of efficient capital markets is losing its popularity. Although financial markets in the developed countries preserve informational efficiency in the short run, vast empirical evidence shows that they tend to develop persistent asset miss-valuations in longer market cycles. This regularity points to the occurrence and the importance of the theories of asset bubbles.

We saw that the OLG framework provides a useful tool for theoretical modelling and analysing the investment/consumption behaviour of rational agents with or without rational expectations. Moreover, as the seminal model of Tirole reveals, in a situation when wealth is abundant, consumers are patient and optimistic and when the economy cannot provide sufficient fundamentally based (productive) investment vehicles to transfer wealth from one period to the next with a satisfactory return, such market economies are prone to financial innovation and are faced with the ease of rational acceptance of bubbly asset instruments. Those instruments could be pure bubbly instruments, or could be a combination of instruments whose price is composed of an intrinsic and a bubbly part such as common equity. The question that Tirole posed at the end of his seminal paper is what characteristics such instruments have to satisfy to become accepted as rationally bubbly instruments? What is so special about gold?

But reviewing the rest of the theory on asset bubbles, which focused not as much on the macro-impact and rationality of bubbles, but on micro-characteristics, we see that it is not so much about the character of the instruments but, rather, about the market micro-structural characteristics of the dynamically inefficient economy. In this respect, a high level of asymmetric information on the market, the prevalence of agency problems within the institutional investors, and a substantial participation of “noise traders” or traders with irrational expectations, are the three main micro structural characteristics that could make it easier for rational agents to accept bubbly instruments of any type and so rationally bid them to their bubbly equilibrium (following the logic of Tirole). We have also seen that in the context of such bubbly capital market dynamics, monetary policy affecting the availability of liquidity could, in turn, affect the returns on the capital markets and impact on the occurrence of asset bubbles. The competitiveness of the market, determined by its structure, also plays a significant role.

In this chapter, we have gained significant understanding of theoretical modelling at the macro market level together with the micro-structural characteristics causing asset bubbles to occur. In the following chapter we develop a specific OLG model economy with an illiquid capital market, to which we add defined contribution pension funds in order to investigate the effect of their influence on equilibrium asset pricing and on the potential occurrence of asset bubbles.

Chapter 2: OLG Model of rational asset bubbles, DC pension fund and an illiquid asset market

2.1. OLG model with productive savings, rational asset bubbles and an investment rule based fully funded DC pension scheme

In this chapter, we present an OLG model with rational asset bubbles developing on illiquid equity markets upon the introduction of influential Defined Contribution (DC) Pension Funds. The Defined Contribution (DC) Pension funds are institutional investors, part of the World Bank led reform of the pension system in many developing countries (World Bank, 1994). They subtract a significant part of the gross salary from every current employee in the country and invest this amount in the financial market for each employee account in order to provide pensions when the employee becomes old. In general we distinguish between defined benefit and defined contribution pension funds, where the second group is becoming more popular than the first (Butrica, Iams, Smith, & Toder, 2009). DC pension funds are part of the so called second pension pillar, the first being the Pay-As-You-Go (PAYG) pillar of direct inter-generational transfers, which with the ageing of the population has proved unsustainable(Ramaswamy, 2012); and the third being the voluntary pension scheme, which is still of marginal importance in the developing countries due to its voluntary nature and the low interest in its utilisation by the actual employed citizens(Antlin, 2008). So the second pillar mandatory DC pension funds now represent the most important pillar of the pension fund reforms in many transition and developing countries.

The aim of this chapter is to theoretically model the systemic financial market implications, with respect to the occurrence of asset bubbles in particular, as a result of the introduction of influential institutional investors such as mandatory DC pension funds that affect economy-wide savings behaviour. These types of pension funds were introduced in some of the South East European (SEE) countries, such as Croatia, during the first decade of the 21stCentury and there are a number of countries where their introduction is considered as part of the pension system reform under the guidance of the World Bank. This fact makes this issue highly relevant for the small financial markets of those developing and transition countries, especially since the occurrence and collapse of asset bubbles may have a significant impact on the functioning of their fragile financial markets and, hence, on their real economies.

We introduce the Defined Contribution(DC)Pension funds in the OLG model framework following their basic institutional characteristic, which is taxing a certain percentage of the gross salary of each employed young agent, investing it directly to the domestic financial market based on some predefined investment rule, and later distributing the value of their investments back to the agents at the time when they arrive at their old age (at their period when "old" using the common characterisation of the OLG model).

At the beginning, before we formally set out the rational asset bubbles general equilibrium model with DC pension funds, we must recall the two main types of the financial market environments in our OLG model, which affect the productive capital accumulation and rationality of the bubbly asset investments. This is important, because it will have consequences for the sustainability of the bubbly asset equilibrium after the DC pension funds are introduced and established. The two types of financial market environments are the following:

- First, are the "dynamically efficient" types of financial markets, where rational asset bubbles are non-sustainable and Pareto sub-optimal. In this case, there is a lower systemic incentive for bubbly assets to appear and develop, and if they appear, they shortly after collapse in a speculative episode to the financial market Diamond Steady State (SS) (Diamond, 1965); and

- Second, the "dynamically inefficient" types of financial markets where rational asset bubbles could appear, sustain for as long as the financial market is dynamically inefficient, and bring the economy to its Pareto optimal steady state defined as the "Golden Rule" SS (Tirole, 1985).

Intuitively explained by recalling our discussion in the previous chapter, and this intuition is of crucial importance, the distinction between the two states of the financial markets lies in the systemic disparity between the amount of investible wealth in the economy from the one side, and the availability and return of the productive capital investment opportunities on the other. When the investible wealth and its growth are higher than the productivity of capital investments, then agents have no option but to accept bubbly instruments in order to achieve higher required returns, avoid productive capital over accumulation and suboptimal steady states with lower than optimal long term consumption and utility. In other words, they would prefer investing in any financial instrument that could bring the economy to its Pareto optimal "Golden Rule" steady state (Phelps, 1961). This disparity between the growth of the investable wealth and the available return on productive investments, which creates the environment and a strong incentive for the bubbly assets to occur, could be monotonically achieved through the path of the development of the economy due to the diminishing return of capital investments, but also could be achieved by some fundamental shock, such as the introduction of DC pension funds, which suddenly changes the amount of investable wealth and the savings attitude in the overall economy. One of the more general claims based on this argument is that the introduction of institutional investors in the developed markets, especially the boom of the popularity and the assets under management of the pension fund industry in the OECD countries (Allen, 2001), led to a historically unprecedented rise in the equity asset prices in the last 20 years on their financial markets. This is a much wider topic for discussion focused on the global role of the pension funds to the global financial markets. We will keep our focus on the simpler example of the SEE countries such as Croatia and their illiquid and relatively isolated financial markets.

Abel, Mankiw, Summers and Zeckhauser (Abel et al., 1989), in their critique of the methods used to measure the dynamic efficiency of financial markets, give us an important insight into the criteria that we further use to assess the dynamic efficiency of the Croatian Financial market. Namely, the authors criticise the widespread use of Short-Term Government Interest (STGI) rates, represented by the returns on Treasury Bills, as a misleading benchmark for the return on productive capital in assessing the dynamic efficiency of the financial markets. They suggest that the sum of Dividend and Stock Repurchase (distributions of the corporate sector) as a percentage of the amount of total value of equity is a much better measure of the return of productive capital. Using a data set for the period between 1952 and 1985, they show that the Return on Capital measured in this way leads to that conclusion that the US financial markets were dynamically efficient, as opposed to the STGI rates that suggest the opposite. They also suggest:

> *The competitive return of any other asset can also be useful in determining whether the economy is dynamically efficient or inefficient* (Abel et al., 1989).

The problem with the use of the Equity Repurchase and the Dividend distribution as a percentage of the value of Equity arises due to the volatility of the value of equity. This is issue even strongly present in the models where the market of equity might contain both intrinsic and speculative-bubbly value. Accordingly, we argue that the long-term interest rate on Long- Term Government Bonds is a much better substitute for the criticized Short-Term interest rates as an indicator of the productive capital return rate. Its long-term perspective is a perfect substitute for the long-term perspective of equity investments if such investments are seen as productive capital investments.

This long-term rate of return on Government Bonds will be our key benchmark for determining the dynamic efficiency of the Croatian financial market in our empirical analysis. Moreover, long-term government bonds present one of the key investment benchmarks in the portfolios of the DC pension funds worldwide.

The real return on productive investments, due to the low capital accumulation levels, is high in the emerging economies (Campbell R. H., 1995), but individual representative agents invest relatively little on the financial markets, which might be a sign that they are sufficiently impatient. The fact that most of the savings of representative agents are held in the form of bank deposits, and away from the equity market, supports this assumption.

One other factor characteristic of SEE financial markets is the illiquidity of their young equity markets. Such illiquidity of their equity markets is a consequence of the inelastic supply of equity, which is widely evidenced in the SEE equity markets and confirmed by the low number of Initial Public Offerings (IPOs). Low supply of new equity produces shallow equity markets, on which investors with a desire to buy or sell greater amounts of equity find it difficult to transact on the market without accepting high a illiquidity premium. This by definition classifies their equity markets as illiquid. To model such illiquidity of the equity market in our theoretical OLG structure, we give the theoretically modelled financial market a fixed supply of equity, which may include the characteristics of a bubbly asset corresponding to the lack of new equity emissions on the SEE equity markets. We use the notion of a pure bubbly asset, although its implications could be taken to refer to the potential bubble within the price of the productive investment such as common equity (Tirole, 1982).

Theoretical foundations explained in detail in the previous chapter on asset bubbles theory, put in place the platform for developing our theoretical model and, later on, for the empirical investigation of how DC pension funds induce asset bubbles on the Croatian financial market. Besides the issue of the overall rationality of asset bubbles modelled by the OLG framework, the micro-structural characteristics giving incentives to the occurrence and the initial acceptance of the bubbly instrument, such as the presence of an agency problem within pension funds and the multiplication of this problem when they invest into local open-end equity mutual funds, the extent of informational asymmetry in the SEE markets and the presence of noise traders on the SEE market are important characteristics for our empirical investigation. Especially later in our empirical study of the Croatian experience with the introduction of the DC pension funds, we see that the introduction of local open-end mutual funds as an investment instrument that significantly increase this information asymmetry, could be a great catalyst for the occurrence of asset bubbles systemically developed by the investments of the DC pension funds.

Although those micro-structural characteristics have their importance as stressed by the different schools of the asset bubbles theory, the OLG model presented in the following section is focused only on systemic implications from the introduction of the DC pension funds, and does not include all the micro structural elements in the model, because such complication of the model would not add value to the theoretical investigation of the pure effect of the introduction of the DC pension funds. This does not mean that we should not later investigate the micro-structural characteristics of the financial market, which could give additional explanations for the formation of asset bubbles and the acceptance of certain bubbly investment instruments.

Among those micro-structural characteristics that can affect the occurrence of asset bubbles by stimulating speculative behaviour of DC pension funds is the potential agency problem as discussed by Allen and Gale, which is not specific only to the pension funds but also to other types of institutional investors such as mutual funds (Allen & Gale, 2007). Especially at their onset, pension fund management companies and, especially, more short-sighted open-end mutual fund management companies, could be characterized by a lack of specialized human capital among their investment managers, erroneous asset managers' incentive mechanisms and weak controlling mechanisms, all of which could lead them to overinvest on the domestic illiquid equity markets and accept high risks and bubbly assets due to the significant amount of "risk shifting". This effect of the agency problem could even further multiply if pension funds invest indirectly on the equity market using the local open-end mutual funds[9], because the indirect investment vehicles extend the chain between the principal and the agent and strictly increase the agency problem in the investment process.

As we saw when investigating the implications of asymmetric information on asset bubbles, they present another important characteristic within the market micro-structure potentially leading to a bubbly equilibrium on SEE financial markets. Pension funds could suddenly be perceived as leading market participants and thus as gaining a strong signalling role for the herd of small investors who, in such a context, would react much more aggressively when investing in the bubbly instruments. This characteristic is especially stressed in market specific contexts where there exists a large asymmetry of information on the fundamental value of assets, and where naturally small individual speculators are seeking a leading signalling role from a large institutional investor. So, in this context, when the leading investors' signalling role points to the acceptance of the bubbly asset, it may trigger mass investments by the ignorant speculators.

Finally, the noise trader school on asset bubbles gave arguments about the potential behaviouristic biases among portfolio managers in pension funds, which are especially important for the case of the young SEE financial markets. This could be especially an issue at the early stages of the development of the DC pension fund management companies, due to the lack of knowledgeable and experienced employees who have direct investing responsibilities. As a consequence of the lack of educated investment analysts within the early circles of the professional investment societies in SEE financial markets, such as, for example, internationally recognised chartered financial analysts (CFA Institute charter holders), pension fund managing companies could find themselves lacking in professionals of high prudence, ethics and analytical qualities. This characteristic, from the inception of the DC pension funds in the illiquid SEE financial markets, could create influential institutional investors with significant market power, undertaking investment decisions based on false "misjudging expectations" (DeLong, Shleifer, Summers, & Waldmann, 1990a). Such

[9] Open-end investment funds are institutional investors who collect assets from investors on a voluntary basis and invest them according to a pre-defined investment policy stated in the prospectus of the Fund.

attitudes could easily provoke induction of a wave of bubbly market price dynamics and could play a destabilising role on the financial market.

Although the common OLG models do not distinguish between small and large financial markets, we establish the difference by introducing a constraint on the supply of the bubbly equity, making the equity market shallow which is a common characteristic of small and underdeveloped equity markets. However, other factors are also characteristic of small financial market economies, which can affect the occurrence of asset bubbles. One among those factors can be the sensitivity to foreign capital inflow shocks, which can trigger a jump to the speculative growth steady state of the market economy. This was very strongly evidenced by the empirical data from some small Asian and Latin American emerging economies going through speculative boom-bust financial market cycles (Krugman, 2008). Although we also believe that foreign capital inflow played a significant role, we present a closed market OLG model for reasons of simplicity. In this respect, during the empirical study, the external or global financial market influence is controlled by introducing an external (exogenous) factor variable in the model.

The best way to see the effect of the introduction of DC pension funds on the steady state and the dynamics of the OLG model with rational bubbles is to start with the canonical model developed by Tirole (Tirole, 1985)and then augment this model by introducing the institutional reform of DC pension funds and enforcing a capital saving scheme within the budget constraint of each consumer. The way we do this is by adding an external institution that taxes the wealth of the young generation in the period when they are young. Then it uses this collection of real wealth taken from all young consumers to invest in the bubbly assets by following a predetermined investment rule (in the simplest and most extreme way, investing all taxed real wealth in the bubbly asset).This way, the DC pension fund actually significantly decreases the savings and inter-temporal consumption decision discretion of the individual agents. Later, this investment rule could be actively manipulated, to bring it closer to the active portfolio management strategies introduced by different pension funds or by the regulatory bodies. But the outcome and the insights about the occurrence of the bubbly equilibrium on the financial market, caused as a result of the introduction of the pension funds, would not change by introducing a specific investment rule defined in the strategy of each fund, unless that rule were to correspond to the exact individual saving decisions of each young person in the OLG structure (in which case no first-order difference is made by pension reform). We logically accept that the primary reason for introducing a mandatory DC pension scheme is found in the mismatch between the aggregate outcomes of otherwise individual voluntary pension investment decisions as compared to the investment decision rule introduced by the pension funds. The presence of this mismatch can be seen in the experience of countries that introduced only the voluntary, third pillar of the pension funds reform, such as Serbia, which was able to voluntarily attract only a marginal amount of popularity and assets under management. This implies that there would be an inherent shock in the amount of investible wealth in the market economy caused solely by the introduction of the mandatory DC pension funds, in turn causing a shock on the illiquid financial market with a limited supply of equity.

A further important characteristic of the model is that individual agents are aware of the investment rule applied by the DC pension funds which, in turn, affects their own additional investment decisions with the rest of their available income. Based on their knowledge they behave rationally, which means that this knowledge will lead them to anticipate the new equilibrium returns of the bubbly asset and rationally adapt their financial market behaviour. Through the no-arbitrage condition of their portfolio decision when saving and investing, individual agents will adopt investments in the bubbly asset as well. As previously mentioned, we won't shift from the rational expectations assumption of Tirole's model, although the presence of noise traders among individual agents could easily be assumed. Such noise traders could be modelled in a similar way as in the DSSW models, by introducing some misjudgement stochastic variable with a predefined distribution, adding some random erroneous value to individual agents' expected return and investment decision functions. We believe that the introduction of such a feature could only intensify the direction of the formation of asset bubble and market volatility, and the simpler OLG structure used in Tirole's work without additional micro-structural complications would give sufficient insights into the pure effects of the introduction of DC pension funds into the Croatian financial market as a representative of the SEE markets.

By formally solving the model in this chapter, we will show that one of the largest effects from the introduction of the mandatory defined contribution pension funds is shifting the financial market to a higher absolute productive capital accumulation state at which bubbles could boom in the financial market more intensively. Depending on the dynamic efficiency of the economy, this inflation to the higher bubbly valuations could be permanent (stable) or only short lived.

This points toward a conclusion about the main hypothesis of this dissertation; namely, that the introduction of a DC pension fund institution transferring part of the wealth of the young consumers to the illiquid bubbly asset market will induce a higher bubbly steady state of the economy, assuming that the financial market is or becomes dynamically inefficient. In the following section, we present our model of productive saving and rational bubbles in the presence of a mandatory DC pension fund.

2.2. The Model

We define an elementary two-stage OLG model market economy similar to the basic Samuelson's model (complete model solution derivations available in Appendix II), where at the initial period t=1, there are N_0 initial old agents, and $N_1=(1+n)N_0$ first generation young agents. The population of young agents N_t grows at exogenous fixed rate n, which represents the main driver of the growth of the economy in this model and consequently of the investible assets. Later in the empirical model specification, this growth rate of the investible assets is indicated by the growth rate of the new subscribers in the DC pension schemes.

Agents when they are young are endowed with one unit of labour and nothing when they are old, and they have complete knowledge about their actual and future working ability. They must engage in employment when they are young and earn their lifetime salary to be able to derive a positive utility in the two periods which, at the minimum, determines their survival. All agents have a homogenous von Neumann - Morgenstern (vNM) utility function, which provides for them to utilize consumption in both periods. The novelty in the model is that part of agents wage earned when they are young is "taxed" by the introduced DC pension fund, which simultaneously invests the whole taxed amount for their account in the bubbly asset according to a rule. Consumer agents on the other side invest both in the bubbly or/and in the productive asset. The fact that the pension fund invests only in the bubbly asset is made for simplicity. In reality, the pension funds invest in equity or open-end mutual funds, whose investment price consists of both its intrinsic value and its bubbly value (Tirole, 1982). Using the pure bubbly asset in the model gives better insights on the dynamics of an overall bubble in the financial market, although similar effects potentially arise from either a pure bubbly asset or a bubbly part of the valuation of some asset with non-zero intrinsic value such as equity or an equity related open-end mutual fund.

What is characteristic to the DC type of pension scheme, which is introduced in this model, is that the retirement benefits it provides are not predefined and are fully dependant on the market outcome. This increases the agency problem called "risk shifting", as compared to the "Defined Benefit" pension funds where the pension fund management company has a liability for a certain amount of promised asset return. In the case of Defined Contribution pension funds, because the pension fund management company has only a very limited risk of its own private loss if their investments turn out worthless, this makes the defined contribution pension funds a more aggressive risk taker and on average faster to accept bubbly instruments into their portfolio.

Returning to the description of the model and the division of agents' first period salary, the rest of it left after subtracting the contribution given to the defined contribution pension fund is the amount of income available for consumption and/or personal voluntary investment by the young agents in the period when they are young. Based on their personal inter-temporal consumption preferences characterised by the implicit utility function (being patient or impatient consumers) and the knowledge of the investment rule applied by the pension fund which covers them (affecting in return their own lifetime budget constraint), individual agents individually decide how to invest and/or consume the rest of their salary. Their individually chosen investments are realized through investing in capital (productive saving) and/or bubbly asset (non-productive saving), comparing and deciding between the two investment opportunities according to the non-arbitrage condition.

Markets for labour, productive capital and the bubbly asset are perfectly competitive in this model. In the next period, when the initial generation of agents becomes old, the pension fund sells the number of previously invested units of the bubbly asset and transfers this amount as pension compensation to the old agents split according to the share of their initial contribution. In this model all young participants live equally long and survive to consume their mandatory pension fund savings together with their individual savings in the second period when old.

There is also production in this model undertaken by rational entrepreneurs, represented by the constant returns of scale (CRS) production function implicitly defined as $F(K_t,L_t)$.Constant returns of scale implies that, $F_k(k=0)=\infty$ and $F_k(k=\infty)=0$, hence diminishing returns to capital in the economy. This also implies that as the growth of investments in productive capital pushed by the increase of the amount of investable wealth in the economy increases, the profitability of the investments decreases. In the case of the dynamically inefficient economy, this opens space for financial innovation and introduction of bubbly assets keeping the rates of return in the economy at a higher level and avoiding suboptimal productive capital over-accumulation. This assumption is the crucial one, which creates the incentive for introduction of bubbly instruments. Otherwise, without diminishing return on productive investments, bubbly assets would be continuously crowded out by productive investments. This intuitively implies that real sector innovation, keeping productivity rates higher compared to the rate of growth rate of investible assets, would be a permanent alternative to the second best outcome with rational bubbles in place. Producers/Entrepreneurs who govern production are fully rational, so they make decisions on the margin when they participate in the market looking to employ production resources provided by the young consumers.

We will operate in per-young agent terms for all variables in the model. We standardise the production function and all economy/market variables by dividing them by the number of young agents, which equals L_t (Labour in the economy at time t), since each young agent is endowed with only one unit of labour when young.

There are two types of investment assets available for transferring wealth from one period to another.

1) **Productive asset - physical capital,** whose return - solving the producers' first order condition(FOC) for capital per young agent - is defined by:

$$R_{t+1} = f^{'}(k_{t+1}) + 1 - \delta \qquad (2.1)$$

Here k_t stands for the amount of per young agent capital at time t, while δ is the yearly depreciation rate of the capital invested. The total rate of return of capital (RHS in equation 2.1) equals the marginal productivity of capital minus its depreciation rate (LHS in equation 2.1). This is the well-known marginality principle in micro-economics. This represents a small departure from Tirole's result, who did not account for the depreciation of capital now common in OLG structures. Its inclusion does not affect Tirole's result, but brings the economy closer to its real functioning. This may be particularly relevant to economies in transition, such as those in SEE. Introduction of such depreciation cost can be used to apply "transition" costs on capital, which might significantly decrease the expected return on equity and add more pressure on the dynamic efficiency of the equity market.

2) **Bubbly asset – asset which in the fashion of Tirole is intrinsically worthless**

In our model, following the Tirole Rational Asset Bubbles model, we also introduce the second investment vehicle, the pure bubbly asset (b_t). Due to the introduction of the DC pension funds which invest in the bubbly asset, the equation describing the return of the bubbly asset will differ in our model from the equation describing the return of the bubbly asset defined in Tirole's model. We will compare the two after we derive the new equation.

We begin the derivation of the dynamics of the bubbly asset by defining the value of the bubble per young agent b_t (LHS in equation 2.3.1) equal to our fixed supply of units of the bubbly asset "B" multiplied to the price of the unit of the bubbly asset $\prod_t$ and divided by the total number of young agents N_t(RHS in equation 2.3.1):

$$b_t = \frac{\{B\}\Pi_t}{N_t} \tag{2.3.1}$$

Now we introduce the DC pension fund investments to the market of the bubbly asset impacting the residual supply of the units of the bubbly asset in the RHS of equation 2.3.1. This is done by considering the net effect of the investment rule of the DC pension fund at each time period. The investment rule of the DC pension fund operates in the following way. The DC pension fund invests/buys a certain amount of units of the bubbly asset which equals to the amount of the tax gathered from the total number of young agents (τ*N_t) divided by the price of the bubbly asset in the current period $\prod_t$ (equation 2.3.1a)

$$Pf\,(buy) = \frac{\tau}{\Pi_t} N_t \qquad (2.3.1a)$$

Simultaneously, the DC pension fund also sells a number of units of the bubbly asset to provide pension to a number of agents N_{t-1} who were young in the previous period when they were taxed by the pension fund, and who are now old in the current period and should receive pension income. The number of units of the bubbly assets that has to be sold equals the tax of the DC pension fund multiplied by the number of old agents and divided by the price of the bubbly asset from the previous period $\prod_{t-1}$when this investment was made(equation 2.3.1b)

$$Pf\,(sell) = \frac{\tau}{\Pi_{t-1}} N_{t-1} \qquad (2.3.1b)$$

When we subtract and add the investments made by the DC pension fund from the total supply of the bubbly asset B in equation 2.3.1, introducing the net investment impact of the DC pension fund, we derive equation 2.3.2.This represents the value of the bubbly asset at time t, including the net effect of the DC pension funds investment rule.

$$b_t = \frac{\left\{ B - \frac{\tau}{\Pi_t} N_t + \frac{\tau}{\Pi_{t-1}} N_{t-1} \right\} \Pi_t}{N_t} \qquad (2.3.2)$$

In the brackets on the RHS of the equation 2.3.2, we introduce the mechanics of the investment of the DC pension fund on the market with the bubbly asset affecting the constant supply of the bubbly asset “B”. Applying the dynamics of the population growth “n”, knowing that “N_t=(1+n)N_{t-1}“we substitute for N_t in equation 2.3.2 to get to equation 2.3.3.

$$b_t = \frac{\left\{ B + \frac{\tau}{\Pi_{t-1}} N_{t-1} - \frac{\tau}{\Pi_t} N_{t-1}(1+n) \right\} \Pi_t}{N_t}$$

$$b_t = \frac{B\Pi_t + \tau \frac{\Pi_t}{\Pi_{t-1}} N_{t-1} - \tau N_{t-1}(1+n)}{N_t} \qquad (2.3.3)$$

We still need to apply the impact of the consumer agent investment solution in order to establish the clearance of the market of the bubbly asset.

Now we apply the impact of the consumer agent's investment decisions in order to reach the clearing condition of the market with the bubbly asset. In order to introduce their impact, we define their inter-temporal consumer problem, solve it and apply the result of their inter-temporal optimisation of consumption and saving using both productive and bubbly assets as investment vehicles. This yields the Non-Arbitrage condition, which connects and simultaneously clears the markets of the two investment vehicles in our financial market. The agents' inter-temporal utility problem together with its first order conditions of the utility maximisation solution are presented in the equations 2.3.4.

$$\max U\{c_{1t}, c_{2t+1}\} s.t. \left\{ \begin{array}{c} w_t = c_{1t} + \tau_t + k_{t+1} + b_t \\ R_{t+1}k_{t+1} + \frac{\Pi_{t+1}}{\Pi_t}[b_t + \tau_t] = c_{2t+1} \end{array} \right\}$$

$$\max \Lambda = U(c_{1t}, c_{2t+1}) + \lambda_1 [w_t - c_{1t} - \tau_t - k_{t+1} - b_t] + \lambda_2 \left[R_{t+1}k_{t+1} + \frac{\Pi_{t+1}}{\Pi_t}[b_t + \tau_t] - c_{2t+1} \right]$$

$$\begin{aligned} & c_{1t} : U_1(.) = \lambda_1 \\ & c_{2t+1} : U_2(.) = \lambda_2 \\ & k_{t+1} : -\lambda_1 + \lambda_2 R_{t+1} = 0 \\ & b_t : -\lambda_1 + \lambda_2 \frac{\Pi_{t+1}}{\Pi_t} = 0 \end{aligned} \qquad (2.3.4)$$

Finally, the investment solution of the consumer agent is derived from the first order conditions of his/her inter-temporal utility maximisation. This result is called the Non-Arbitrage condition, which we present in equation 2.3.5 and shows that in the interior solution, when both investment vehicles are used on the financial market to transfer wealth from one period to the other, market clearing conditions should simultaneously drive the returns of the both investment assets to be equal.

$$R_{t+1} = \frac{\Pi_{t+1}}{\Pi_t} \tag{2.3.5}$$

Finally, we apply the Non-Arbitrage condition (equation 2.3.5), and thereby the impact of the rational consumers, to the bubbly asset market clearing condition represented in equation 2.3.3. We further solve and we obtain equation 2.3.6.

$$b_t = \frac{B\Pi_t + \tau R_t N_{t-1} - \tau N_{t-1}(1+n)}{N_t}$$

$$b_t = \frac{B\Pi_t}{N_t} + \left\{ \frac{\tau N_{t-1}\left(R_t - (1+n)\right)}{N_{t-1}(1+n)} \right\}$$

$$b_t = \frac{B\Pi_t}{N_t} + \left\{ \frac{\tau\left(R_t - (1+n)\right)}{(1+n)} \right\} \tag{2.3.6}$$

In order to derive the Equilibrium Law of Motion of the bubbly asset we first use the equation 2.3.6 describing the market value of the bubbly asset per young agent at period t and, describe a system of two consecutive periods of market value of the bubbly asset at periods t and t+1 as shown in equation 2.3.7

$$\begin{cases} b_t = \dfrac{B\Pi_t}{N_t} + \tau \dfrac{\left[R_t - (1+n)\right]}{1+n} \\ b_{t+1} = \dfrac{B\Pi_{t+1}}{N_{t+1}} + \tau \dfrac{\left[R_{t+1} - (1+n)\right]}{1+n} \end{cases}$$

And, after manipulation:

$$\begin{cases} \Pi_t = \dfrac{N_t}{B}\left\{ b_t - \tau \dfrac{\left[R_t - (1+n)\right]}{1+n} \right\} \\ \Pi_{t+1} = \dfrac{N_t(1+n)}{B}\left\{ b_{t+1} - \tau \dfrac{\left[R_{t+1} - (1+n)\right]}{1+n} \right\} \end{cases} \tag{2.3.7}$$

Dividing the two equations(t+1 by t) presented by the system in 2.3.7, and substituting the market clearing condition represented by the consumer's solution of the non-arbitrage condition, we obtain the Equilibrium Law of Motion (ELM) of the bubbly asset presented with the rate of return on the LHS in equation 2.3.8.

$$\frac{\Pi_{t+1}}{\Pi_t} = (1+n)\left\{\frac{b_{t+1} - \tau\frac{\left[R_{t+1} - (1+n)\right]}{1+n}}{b_t - \tau\frac{\left[R_t - (1+n)\right]}{1+n}}\right\}$$

, use the Non-arbitrage condition $\frac{\Pi_{t+1}}{\Pi_t} = R_{t+1}$

$$R_{t+1} = (1+n)\left\{\frac{b_{t+1} - \tau\frac{\left[R_{t+1} - (1+n)\right]}{1+n}}{b_t - \tau\frac{\left[R_t - (1+n)\right]}{1+n}}\right\}$$

$$R_{t+1} = (1+n)\left\{\frac{b_{t+1}(1+n) - \tau\left[R_{t+1} - (1+n)\right]}{b_t(1+n) - \tau\left[R_t - (1+n)\right]}\right\} \qquad (2.3.8)$$

Comparing equation 2.3.8, with Tirole's dynamic market equilibrium solution of the bubbly asset presented in equation 2.3.9, we see the difference implied by the introduction of the DC pension funds. In a hypothetical case when we exclude the DC pension fund from the system by setting $\tau=0$ in equation 2.3.8, we reduce our ELM in equation 2.3.8 to the solution to the Tirole's dynamic market equilibrium for the bubbly asset presented in equation 2.3.9.

$$R_{t+1} = (1+n)\frac{b_{t+1}}{b_t} \qquad (2.3.9)$$

Finally, we substitute in equation 2.3.8 the return of the productive $f'(k_{t+1}) - 1 - \delta$ investments in order to connect the market evolution of the two investment assets, which is required by the non-arbitrage condition. The return of the productive asset is derived from the producer's first order condition as presented in equation 2.1. Applying some additional algebraic transformations to equation 2.3.8(details in Appendix II) we obtain the first difference equation of the value of the bubbly asset representing its Equilibrium Law of Motion (equation 2.4)

$$b_{t+1} = \frac{\left[f'(k_{t+1}) + 1 - \delta\right]\left[b_t(1+n) + \tau\left[\left(f'(k_t) + 1 - \delta\right) - (1+n)\right]\right]}{(1+n)^2} + \frac{\tau\left[\left(f'(k_{t+1}) + 1 - \delta - (1+n)\right]\right.}{1+n} \qquad (2.4)$$

This is our first dynamic first order difference equation representing the equilibrium law of motion (ELM) of the value of the bubbly asset per young agent (b_t) in our OLG modelled economy (equation 2.4). For the ease of comparison, we reproduce Tirole's bubbly asset ELM

$$b_{t+1} = \frac{(f^{'}(k_{t+1})+1)b_t}{(1+n)} \quad \text{- Tirole's bubbly asset ELM} \qquad (2.5)$$

Comparing our modelled ELM for the bubbly asset (equation 2.4) with the result obtained by Tirole(1985) (equation 2.5), we come to the conclusion that the introduction of a positive defined contribution pension fund tax "τ" invested on the bubbly asset market significantly impacts the evolution of the bubbly asset. Whenever the return of the productive asset is greater than the growth rate of the population (investible assets), the term in $f'(k_{t+1})+1-\delta>1+n$ the brackets multiplied by "τ" on the RHS of equation 2.4 will be positive, causing more rapid positive evolution of the value of the bubbly asset with every positive DC pension fund investments in the bubbly asset. The economic mechanism proceeds from the bubbly asset to real investment via the non-arbitrage condition: a real rate of return greater than the rate of population growth induces investment in productive assets together with – via the non-arbitrage condition – investment in the bubbly asset. The introduction of the defined contribution pension fund brings a faster dynamic to the value of the bubbly asset toward its equilibrium. This implies the following hypothesis:

Hypothesis 1: F*aster and more aggressive rise in the value of the bubbly asset could be expected with the introduction of the DC pension fund on the financial market.*

Now in order to derive the second dynamic equation describing the equilibrium law of motion of the productive capital per young agent (k_t), we turn to the clearing condition of the savings market in our modelled economy. The savings market embraces all consumer agents with their own investment decision.

$$N_t S_t(w_t(k_t)-\tau;\tau; f^{'}(k_t)+1-\delta) = K_{t+1} + b_t N_t \qquad (2.6)$$

On the left hand side of the equation (2.6) is the demand for savings of all agents in the economy, determined by the number of young agents' at time t (N_t), multiplied by their individual implicit savings function S(.). Their individual savings function S(.), as an implicit solution of their consumer problem FOC defined at (2.3.4), is dependent on their income in the first period (from which the DC pension fund subtracts an amount "τ" to achieve the individual pension investment), their income in the second period (when the pension fund adds to their income the amount subtracted in the first period and multiplied by its investment return) and the third component of the saving function is rate of return on savings. We assume the implicit savings function of every agent is "normal", which means that its partial derivative with respect to the wealth in the first period is positive – meaning that if having more wealth when young, agents will strictly save more; its partial derivative with respect to wealth in the second period is negative – meaning that if agents know they will receive higher wealth when old, they strictly save less when they are young; and the partial derivative with respect to the interest rates is positive – meaning that higher interest rate makes strictly positive effect on savings:

$$S_1'(.) > 0; S_2'(.) < 0; S_r'(.) > 0$$

In other words, this so called "normality" of the savings function presupposes that rational young agents with more expected income in the future period would have less incentive to save today, but as the today's income and expected return from saving increase, agents would have incentive to add more to their saving to transfer some of their consumption in the later period.

Going back to the savings market, the right hand side of the market clearing equation (2.6) represents the supply of savings instruments available to the consumer agents in the economy. It consists of the sum of the productive capital and the sum of the bubbly instrument, which are the instruments that could satisfy the savings needs of the consumer agents. Here K_{t+1}, represents the economy's productive capital investment opportunities at time t which turn into capital in t+1, while (b_tN_t), represents the total market value of the bubbly asset at time t. Their sum gives the total savings instrument vehicles supply at the savings market at time t in our modelled financial market. Dividing the savings market clearing condition (equation

2.6) by the number of young agents N_t from the both sides in order to represent the dynamics of the productive capital and the bubbly asset in per young agent terms, we get the equilibrium law of motion for the productive capital represented in equation 2.7. The savings function (the first component in the numerator on the RHS of equation 2.7) is represented in its implicit form with its components in the brackets: the explicit definition of the wealth in the period when young; the wealth in the period when old; and the rate of return from saving.

$$k_{t+1} = \frac{S_t(w_t(k_t) - \tau; \tau; f'(k_t) + 1 - \delta) - b_t}{1+n} \tag{2.7}$$

Equation (2.7) describes the evolution of the value of productive capital per young agent k_{t+1}, which increases with the increased savings demand S(.) but decreases with the growth of the bubbly asset b_t, as seen from the RHS of equation 2.7.This means that the bubbly asset crowds out part of the productive capital investment. Savings demand tends to increase at a decreasing rate, due to the diminishing productivity of capital which, in turn, decreases the return of capital. The bubbly asset prevents the suboptimal productive capital over-accumulation by crowding out part of the investments in productive capital.

Thus far we have derived the two first order difference equations describing the dynamics of the financial market in the fashion of Tirole’s settings augmented by a DC pension scheme. If we recall the result derived by Tirole (equation 1.10 in Chapter 1), we notice that if we cancel the DC pension scheme in our model by making $\tau = 0$, then we get the same dynamic first order difference equation for the per young agent productive capital (k_t) and per young agent bubbly asset (b_t) as in Tirole's result (Tirole, 1985, p. equation 16' and 16") as shown below:

Our economy’s two ELM equations with DC pension scheme:

$$b_{t+1} = \frac{[f'(k_{t+1}) + 1 - \delta][b_t(1+n) + \tau[(f'(k_t) + 1 - \delta) - (1+n)]]}{(1+n)^2} + \frac{\tau[(f'(k_{t+1}) + 1 - \delta - (1+n)]}{1+n}$$

$$k_{t+1} = \frac{S_t(w_t(k_t) - \tau; \tau; f'(k_t) + 1 - \delta) - b_t}{1+n}$$

Setting the DC pension scheme “tax” τ=0, which means cancelling the pension scheme, we obtain Tirole's result (Tirole, 1985):

$$b_{t+1} = \frac{(f^{'}(k_{t+1})+1)b_t}{(1+n)}$$

$$k_{t+1} = \frac{S_t(w_t(k_t); f^{'}(k_t)+1) - b_t}{1+n}$$

This implies our second hypothesis:

Hypothesis 2: *Manipulating the value of τ, which could be in the hands of the law setting body responsible for the introduction of the DC pension fund in the system, affects the intensity of the effect of the DC pension fund on the financial market and on the dynamics of the productive and bubbly investment assets in the economy.*

In the extreme, setting τ=0, would mean cancelling out the influence of the DC pension scheme on the financial market, obtaining the equilibrium demonstrated by Tirole (1985). Conversely, if we introduce a tax “τ” close to the whole amount of young agents’ wage, then the regulator would account completely for the dynamics of the productive and bubbly investment asset in such an economy.

In order to understand the impact of the introduction of the DC pension funds on the dynamics of the financial market, described by the two equilibrium laws of motion for value of the productive capital and the value of the bubbly asset, we will continue with Phase diagram analysis. The impact of the introduction of the DC pension fund on the shapes of the two steady states loci describing the steady state points of the ELM of the bubbly asset and of the productive capital have a crucial importance on the difference in the dynamics of the system affected by the introduction of the DC pension funds.

2.3 Phase diagram analysis

In order to analyse the dynamics of the financial market governed by the two equilibrium laws of motion describing the dynamics of the value of productive capital and the value of the bubbly asset, we use Phase diagram representation. Phase diagrams are graphical presentations of the dynamics of a system of two or more dynamic variables interacting among each other, represented in two or more dimensional space. Phase diagrams graph the dynamics from every starting state of the system, described by the two (or more) dynamic variables and picture its dynamic path to some steady state point governed by the temporal evolution of the two (or more) equilibrium laws of motion. In our case, we have two dynamic variables, which are represented by the ELM of the value of productive capital instrument and the ELM of the value of the bubbly asset.

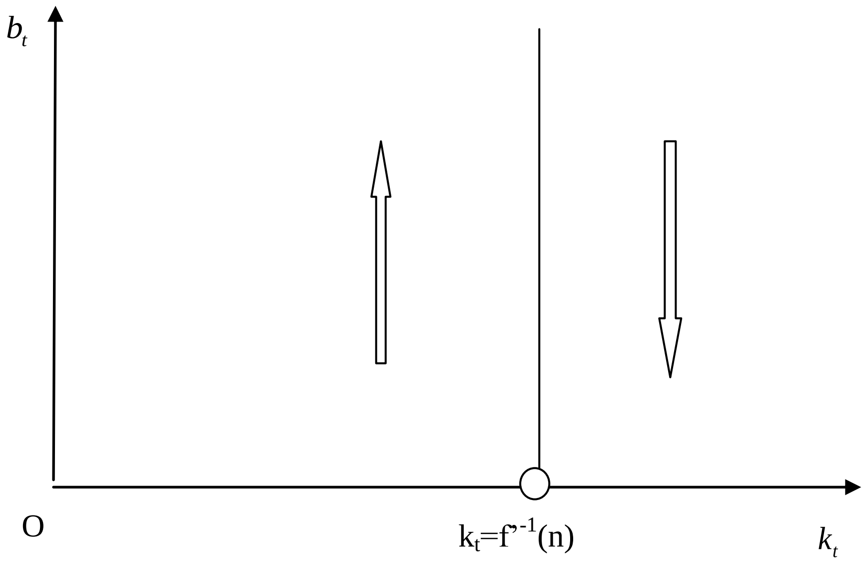
By the use of a Phase diagram, we gain more intuitive understanding about the implied impact of the introduction of a DC pension fund into the financial market with respect to the joint dynamics of productive capital investments and of the bubbly asset. This would be accomplished by analysing its joint impact on the evolution of the financial market through the analysis of the Phase diagram simultaneously analysing the two equilibrium laws of motion.

We first begin in Figure 2.1 by graphing the locus of SS points for the bubbly asset equilibrium law of motion, finding its locus of steady state points and thus identifying the dynamics in the environment close to the locus of steady state points. For this purpose, we work with the already derived ELM function for the bubbly asset (equation 2.4):

$$b_{t+1} = \frac{[f'(k_{t+1}) + 1 - \delta][b_t(1+n) + \tau[(f'(k_t) + 1 - \delta) - (1+n)]]}{(1+n)^2} + \frac{\tau[(f'(k_{t+1}) + 1 - \delta - (1+n)]}{1+n}$$

The locus of steady state points is a set of points where the dynamic variable has no dynamics. This means that on the locus of SS points (b_t=b_{t+1}), meaning that, ceteris paribus, there will be no further dynamics in the value of the bubbly asset per young agent once the SS locus is reached from any direction. In our model of the financial market, it will be represented by some continuous line on the Phase diagram (Figure 2.1). Analysing the Phase diagram for the bubbly asset ELM (Figure 2.1), we notice that there is no trivial SS for the bubbly asset, where $b_{t+1} = b_t = 0$, as in Tirole's model (Tirole, 1985). This is because of the existence of the pension scheme, which accepts the bubbly asset in its portfolio and precludes the corner solution as long as the pension fund is in place and the rule $\tau > 0$ strictly holds. There are indefinitely many non-trivial solutions for the steady state of the bubbly asset ELM, represented by the vertical line on the graph (Figure 2.1). They occur only when the return of capital decreased by its depreciation rate equals the growth rate of the economy or its investable assets (population growth rate) $(1+n=1+f'(k_t)-\delta)$.(The growth rate of the population will be replaced by the growth rate of the pension fund subscribers in the later empirical study, since it directly affects the growth rate of economy's financial market investable wealth.) The result for the locus of steady state points for the bubbly asset is the same as the one present in Tirole's model except for the exclusion of the zero point, which is excluded by construction because the introduced DC pension fund invests a non-zero amount in the bubbly asset. This means that the main criterion for the rationality of asset bubbles on the financial market, the dynamic efficiency threshold, does not change with the introduction of the DC pension funds. The criterion stays the same and is defined by the relationship between the return of the productive capital and the rate of growth of the economy (defining the rate of growth of the investible assets).

Figure 2.1: Phase diagram representation for the ELM of the bubbly asset (b_t) and is SS locus



b_t – represents the value of the bubble in per young agent terms
k_t – represents the value of the capital in per young agent terms
kt=$f'^{-1}(n)$ – the inverse of the marginal product of capital at the rate of return equal to the growth of the population, derived as follows: $f'(k)=r$ => take inverse =>$f'^{-1}(r)=k$ and, as our condition is r=n, it follows that $f'^{-1}(n)=k$.
Source: Author's own graphical representation of the bubbly asset ELM

The intuition of the Phase diagram of the bubbly asset ELM is given by the arrows on the left and on the right side of the vertical steady state locus of the bubbly asset ELM. If the economy and its financial market is found on the left side of the SS locus, then due to the lower productive capital accumulation and its consecutive higher marginal productivity rate, and due to the non-arbitrage condition, the ELM of the bubbly asset would produce a growing upward force on the value of the bubbly asset per young agent ($b_{t+1}>b_t$). This could be seen by looking at ELM of the value of the bubbly asset presented in equation 2.4. When the system is positioned left from the vertical line in Figure 2.1 denoted by $1+n<1+f'(k_t)-\delta$, the term $\frac{f'(k_{t+1})+1-\delta}{1+n}$ multiplying b_t on the RHS of the equation 2.4 is greater than one producing increasing dynamics of the value of the bubbly asset b_{t+1} . This is why the arrow on the left is pointed upwards. Conversely, to the right of the locus of steady state points, when we have higher states of productive capital accumulation per young agent, and lower marginal productivity of capital, due to the non-arbitrage condition we would have a falling value of the bubbly asset per young agent ($b_{t+1}<b_t$). In this opposite case right from the vertical line in Figure 2.1 we have $1+n>1+f'(k_t)-\delta$ that makes the term multiplying b_t on the RHS of the equation 2.4 smaller than one producing decreasing dynamics of b_{t+1}. This is why the arrow representing the dynamics of the bubbly asset per young agent on the right side of the locus of steady state points downward. However, the economy and its financial market are not governed solely by the bubbly asset per young agent but also by the dynamics of the productive capital accumulation per young agent. Consequently, we need to also graph the locus of steady state points for the previously derived first order difference equation representing the evolution of the accumulation of productive capital per young agent in the economy. Due to the inclusion of the implicit savings function in the ELM representing the accumulation of productive capital per young agent (k_t), the locus of its steady state points is not a straight line. We recall the previously derived productive capital ELM (equation 2.7) in the following line:

$$k_{t+1} = \frac{S_t(w_t(k_t)-\tau;\tau;f'(k_t)+1-\delta)-b_t}{1+n}$$

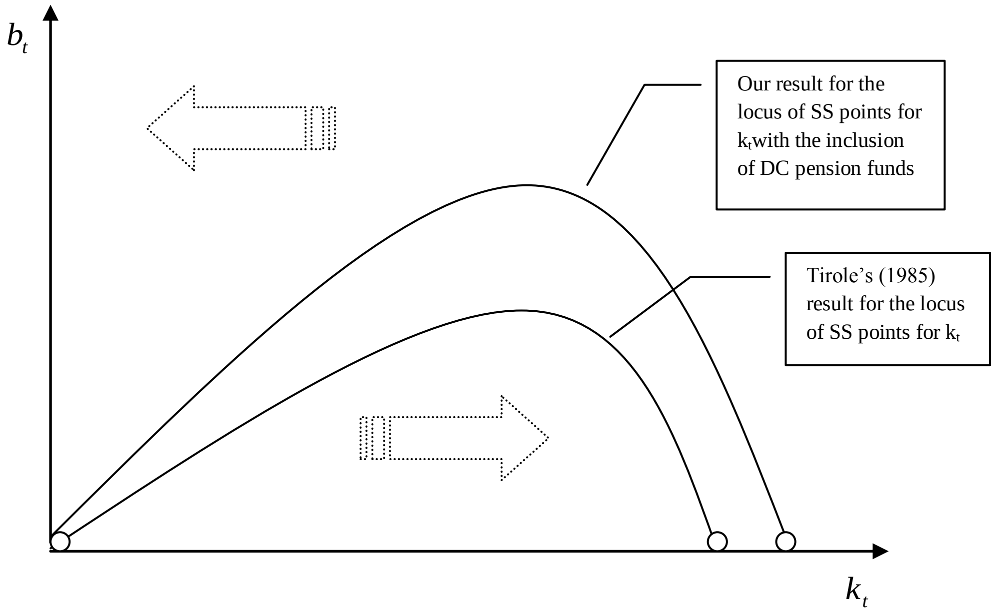
We take a heuristic approach when drawing the locus of steady state points for the level of productive capital per young agent. This means that we could find only the intercept points on the x-axis, when the value of the bubbly asset is zero, and the shape of the curvature which is based on the assumed normality of the savings function and the non-arbitrage condition. In Tirole's model, which uses the same "normal" savings function, there is one trivial steady state point, where the value of capital accumulation equals zero. This point implies that the economy is not existent, since there cannot be any production without any capital accumulation. Moreover, this point is impossible in our model with DC pension funds, due to the assumption that the pension fund immediately accepts the bubbly asset in its portfolio and the non-arbitrage condition has to hold at all times, which implies immediate productive capital accumulation in the individual consumer agents' portfolios. Consequently, we begin depicting the locus of steady state points for the productive capital accumulation immediately after the origin excluding the origin point. Next, we determine the direction of the SS locus by analysing the direction of the productive capital accumulation close to the inception of the economy when there is very little capital invested. At this point, the marginal productivity of capital is extremely high- in fact it tends to infinity - as we are close to zero capital per young agent, and so accumulation increases rapidly. Due to the non-arbitrage condition, this means that the bubbly asset must be accumulating in value at the same speed in order to provide the same return as the productive capital in the interior solution. So the locus of steady state points for the productive capital per young agent increases in the positive direction for both k_t and b_t.

At this point it is important to distinguish between the rates of return of the productive capital established when the financial market is in dynamic equilibrium, and the rates of return of the productive capital along its path to the financial market dynamic equilibrium. Along the path of productive capital accumulation, it is normal for the financial markets to pass through the phase of higher interest rates compared to the rate of growth of investable assets. In this phase of productive capital accumulation, bubbly assets might occur and rise in value together with the accumulation of productive capital. However, the bubbly assets will not be sustainable on the market if the productive capital continues providing competitive rates of return – i.e. higher than the rate of growth of the wealth that has to be transferred from one period to another on the financial market to keep pace with population growth – which defines the financial market as being dynamically efficient. In this case bubbly assets will disappear, being crowded out by the productive asset, and the financial market will stay dynamically efficient without the presence of bubbly assets. On the other hand, when the return of the productive assets tends to decline below the rate of growth of the investible assets, then the bubbly asset would sustain on the financial market. In this case, the presence of the bubbly asset will prevent the occurrence of productive capital over-accumulation, which is a Pareto inferior state compared to the one with the presence of the bubbly asset on the financial market and lower productive capital accumulation. It is assumed in this model that financial markets move rapidly towards their dynamic equilibrium point.

The line of the steady state locus for the productive capital per young agent has a decreasing slope (the second derivative of the slope is negative), due to the diminishing rate of return of capital and the "normality" of the savings function (decreasing investment return causing decreasing savings). So, finally, the SS line hits the x-axis again at the "Diamond" steady state, where bubbly assets per young agent are fully crowded out by productive capital accumulation. In Figure 2.2, we represent the locus of steady state points for the productive capital, comparing Tirole's result with the one that we get including the DC pension funds in the economy. The two biggest differences are the shape of the locus of steady state points and the level of capital accumulation representing the Diamond steady state. The immediate impact of the introduction of the DC pension fund, especially in an economy where agents are not investing a great part of their incomes, which implies that impatient consumers dominate the economy, is to crowd out a significant part of the productive capital investments and add to investment in the bubbly asset. In Figure 2.2, this immediate impact is represented by a higher value of the bubbly asset per young agent (bt) and a lower value of productive capital per young agent (kt). This translates into a higher sloped steady state locus of the productive capital accumulation in the economy, where for every steady state for the productive capital accumulation, as compared to the state of the economy without the existence of the pension funds, a higher level of bubbly asset accumulation in the economy is required when the DC pension fund is introduced.

The introduction of the DC pension fund in effect is the institutional capturing of a significant part of the discretionary representative agent inter-temporal consumption decision. This means that, as the pension scheme captures higher amounts of agents' income and invests it by its own decision rule, it lowers the impact of the individual representative agent's inter-temporal consumption preferences on the economy's savings and investment outcomes. More practical understanding would be to imagine an economy where all agents would prefer to spend most of their income today, but the introduction of the pension scheme captures most of their current income and invests it. This way the institution commands them to change their inter-temporal consumption attitude from being extremely impatient to being extremely patient. This causes a shift of the steady state locus for productive capital accumulation pushing the Diamond steady state point further to the right making the economy potentially more dynamically inefficient or bubbly as shown in the previous chapter (Figure 1.2.6). This also implies that the bubbly asset could impact the system more strongly than the capital accumulation due to its acceptance by the DC pension fund (Figure 2.2).

Figure 2.2: Phase diagram representation for the ELM of productive capital (k_t) and its SS locus



*the steady state loci are a heuristic presentation to demonstrate the relative positions of the curves, whereas the relative position of the peaks is not explicitly determined within the model (because of the use of an implicit savings function).
Source: Author's own graphical representation of the productive capital per young agent ELM

Analysing Figure 2.2, we can see that if the financial market is below the locus of steady state points for the productive capital accumulation per young agent ELM (k_t), this means that the lower level of the value of the bubbly asset creates a lower drag on the accumulation of the productive capital (it crowds out less from the productive capital). This translates into more productive capital investments, which drives the dynamics of the system below the productive capital SS locus to the right. The opposite holds above the locus of the steady state points of the productive capital accumulation, where the accumulation of the bubbly asset is higher and crowds out higher amount of the productive capital investment that decreases the amount of productive capital. This effect is due to the presence of the non-arbitrage condition, and this is the reason why the arrow showing the dynamics of the productive capital accumulation above the locus of SS points to the left in Figure 2.2. The most important insight for our analysis at this point is the change of the shape of the locus of steady state points for the productive capital accumulation (k_t), created due to the introduction of DC pension funds in the system. If we observe the productive capital ELM in equation 2.7, and determine the steady state locus for the productive capital accumulation as the line where $k_{t+1}=k_t=k$, we describe the locus by the following equation:

$$k = \frac{S_t(w_t(k)-\tau;\tau;f^{'}(k)+1-\delta)-b_t}{1+n}$$

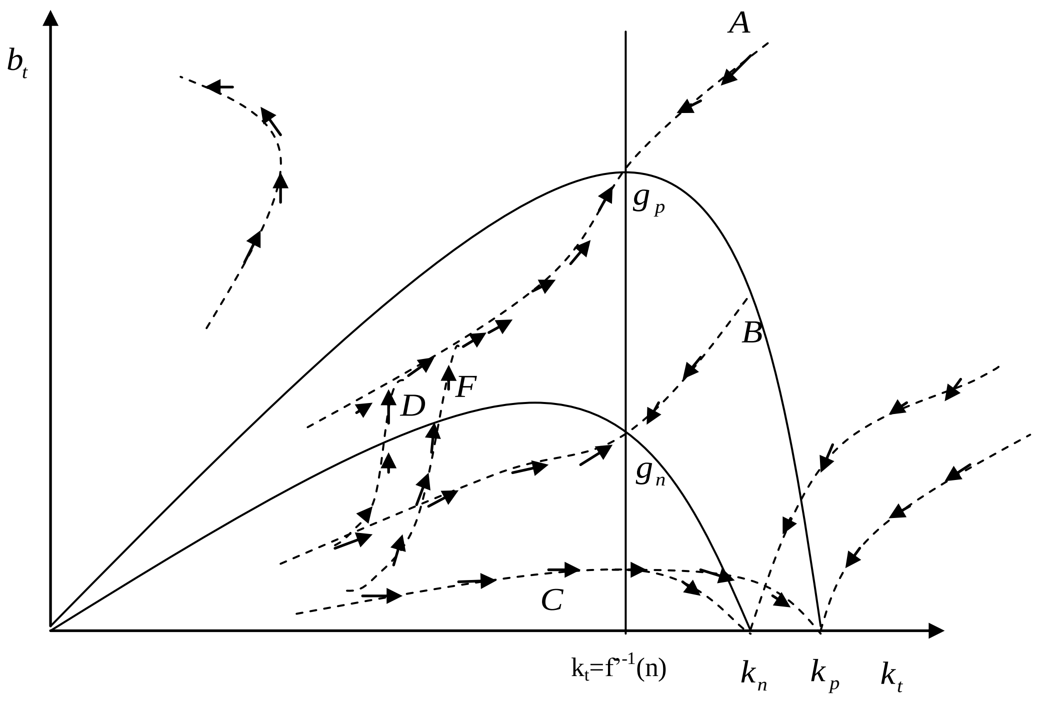
Now if we increase the DC pension fund investments by increasing the tax or, as previously discussed, by capturing part of the investment discretion from each individual agent, we will decrease its first period available income defined by "$w_t(k)$-τ" (the first component of the savings function), which negatively affects her savings function and the investment in the productive capital. In addition, the rational agent considers receiving higher income in the next period from the pension fund (the second component of the saving function), which additionally decreases its incentive to save in the productive asset. On the other side, the investment of the DC pension fund will positively affect the value of the bubbly asset, which also has a negative effect on the level of productive capital investments. The joint outcome of the introduction of the DC pension fund is a higher value of the bubbly asset at each state of the productive capital accumulation, or in other words, a strictly higher positioned locus of steady states for the ELM of the productive capital accumulation.

The impact of the introduction of the pension funds on the representative agent's savings function is to increase the overall economy-wise investable assets and so move the Diamond equilibrium to the right, making the sustainability of the bubbly assets higher, ceteris paribus. To understand this difference, taking an extreme situation into consideration, if the Pension fund taxes close to the amount of whole of the first period wage of the individual agents, it automatically commands its saving function making it similar to an extremely patient agent, which leads to the acceptance of even negative interest rates and extremely suboptimal productive capital over-accumulation. In other words, the introduction of DC pension funds, by greatly increasing the amount of saving and investments, pushes the financial market toward its dynamically inefficient state, where asset bubbles become a rational outcome. This same logic, explained from the point of view of Tirole's model, implies the following hypothesis:

Hypothesis 3: *The introduction of the mandatory DC pension fund to a closed illiquid financial market increases significantly the probability that the economy becomes dynamically inefficient and that it requires a bubbly asset to be introduced to overcome the potential issue of productive capital over-accumulation.*

Now that we have explained the effect on the two separate equilibrium laws of motion for the productive capital k_t and for the bubbly asset b_t, we merge the effects of the two dynamic equations to analyse the dynamics of the economy and its financial market governed simultaneously by productive capital and the bubbly asset accumulation. Drawing the joint phase diagram of the economy in b_t/k_t space, with the consequences of the introduction of the DC pension funds, we get the dynamics as depicted in Figure 2.3:

Figure 2.3: Phase diagram representation for the joint effect of the ELM of the productive capital (k_t) and the ELM of the bubbly asset (b_t) in the dynamically inefficient economy before and after the introduction of the mandatory DC pension fund



Source: Author's own graphical representation of the locus of steady state points for the productive capital per young agent(k_t) ELM,and the bubbly asset per young agent (b_t) before and after the introduction of the DC pension fund

Now we analyse the overall effect of the introduction of the DC pension fund on the equilibrium of the financial market. The fact that the k_n, or the Diamond steady state reached without the presence of the DC pension fund, is already at a level of productive capital accumulation where the marginal rate of return (interest rate) is lower than the rate of growth of the population "n", shows that we are initially in the dynamically inefficient economy, where bubbly assets are required to bring the financial market back to its Pareto efficient Golden Rule state as described by the steady state point (g_n). Although bubbly assets could bring the financial market to its dynamically efficient state, still they might not be introduced or they might be only marginally accepted, driving the dynamics of the system through the dynamic path C (Figure 2.3). This is avoided by construction in the case of our financial market with the DC pension fund, because we introduce the bubbly asset in the pension fund's portfolio.

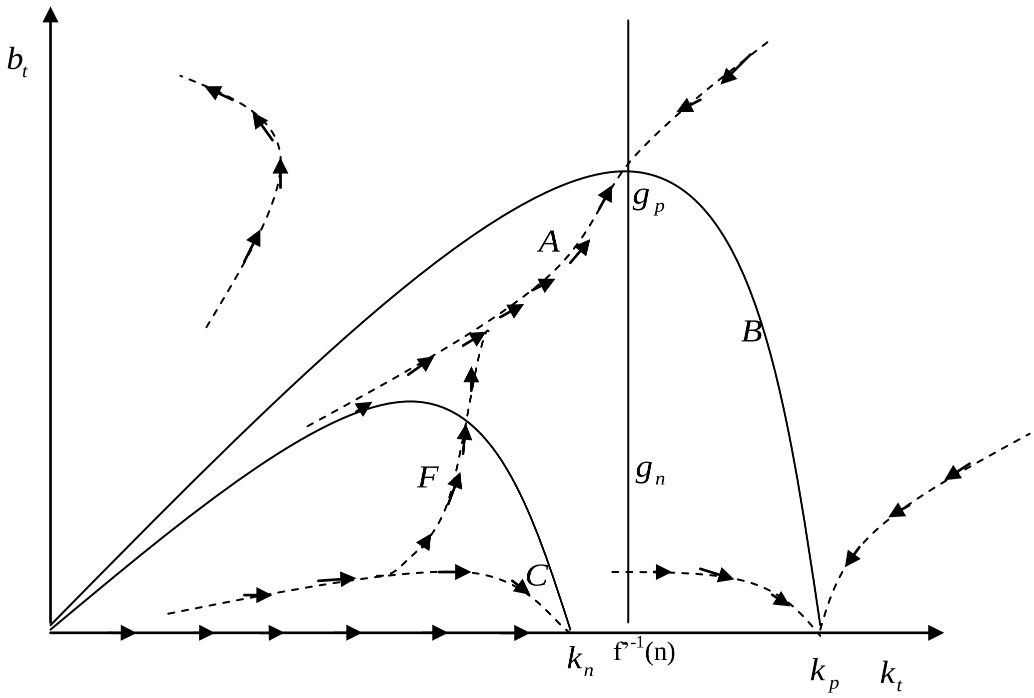
The greatest change caused by the introduction of the DC pension fund arrives by shifting the equilibrium productive capital over-accumulation point, called the Diamond steady state point, to the right, from the point k_n to k_p (Figure 2.3). This is a consequence of the fact that the introduction of the pension fund changes the economy's savings attitude and commands part of agents' inter-temporal consumption preferences toward those characteristic of more patient consumers. We previously explained, when analysing the locus of steady state points for of the ELM of the productive capital, that the introduction of the DC pension fund assigns a higher value of the bubbly asset for every steady state level of the productive capital accumulation, shifting the steady state locus upward. This ultimately means that the steady state locus will intercept the x-axis at a point further to the right. Ultimately, this increases the probability that the economy, without sufficient innovation in the productive sector required to sustain the marginal productivity of capital at a higher level, would become dynamically inefficient from the financial market perspective and, without the introduction of some bubbly asset, would lead to even higher suboptimal productive capital over-accumulation represented by the new Diamond steady state k_p. Such a shift of the productive capital steady state locus upwards and to the right intersecting the x-axis of the productive capital accumulation at a higher Diamond steady state indicated in Figure 2.3 by the point k_p.

With respect of the dynamics of the bubbly asset, this would mean that the Golden rule point would move to a higher value of the bubbly asset per young agent, defined at the point g_p. Intuitively, this means that as the overall "patience" of the representative agent is increased by command of the pension fund investment rule, the investable amount in the economy increases and the financial market requires a higher value of the bubbly asset to bring the dynamically inefficient economy to its Pareto optimal Golden Rule steady state defined by g_p (Figure 2.3). At this point, the additional value of the bubbly asset generated as a consequence of the introduction of the defined contribution pension fund equals the difference between g_p and g_n multiplied by the number of young agents N_t.

The immediate effect of the introduction of the DC pension fund to the other case of dynamically inefficient economy is depicted by Figure 2.3. The immediate consequence of such a change would be an upward shift in the value of the bubbly asset and a shift of the dynamics from the asymptotically bubbly path of the economy without the DC pension fund as described by (B) to the asymptotically bubbly path of the economy with the DC pension fund as described by a higher bubbly level path (A) (Figure 2.3). The immediate effect of the introduction of the defined contribution pension fund to the dynamically inefficient economy would be an increase in the valuation of the bubbly asset on the financial market. An important implication to discuss is the effect of future potential termination of a DC pension fund that was previously introduced. The consequence of the termination of the DC pension fund to the dynamics of the financial market and the economy will be discussed at the end of this section, when discussing the causes of the collapse of the bubble.

Now, we need to investigate the other more interesting case, which is potentially more characteristic for the Croatian and other SEE financial markets; namely, the potential effect of the introduction of the DC pension fund to the dynamically efficient financial market. This is the financial market, as previously described, where agents are predominantly impatient consumers who are not very keen on saving most of their wage when young, so that the economy's Diamond bubbly-less steady state of the productive capital accumulation (k_n) brings in a dynamic equilibrium where the marginal rate of return (interest rate) of the productive capital investments is higher than the growth rate of the investible assets as described by the growth rate of the population. This means that the Diamond steady state is to the left of the locus of stability points of the bubbly asset ELM and there is no systemic attraction toward higher values of the bubbly asset. This case is depicted by the Figure 2.4.

Figure 2.4: Phase diagram representation for the joint effect of the ELM of the productive capital (k_t) and the ELM of the bubbly asset (b_t) in the dynamically efficient economy before and after the introduction of the mandatory DC pension fund



Source: Author's own graphical representation of the locus of steady state points for the productive capital per young agent(k_t) ELM,and the bubbly asset per young agent (b_t) before and after the introduction of the DC pension fund

Here in the case when representative agents are predominantly impatient consumers and require a lot of liquidity, and the economy is dynamically efficient. In this case, there is no need for the existence of the bubbly asset to bring the economy to its Pareto optimal steady state prior to the introduction of the DC pension funds, because such an optimal state is already reached at the equilibrium point k_n. We later see that the speculative introduction of the bubbly asset into such economy could be only short lived (C), again leading to the ultimate bubbly-less Diamond steady state point k_n. Here, in this type of the economy, the introduction of the defined contribution pension fund could have even more disturbing effects on the market. The introduction of the defined contribution pension fund to the system could significantly change the relative savings and investment attitude on the financial market to an extent causing a shift of the dynamically efficient state to the dynamically inefficient state of the financial market. In such a state the new Diamond steady state equilibrium point is represented by k_p (Figure 2.4), a steady state point which is Pareto sub-optimal to the bubbly steady state equilibrium point g_p because it leads to productive capital over-accumulation and lower consumption and utility. In this economy, the introduction of the DC pension fund would by itself make the occurrence of the bubbly asset a rationally likely outcome, because of its impact on the economy-wise savings demand and its misbalance with respect to the productive investment opportunities. This means that the financial market would eventually produce the asymptotically bubbly path (A) leading to the bubbly steady state g_p from a much lower bubbly-less state of the market. In this case, the introduction of the DC pension fund would initially produce an even higher relative increase of the value of the bubbly asset on the financial market bringing the economy to the dynamic path F as depicted on Figure 2.4.

We saw from the analysis of the Phase diagrams for both dynamically efficient and dynamically inefficient economies, investigating the impact of the introduction of the DC pension funds, that in both cases we could expect an increase in the value of the bubbly asset in the economy. Comparing the two cases, we arrive at another hypothesis.

Hypothesis 4: *The increase of the value of the bubbly assets, consequent upon the introduction of DC pension funds, would be relatively much more dramatic and unsustainable in the case when the starting point is a dynamically efficient economy, if the capturing effect on income is so intensive that the introduction of the pension fund shifts the financial market to becoming dynamically inefficient.*

This could be an expected consequence when the economy has a small, illiquid and closed financial market, when DC pension funds at the beginning of their functioning tend to predominantly invest at home, and when they dramatically affect the overall investible assets and investing attitudes on the financial market. Next, we turn to the investigation to some of the potential triggers for the crash of the rational bubble.

2.4. Potential triggers causing the Crash of the bubbly asset

In our theoretical model, there are two major factors causing bubbly asset price volatility and potentially its crash: the first one has its origin in the change of consumer inter-temporal preferences; and the second one in the changes in the fundamentals determining economic growth (and the growth of investible assets). This widely corresponds to the intuitive understanding of the main factors causing market volatility and the crash of a pricing bubble often commented on by market professionals. Inter-temporal consumer preferences, as a primary reason for the volatility and ultimately for the crash of the price of the bubbly asset, are in part highly reflected in the psychology of the mass of investors. Not every generation lives with equal optimism for the prospects of its living standard, future income and overall expected utility, which defines its inter-temporal consumption preferences and its investment attitude. In other words, if we suddenly face fear that tomorrow we won't exist, then we would like to consume everything today, which would collapse even the ultimately bubbly asset, fiat money, leading to the collapse of the overall economy. Analysing this argument in the light of the Global Financial Crisis in 2008, we could connect the sudden investors fear with the higher risk premiums and higher required rates of return, which in our model could make many financial markets suddenly become dynamically efficient without any rationality for sustainable asset bubbles.

Robert Shiller (2000), in his seminal work on investor animal spirits and asset bubbles on financial markets, describes 12 factors determining equity market dynamics that correspond to and stimulate investors "irrational exuberance". However, our model suggests that the "irrational exuberance" of consumer agents is not the only potential source of financial market asset bubbles. Our model suggests that institutional reform could be another such source. When we introduce the defined contribution pension fund to the economy, the institution actually takes command to an extent of the consumer investment discretion typically promoting a higher level of financial market investments in the less developed economies. This could have a relatively strong effect on the market-wise investment attitude that is especially strong when defined contribution mandatory pension funds are applied to the dynamically efficient economy, in which most consumers had no investment experience prior to the introduction of the DC pension fund.

As indicated above, the fundamental drivers of financial market dynamics are technology and consumer preferences. Our model is not designed to pursue the implications of technical progress. However, our model, focused on institutional change, can illuminate the implications of two main factors liable to cause the crash of a rational bubble: a change in the inter-temporal consumer preferences and saving behaviour; and the termination of the institutional reform and the DC pension funds. The relative impact of both of these two factors depends on the relative control over aggregate saving behaviour by the pension funds. This means that not only changes in consumer preferences but also the termination of DC pension funds could return the financial market to a bubble-less steady state.

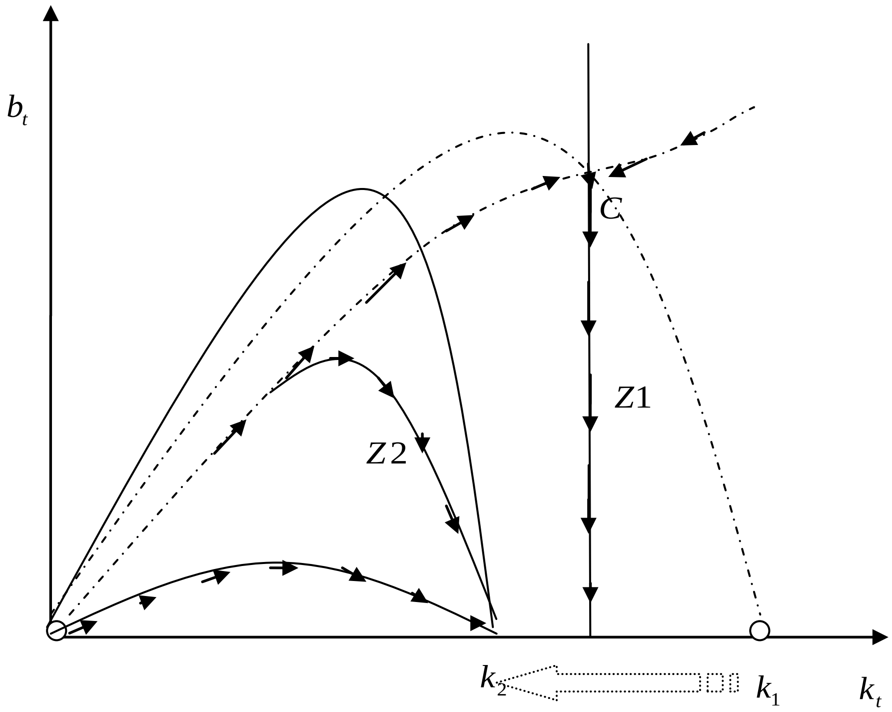
We consider two cases. The first is characteristic of the transition and developing markets. Consumers in such markets are predominantly impatient and consumer inter-temporal preferences are only of secondary importance to the primary impact of DC pension fund investment decisions on the dynamics of the bubbly assets. The second case is characteristic of developed markets. Consumers in such markets are relatively more patient, possessed of higher discretionary income and have relatively greater influence on market saving behaviour; hence the DC pension fund impact is correspondingly lower.

In both cases, the termination of the DC pension funds will affect the financial market, although this effect will be stronger in the case of less developed financial markets; for example, in the SEE financial markets, which are characterised by being very young with very limited supply and demand for investment instruments. In this environment, the experience of the termination of the DC pension fund in Hungary in 2010 is instructive. In the 12 months following the nationalisation of the DC pension funds in Hungary at the end of the 2010, equity markets suffered a loss of over 20%. Conversely, in developed markets, which are more commonly dynamically inefficient and bubbly, the termination of the DC pension fund would have smaller impact on the dynamics of the bubbly asset.

On developed financial markets which are already bubbly, or which are already dynamically inefficient before the introduction of defined contribution pension funds, individual investors are already significantly impacting system-wise investment attitudes on financial markets. In this context, although the introduction of the DC pension funds, as previously seen in Figure 2.3, has an increasing impact on the value of the bubbly assets, stochastic change of the inter-temporal consumer preferences of the representative agents as a group would have a primary role vis-a-vis the pension funds. In this case, changes in consumer preferences - such as might be induced by fear in periods of crisis -could be the primary cause for the crash of the bubbly asset.

In this respect, to answer the question of how inter-temporal consumer preferences defining the investment attitude (controlled to an extent by the DC pension fund) can crash the bubbly asset, we will refer to our phase diagram derived for the two cases of the economy. Inter-temporal consumer preference, as we already mentioned, determines whether the economy is dynamically efficient or dynamically inefficient. When consumers are “scared” about their future income and prospects, they become more impatient consumers and require immediate liquidity to maximize their lifetime utility function. This drives the dynamically inefficient economy very fast to a state of being dynamically efficient, which translates to a market state of increased supply surplus of long-term financial assets. In such a state, the bubbly instrument is no longer rationally required to provide the Pareto optimal outcome, and the bubble crashes rapidly. The generation which gets “scared” consumes more today, but loses a significant amount of lifelong utility due to its reaction and due to the crash of the bubble. This situation is depicted in Figure 2.5.

Figure 2.5: Phase diagram representation for the effect on the asset bubble when the dynamically inefficient financial market suddenly becomes dynamically efficient triggered by change in inter-temporal consumer preferences or a reversal of DC pension fund investments



Source: Author's graphical representation of the locus of steady state points for the productive capital per young agent(k_t) ELM,and the bubbly asset per young agent (b_t) before and after the change of consumer preferences or after the termination of the DC pension funds

In Figure 2.5, sudden change in consumer preferences, towards being extremely impatient consumers, or termination of the DC pension fund investments, have qualitatively similar effects. In both cases, the intersection of the stability points locus for productive capital accumulation is driven from point $k_1$to point k_2. The market economy changes from being dynamically inefficient to being dynamically efficient. The bubbly asset is no longer rationally needed, which breaks the non-arbitrage condition. Consumer agents become aware of the speculative character of the bubbly asset and are no longer risk indifferent between the bubbly asset and the productive capital. In such a case, the market will instantly collapse the bubble, following the path Z_1 or Z_2 depending on the starting point, and will soon reach the new bubble-less steady state k_2. In the new steady state, productive capital accumulation is at a lower level, finding its new stability point depicted at the point k_2, which represents the new bubbly-less Diamond dynamic equilibrium, which is Pareto-optimal for such a dynamically efficient economy. This way we arrive to our final hypothesis:

Hypothesis 5: *The crash of the bubbly asset could result from the sudden change of financial market dynamic efficiency caused either by a termination of the DC pension scheme or a shift in the inter-temporal consumption preferences of individual investors affected by some domestic or foreign risk event such as the Global Financial Crisis.*

Conclusion

In this chapter adapting the canonical rational asset bubbles OLG model as our modelling platform (Tirole, 1985), we developed and analysed a dynamic financial market model with DC pension funds and illiquid supply of financial assets. This model analyses the dynamics consequent upon introducing the DC pension fund reform into young and illiquid financial markets such as those of the SEE economies.

The model describes the dynamic properties of the financial market with its productive and bubbly assets before and after the introduction of DC pension funds as influential institutional investors as well as upon the impact of the Global Financial Crisis. Based on the dynamics simulated by phase diagrams, we derived five hypotheses to be analysed and tested in the following empirical part of the dissertation.

Our first hypothesis suggests that the introduction of the DC pension fund on the financial market would induce faster and more rapid dynamics into the value of the bubbly asset. The intensity of such impact, as suggested by the second hypothesis, is governed by regulating bodies by setting the extent to which the DC pension fund captures the saving discretion from the consumer agents, which is defined by the height of the pension tax on their income.

The third hypothesis suggests that the introduction of the DC pension funds could affect the dynamic efficiency of the financial market, creating a fertile platform for the occurrence of a rational asset bubble. This is followed by the fourth hypothesis, which suggests that the increase in the value of the bubbly asset will be especially strong when the influential DC pension fund is introduced to a small, underdeveloped and dynamically efficient financial market. Finally, the fifth hypothesis derived from our theoretical model suggests that events causing significant change in the dynamic efficiency of the financial market and changing the saving attitude of the economy, such as the Global Financial Crisis or the cancelation of the already introduced DC pension scheme, could cause a sudden and rapid crash of the already developed rational asset bubble.

In the second empirical part of the dissertation we will test these hypotheses with data from the Croatian financial market.

Part II: Empirical Testing based on data from the Croatian financial market

Introduction

In this second part of this Dissertation, we test the hypotheses derived in Chapter 2 about the effect of the introduction of the DC pension funds on the value of the speculative/bubbly assets in an illiquid and underdeveloped financial market. We focus on analysing data from the Croatian financial market. In order to test the theoretically hypothesized consequences of the introduction of the DC pension funds to the Croatian financial market, we proceed through the following chapters.

Chapter 3 is focused on describing the data used. Most of the variables required by our theoretical model are not readily available. Accordingly, we explain in detail how the variables in our theoretical model are translated into observable counterparts for econometric analysis. For example, we had to measure the relative value of both the Croatian and US equity bubble and to establish a measure of the dynamic efficiency of the Croatian financial market. To construct our dataset, we had to derive data values from different data sources and apply transformations. In particular, we had to correctly assign values from the numerous and diverse financial reports of the DC pension fund's investments and, for certain variables, make interpolations from lower to higher frequencies. This whole process of collecting and explaining the characteristics of the dataset is undertaken in Chapter 3, preparatory to the econometric modelling reported in Chapters 4 and 5.

In Chapter 4, we use the data set to test our hypotheses by estimating a model with two investment variables representing the activity of the DC pension funds - respectively, their direct equity and indirect equity investments - derived from the pension funds' audited financial statements and interpolated from their genuine yearly to a monthly frequency. We used a Vector Error Correction Model with two co-integrating long-term relationships, which confirmed the significant positive impact of the DC pension fund investments on the creation of the Croatian equity market asset bubble and identified the indirect DC pension fund investments through the Croatian and SEE equity focused open-end mutual funds as the bubbly asset accepted by the Croatian DC pension funds. However, this model suffered from structural instability of the estimated coefficients and from the cross-correlation of the two investment variables produced by their interpolation method. These issues motivated our estimation of a much simpler model in Chapter 5.

Because of the drawbacks of the model presented in Chapter 4, in Chapter 5 we estimate a VEC Model with a single investment variable and two long-term co-integrating relationships. This model overcomes the issue of cross-correlation of the DC pension fund investment variables by using a single investment variable, has stable estimation coefficients and is much simpler compared to the model presented in Chapter 4. Yet its results are consistent with the results of the model reported in Chapter 5, again confirming the positive impact of the introduction of the DC pension funds on the development of the Croatian equity bubble. It also confirms that the dynamic inefficiency of the financial market creates a positive platform for the development of asset bubbles.

Finally, both empirical models in Chapter 4 and Chapter 5 confirm that the Global Financial Crisis, which was introduced as a structural break variable in the co-integrating vectors in both models, changed the dynamic efficiency of the Croatian financial market and was the major trigger for the rapid crash of the Croatian equity bubble.

Chapter 3: Data collection, derivation and description

Introduction

This Chapter is focused on explaining the data used for testing the theoretical OLG model derived in Chapter 2. We used data from the Croatian financial market, which after the introduction of the DC pension reform in 2002 experienced a strong speculative episode on its equity market in the period 2006/2008. Moreover, the Croatian equity market by its size and volume dominates the financial markets in the surrounding SEE countries, which were also affected by the speculative episode. In order to determine the data set we need to extract, we first recall the implicit function defining the dynamics of the bubbly asset and the variables having an effect on the value of the asset bubble. Then, we derive and measure the value of the bubble on the Croatian equity market and we analyse its properties. Further, we also measure the value of an asset bubble on the US equity market using the same methodology. This “external market effects variable” is to be included in the model in order to control the effects of foreign financial markets. Next, we define the Dynamic Efficiency variable of the Croatian financial market describing the necessary condition for the appearance and sustainability of the bubble and we analyse its properties. Finally, we derive and explain the vector of variables describing the investments of the DC pension funds into our two potentially bubbly assets: Croatian Equity; and Croatian and SEE equity-focused Open-end Mutual Funds. We divide this vector of variables explaining the DC pension fund investment into three categories: First, are the dummy variables signifying some important legislative changes affecting the DC pension funds’ introduction and investment decisions in the bubbly assets; Second, are the annual frequency variables representing net investment into the bubbly assets, which are used to derive higher monthly frequency variables; and, Third, are the monthly data on the estimated new investments in the Croatian and SEE Equity-focused Open End Mutual Funds made by the DC Pension funds. We describe the properties of all variables before we move to the next chapter, which uses them in econometric analysis.

3.1. Implicit function and the explanatory variables describing the dynamics of the rational asset bubble

Based on the theoretical model of rational asset bubbles with DC pension funds that we developed in Chapter 2, we determine the implicit function explaining the occurrence of the equity asset bubble on the Croatian financial market in the period of 2006-2008. Following our OLG theoretical model, B_m represents the value of the pure bubble in the financial market at the month *m* (B_Market_m). This value can be observed as an attached value to the intrinsic value of the bubbly asset, representing the relative difference between its market and its intrinsic value expressed in percentage points. The dynamics of the value of the bubble at time *m* (B_m), based on the OLG model, are explained by an implicit function including the following set of variables (Equation 3.1).

$$B_m = f\left(B_{m-1}, DYNEF_m, \tau_m, External_m\right) \tag{3.1}$$

We recall from our result in Chapter 2, that the dynamics of the bubbly asset B_m was defined by a first-order difference equation depending on the state of factors such as the dynamic efficiency of the financial market and the DC pension funds' investments into the bubbly asset. In addition, the influence of external/global financial markets could affect the emergence or the crash of the asset bubble, and because of this– in spite of the fact that our theoretical model represents a closed financial market economy – we control for this external factor in our model. To describe those factors in our implicit function specified by our theoretical model as determining the dynamics of the bubble, we define:

- a variable "$DYNEF_m$", which represents the dynamic efficiency of the financial market at the month *m*, and
- the $\boldsymbol{\tau}_m$ vector of variables representing
  - the introduction of the Defined Contribution Pension Funds and
  - the investments in the bubbly asset by the DC Pension Funds (Equation 3.1).

As we already briefly mentioned, although our OLG theoretical model of rational asset bubbles with DC pension funds was specified for a closed market economy, in order to take account of the inevitable effect of the international capital markets we also introduce

- a vector of external factor variables (*$External_m$*).

As discussed in Chapter 2, external factors, especially through the information channel, could spread pessimistic news about the future from the global financial market and give rise to sudden shift in inter-temporal consumer preferences, thereby changing the dynamic efficiency of a small financial market. This could trigger a rapid crash of a developing asset bubble if it turns the financial market environment into the state of being dynamically efficient, in which case rational asset bubbles become unsustainable. We also suspect that those external global financial influences could fuel the exuberant attitude on a specific market and could support the bubbly asset price dynamics independently or jointly with the internal factors. Consequently, we had to isolate this potential external financial market effect in our empirical analysis of the Croatian equity bubble and so we introduced the "$External_m$" vector as potential explanatory variables.

Regarding the role of our variables representing the driving forces of the savings market equilibrating mechanism, we recall the functioning of the financial market as depicted earlier in Figure 1.2.2 in Chapter 1. There we had the supply function of savings vehicles, which in our case represents the supply of both productive and bubbly assets, where productive assets are subject to diminishing returns and where the rates of return of both asset types are aligned by the non-arbitrage condition.[10] On the other side, we have the demand, which arises from both the productive and the bubbly asset investments of the representative agents in the economy, but also from the bubbly asset investments of the DC pension funds. The interaction of the supply and demand determines the market interest rates or the rates of return of both the productive capital and the bubbly asset through the non-arbitrage condition. In our empirical model, the interest rate is contained within the dynamic efficiency variable, where the long-term interest rate of the Croatian government bond is used as a benchmark. In turn, our main interest is the impact of the investments of the DC pension funds in the bubbly asset, which represents part of the demand for investment vehicles. Under the assumption that the price of a productive asset could be composed of both intrinsic and bubbly value

[10] Figure 1.2.2 is general for OLG models. In Tirole's model in particular, and in our extension, non-productive saving takes place only by means of the bubbly asset. In the model, productive assets are supplied according to the objective function of the representative entrepreneur. Yet there is no such objective function for the bubbly asset; rather, once the productive asset is supplied, it has the potential to develop a bubbly value in addition to its productive value (Tirole, 1982). However, in the model under consideration, for analytic convenience assets are separated into two types, productive and bubbly.

components, then the DC pension fund investment into the bubbly asset could correspondingly be channelled through different routes to the bubbly equity assets. One such route is direct equity investment and another could be indirect equity investments. Each of the two could have a more or less speculative character, but in total they should account for the impact of the DC pension funds on the relative value of the equity bubble on the Croatian equity markets.

Having specified our implicit model and variables affecting the value of the bubble, based on our theoretical model assumptions developed in detail in the previous chapter, next we have to work on measuring and explaining our dependent and independent variables.

3.2. Defining, measuring and describing the value of the asset bubble *"B_Market$_m$"*

Our first task is to define a measure for the value of the bubbly asset on the specific financial market, *Bm*. This is not an easy task, since there is no standard accepted measure published by the stock markets to show the extent of speculative investment and its value on the financial market. So we had to produce our own indicator of the value of the pure bubble attached to the price of equity.

We begin with the plain vanilla definition of the asset bubble representing a deviation of the price away from the intrinsic value of the underlying asset. Consequently, we had to define the connection between the intrinsic value of equity as a potentially bubbly asset (using the broad equity market index in our definition) and the fundamental factors of the economy such as the Gross Domestic Product (GDP). We begin with the following equation (3.2) connecting the value of the broad market equity index (P) with GDP, which is then decomposed into two ratios:

$$\frac{P}{GDP} = \frac{P}{E}\frac{E}{GDP} \tag{3.2}$$

Here, *P* represents the absolute price of the broad market equity index such as the CROBEX or the SP500 index; *E* represents the total end of period earnings of companies comprising the market index; and GDP represents the Gross Domestic Product of the economy represented by the equity market index. And the decomposition is accomplished by taking the P/GDP ratio and then dividing and multiplying by Earnings. This trick is borrowed from the analytical toolset used in the CFA Level 2 literature (CFA Institute, 2013, p. 587 Book 1), with a slightly changed order in the equation. Taking natural logarithms on both sides of the equation yields Equation 3.3.

$$\ln\left(\frac{P}{GDP}\right) = \ln\left(\frac{P}{E}\right) + \ln\left(\frac{E}{GDP}\right) \tag{3.3}$$

This "ratio elasticity" equation (3.3) gives an insight to the factors defining the change of the P/GDP ratio. The percentage change of the ratio between the price of the equity market index (P) and the GDP of the economy is decomposed into the sum of the percentage change of the P/E ratio and the E/GDP ratio. In the long run, both of those ratios are expected to be stable, which implies that the relationship between price of the equity market index (P) and the GDP is also expected to be stable in the long run (CFA Institute, 2013, pp. 587, Level 2 Book 1). The exception exists only when we have a market abnormality such as an asset bubble. Then, we could have short to medium run fluctuation of the P/E ratio around its mean and consequently an abnormal P/GDP ratio, while the E/GDP ratio is relatively much more stable (CFA Institute, 2013). Many authors, such as Nobel Prize winner Shiller, recognize the mean-reverting character of P/E ratio (Campbell & Shiller, 1988; 2001). Shiller's CAPE (Cyclically Adjusted P/E) is often referred as a good long-term market predictor in the finance industry.

This way, we could say that when the equity market index is valued at its long-term average P/GDP ratio, then it is fairly valued and the price of the index consists solely of the price characterized by the intrinsic value of its constituent companies. Any positive deviation from the index price consistent with the long-run P/GDP ratio corresponds to the extent of the bubbly value within the observed price of the market index. This way of defining an indicator of the value of the bubble is also in line with the theoretical approach of Tirole, who assumes that within the price of an asset such as common equity, which has a certain determinable intrinsic value, could also exist a bubbly value attached, represented by the difference between the current market price of that asset and its current intrinsic value (Tirole, 1982).

Joining those two approaches together we use the following formula in order to derive the indicator for the value of the bubbly asset within the price of the equity market index (Equation 3.4)

$$B_m = \left\{ \frac{\dfrac{MarketIndex\,Price_m}{\dfrac{1}{11}\sum_{i=0}^{10} GDP_{m-i+5}}}{mean\left(\dfrac{MarketIndex\,Price_m}{\dfrac{1}{11}\sum_{i=0}^{10} GDP_{m-i+5}} \right)} - 1 \right\} \bullet 100 \qquad (3.4)$$

In the numerator on the right-hand side of equation (3.4) we calculate the ratio between the current price of the market index (P) and its corresponding eleven month average GDP compared to its long-term mean. We smooth the GDP values first by assigning the yearly value of GDP for every month in the particular year of the analysed period, and then by creating a moving average value as an average of the current month value and all five previous and consecutive monthly values of yearly GDP. This way, we smooth the yearly GDP kinks, which otherwise appear between December of the current and January of the consecutive year, and we also include the GDP information about the previous and future months in the smoothed GDP series, which brings a forward looking approach to the index pricing. We use the eleven month moving average for preserving symmetry where, besides the current month GDP, we include five previous and five consecutive months in the calculation. We compare the current monthly ratio of the index price to the smoothed GDP with the long-term all-period mean of the same ratio (the denominator in Equation 3.4). This gives us the measure of the relative value of the asset bubble B_m, expressed in percentage points, representing the relative deviation of the current P/GDP ratio from its long-term mean value.

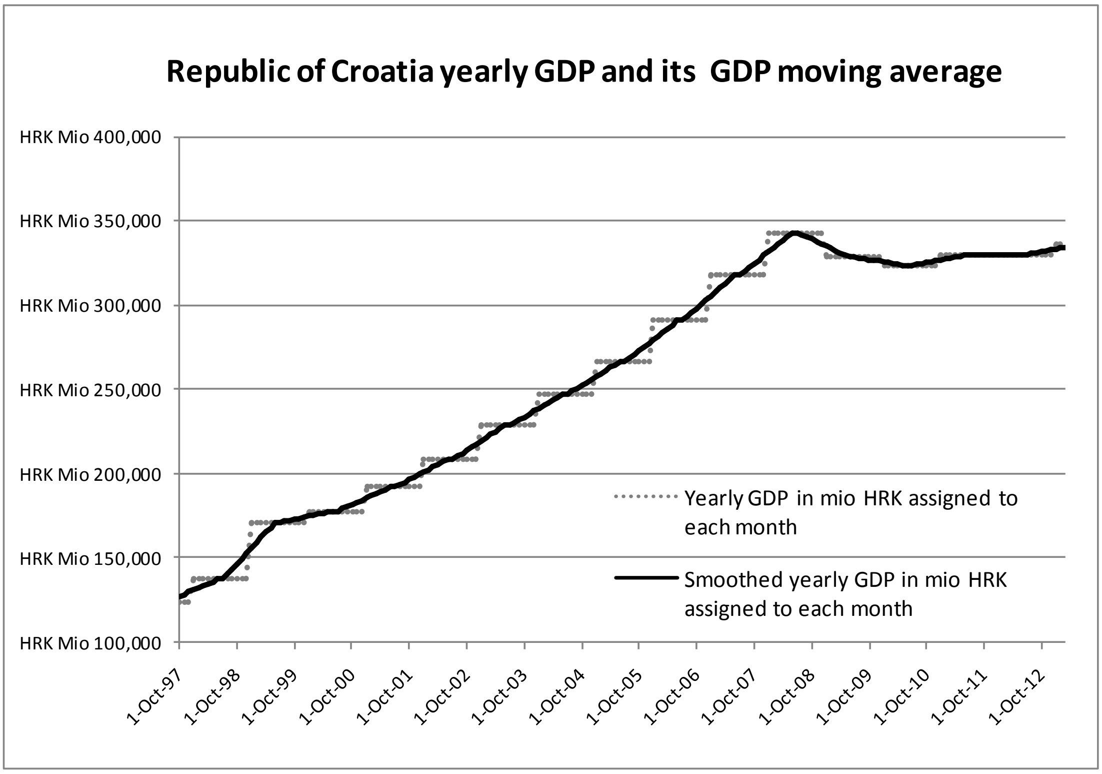
Both the price of the equity market index and the GDP should be measured in the same currency to avoid disturbances caused by the dynamics of the currency market. Based on the efficient capital markets hypothesis, in the long run, this ratio should be solely based on the fundamental value of assets and should not significantly change on other than fundamental grounds. In other words, the price dynamics of the broad equity market index should follow the potential or long-term rate of growth of GDP. Deviations of the ratio should represent market mispricing and the corresponding relative value of the equity asset bubble.

Having defined the indicator for the relative value of the asset bubble on a specific equity market, we continue by measuring the data series with the values of the relative equity asset bubble in Croatia and in the US, which are two variables important in our empirical model.

3.2.1. Equity Asset Bubble in Croatia “B_m(Croatia)” and its first-order difference

We used this approach to measuring the absolute value of the equity market asset bubble with data from the Republic of Croatia. Using the Croatian equity market index values (CROBEX) and the Croatian GDP, we determined the following data series. We used the IMF public database for extracting the yearly GDP time series for the Republic of Croatia in HRK (Croatian Kuna currency) for the period 1997-2013 and we smoothed the yearly value by the previously explained formula (Figure 3.2.1.1).

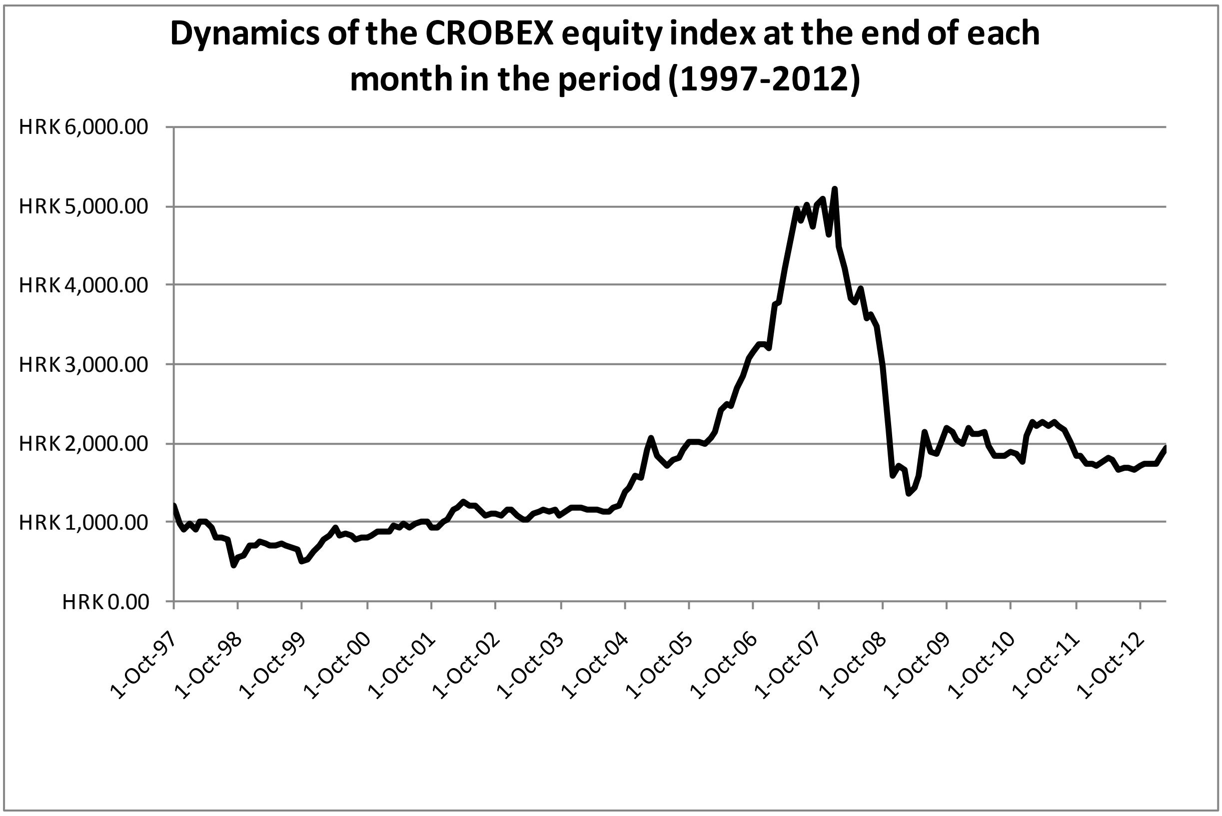
Figure3.2.1.1 Republic of Croatia yearly absolute GDP values in million HRK smoothed at monthly frequency using Equation 3.4.



Source: Data extracted from the IMF public database; own calculation

Next, we collected data on the broad equity market index of the Croatian Equity Zagreb Stock Exchange (CROBEX). The CROBEX equity market index was established at the beginning of September 1997 with an initial value of 1000 HRK and represents the portfolio of the 25 most liquid companies representing the highest market capitalization on the Croatian Equity Market. We used price data from the public database of the Croatian Stock Exchange available at the following webpage (www.zse.hr). The CROBEX Index dynamics can be seen in Figure 3.2.1.2.

Figure3.2.1.2: Dynamics of the value of the Croatian main equity market index CROBEX (in HRK)



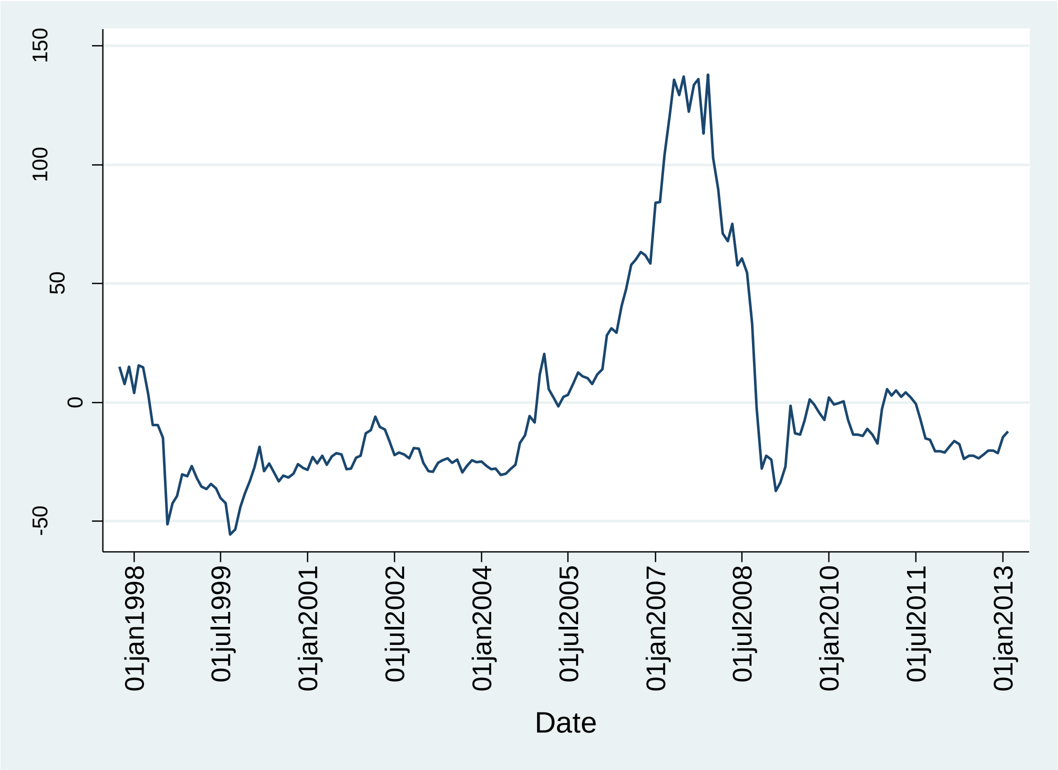
Source: Croatian Stock Exchange (Zagreb Stock Exchange www.zse.hr)

Applying our equation for the calculation of the relative value of the asset bubble on the Croatian equity market (Equation 3.4), we obtained the following data series represented in Figure 3.2.1.3. It represents the monthly percentage point deviation of the actual ratio of the price of the CROBEX market index to the smoothed Croatian GDP (numerator) to its fundamental long-term average value (denominator) calculated following Equation 3.4. The fundamental long-term average of the P/GDP ratio for the Croatian equity market was 0.006628 based on our data set, which according to Equation 3.4 would represent the 0 value of the measurement variable of the relative value of the Croatian equity bubble "B_Croatia". During the peak of the equity market asset bubble, in 2007, the P/GDP ratio reached 0.009152 or a value of 138 in terms of "B_Croatia", representing 138% overvaluation of the market index estimated fundamental value.

We would like to stress that the CROBEX index includes within its 25 constituent companies the Croatian Telecom (HT) stock, which we later exclude from the investment variables of the DC pension funds, because it was the only stock that undertook IPO during the speculative episode on the Croatian equity market. Although HT stock accounted for less than 10% of the Crobex index, considering the fact that HT stock price was much less volatile its inclusion might cause a slight negative bias in the actual value of the Croatian equity bubbly as measured on the rest of the 24 index participants.

The dynamics of the relative value of the asset bubble on the Croatian equity market, especially in the years 2005-2008, are reflected in the dynamics of the "B_Croatia" indicator presented in Figure 3.2.1.3. Looking at the figure, we notice that just as the equity prices might move over their intrinsic value in a positive direction, there might also be periods, and especially after a crash of the bubble, when the actual P/GDP ratio may indicate the existence of a negative bubble when compared to its long-run average value. As we see from the Figure 3.2.1.3, this is of a smaller intensity compared to the intensity of the positive bubble.

Figure 3.2.1.3: Relative value of the Croatian equity bubble "B_Croatia" measured at a monthly frequency using Equation 3.4



Source: Computed based on IMF and Zagreb Stock Exchange Data in Stata 12

This variable indicating the value of the bubbly asset within the price of the Croatian market index “B_Croatia”, is the variable that we try to explain by relating it to the other variables specified by our theoretical model.

Relating to our theoretical model developed in Chapter 2, we need to stress that our single bubbly asset is measured as the value of the equity market over its intrinsic value as explained above. Such a pure bubbly asset is part of the price of the equity itself, represented by the part above its intrinsic value and measures with our “B_Croatia” variable. Based on this intuition, when a pension fund invests into Croatian equity, if the price of the equity is below or equal to its intrinsic value, then the investor invests only in the productive capital. But if the price is above the intrinsic value, the investor invests both in the productive and in the bubbly value of equity based on their ratio in the total price of the equity. Consequently, the units of equity or stocks, are the carrier of the value of the Croatian equity bubble.

In this respect, relating to our theoretical model in Chapter 2, investments in the bubbly asset attached to the intrinsic value of Croatian equity could be undertaken through direct purchase of equity and/or through indirect purchase using intermediary investment vehicles. Such a channel of indirect equity investments is provided by the Open End Mutual Funds, which are focused on investments in Croatian and SEE equity market.

Although these direct and indirect investment vehicles are formally equivalent from the perspective of their ultimate allocation of investments in Croatian equity, there is a crucial distinction from the perspective of how they operate and how are they viewed by the investors. Namely, the indirect investment vehicles such as the Open End Mutual Funds increase asymmetry of information and introduce an agency problem, while investors focus on the value of the Fund rather than on its individual investments. In turn, this veils the fundamental performance of the underlying assets and correspondingly creates an affinity between the indirect investment vehicle offered by the Open End Mutual Funds and the bubbly part of the equity valuation. Accordingly, we treat the indirect investments into the Open End Mutual Funds as the actual channel for investing in the bubbly or speculative part of the Croatian equity market.

Furthermore, we need to stress that the time series of “B_Croatia” and the other variables are used up to the year of 2011 due to the fact that we could collect investment data about the DC pension fund investments only up to this date. In 2011, the transparency in the annual reports of the DC pension funds decreased significantly not allowing us to extend our series.

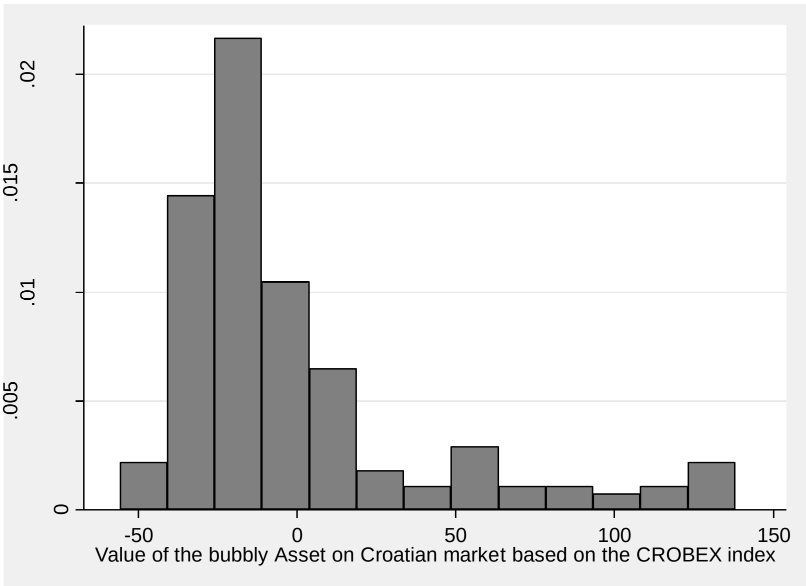
We summarize its statistical properties and check for some important data characteristics, which are of importance when the variable is later used in econometric analysis. For this purpose, we calculated summary statistics for the variable. Test for unit root of the variable will be reported at the last section together with the unit root tests for all endogenous variables presented in this Chapter. As can be seen from the data summary statistics of our "B_Croatia" variable presented in Table 3.2.1.1, its mean by construction equals zero because, using Equation 3.3, this variable represents the deviation of the actual P/GDP ratio from its long-term mean. The data is positively skewed toward more extreme positive values of the bubble, as seen in Figure 3.2.1.4, which is actually the consequence of the massive and intense asset bubble occurring in the period 2006/2008 (Figure 3.2.1.3).

Table 3.2.1.1: Summary statistics of the "B_Croatia" variable

DESCRIPTIVE STATISTICS				
sample range: [1997 M10, 2013 M2], T = 185				
variable	**mean**	**min**	**max**	**std. dev.**
B_Croatia	-2.43E-01	-5.57E+01	1.38E+02	4.20E+01

Source: Author, using JMulti Output 4.24

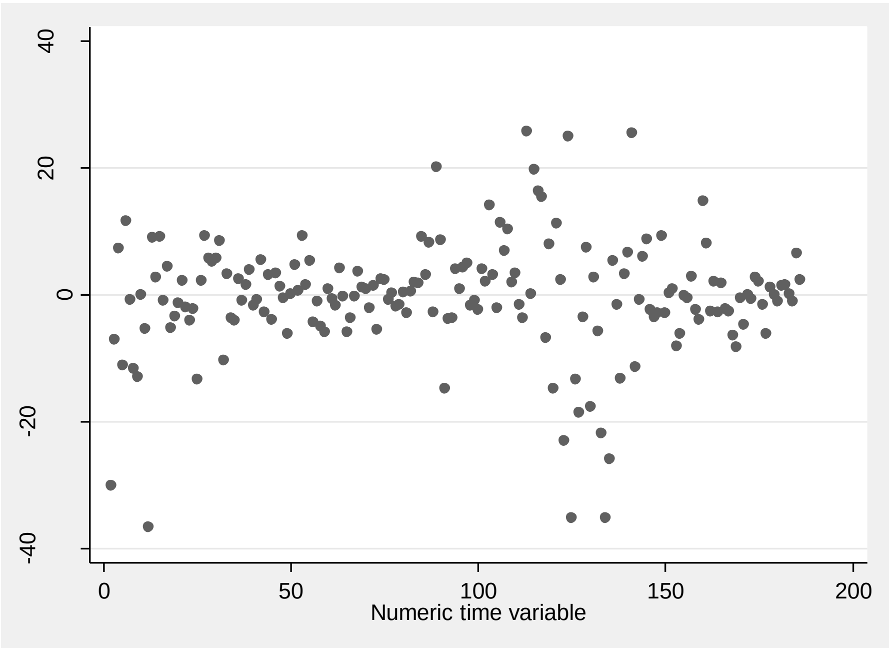
Figure 3.2.1.4: Histogram representing the distribution of "B_Croatia" variable



Source: Author, using Stata 12

As we will see from the last section of this chapter, our variable "B_Croatia" is non-stationary and integrated at level one I (1). Therefore we also generated a new transformed data series consisted of the first-differences of the "B_Croatia" variable, which we named "*FD_B_Croatia*". By first differencing the variable, we keep the information about the shorter period dynamics of the variable, but lose significant information about the mid- to long-term dynamics of the variable such as the potential long-term trend within the process. The dynamics of the "*FD_B_Croatia"* variable are presented in Figure 3.2.1.5. Figure 3.2.1.5 plots the differenced variable, which – allowing for a few outliers – reveals an essentially stationary evolution.

Figure 3.2.1.5: "*FD_B_Croatia"* variable distribution through time, where the x-axis represents the monthly time periods counted from Oct-1997 represented as t=1 to Mar-2013 represented as t=186

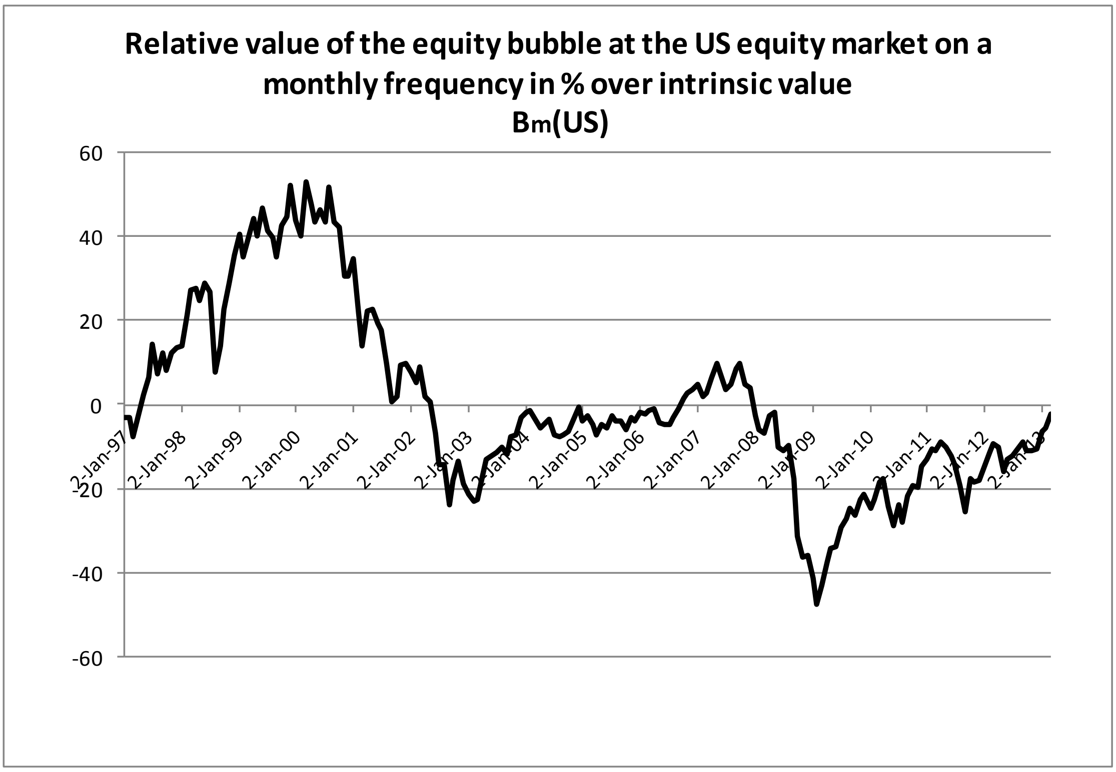


Source: Author, using Stata12

3.2.2. Equity Asset Bubble in the US "*B_USA*" and its first-order difference

Our theoretical model in Chapter 2, assumes that the financial market belongs to a closed market economy, without impact from external financial markets. However, we need to augment our empirical specification by controlling for the effect of the foreign financial markets. In order to control for the potential effect of global financial markets on the creation of the equity asset bubble in the Croatian market, we also need a measure of the equity asset bubble on external equity markets. For this purpose, we measured the value of the equity asset bubble on a representative global financial market; namely, the US market. We collected the value of the SP500 equity market index representing the stock market of the US economy and the variable representing the dynamics of the US GDP. We applied the same approach measuring the relative value of the asset bubble, using equation (3.4), and we created the "*B_US"* time series variable represented in Figure 3.2.2.1.

Figure 3.2.2.1: Relative Bubbly Asset Value within the SP500 Index "*B_US*" measured at a monthly frequency using Equation 3.4



Source: Excel calcuclation based on SP500 and US GDP data

Applying Augmented Dickey-Fuller test to the "*B_US"* variable, we could not reject the null hypothesis that the variable has a unit root (Table3.2.2.1) which was expected for a stock market derived variable. In other words, "*B_US"* is also a non-stationary variable and we continued by first differencing it. By first differencing the values of the "*B_US"* variable representing the relative value of the US bubble we created a new time series variable "*FD_B_US"*, which represents the short term innovations or changes of the value of the relative asset bubble (Figure 3.2.2.2).

Table 3.2.2.1: Augmented Dickey-Fuller unit root test on "*B_US*":

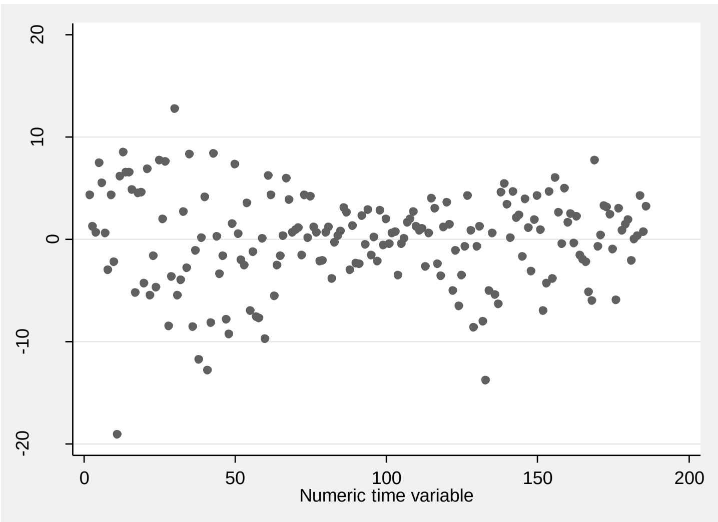
```
ADF Test for series:      B_US
sample range:             [1998 M8, 2013 M2], T = 175
lagged differences:       8
no intercept, no time trend
1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -1.5234

intercept, no time trend
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -1.5197

intercept, time trend
1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -2.2128
```

Source: Author, using JMulti 4.24

Figure 3.2.2.2: "*FD_B_US*", First difference of the "*B_US*" variable where the x-axis represents the monthly time periods counted from Oct-1997 represented as t=1 to Mar-2013 represented as t=186



Source: Author, using Stata 12

We then tested "*FD_B_US"* for stationarity and confirmed that even with 99% level of confidence, similar as with the case of "*FD_B_Croatia"*, we could reject the null hypothesis claiming that the variable has a unit root. Consequently, we consider the first difference transformation variable to be stationary (Table 3.2.2.2).

Table 3.2.2.2: Augmented Dickey-Fuller unit root test on "*FD_B_US*":

```
ADF Test for series:      B_US_d1
sample range:             [1998 M8, 2013 M2], T = 175
lagged differences:       8
no intercept, no time trend

1%          5%          10%
-2.56       -1.94       -1.62
value of test statistic: -3.9209

intercept, no time trend

1%          5%          10%
-3.43       -2.86       -2.57
value of test statistic: -3.8933

intercept, time trend

 1%          5%          10%
-3.96       -3.41       -3.13
value of test statistic: -3.8848
```

Source: Author, using JMulti 4.24

Since the two variables representing the value of the bubble on the domestic (Croatian) and on the global financial market represented by the US market, "*B_Croatia"* and "*B_US"* respectively, are non-stationary and integrated of order one (I(1)), we wanted to analyze whether there is some long-term relationship among them, because by first differencing them and using the first differences, we would lose this information about the potential long-run relationship between the two stochastic variables. For this purpose, we tested for co-integration between the two variables, by making a regression of the original non-stationary variables "*B_Croatia"* and "*B_US"* and then testing whether the regression residuals are stationary (again by performing the ADF test).

Based on the findings of Eagle and Granger, if the residuals of the simple OLS regression between the two variables are stationary, then there exists a vector of coefficient constants which, when multiplied by the matrix consisted of the values of the two or more non-stationary variables, creates a combination which is stationary. This vector of coefficients consists of the OLS coefficients from the regression of the co-integrated variables in levels (Engle & Granger, 1987). Further if such stationarity of the residuals is present, it signifies that there exists a long-term relationship between the two variables and that, in the short and medium term, the dynamics of the variables are partly affected by "error correction" toward the joint long-term co-integrated path of the two variables. This way, the residual could be used as a correcting variable in the short-term effect regression model with first differences, which could explain part of the short-term dynamics in the model, caused by the long-term connection of the two series. This approach is popularly known as an Error Correction Model (ECM), and the dependent variable could have such a relationship with any of the stochastic variables originating from the same level of integration as the level of integration of the original dependent variable. Johansen (1988) extended the idea to a Vector Error Correction Model (VECM) where we have a vector of co-integrating relationships.

Regression coefficients from the model, regressing "*B_Croatia*" on "*B_US*" using simple OLS, were insignificant at 90% significance level (Table 3.2.2.3) and the residuals failed to display stationarity according to the ADF test (Table 3.2.2.4). We conclude that the dynamics of the bubbly value in the two markets do not follow a consistent jointly related long-term path. Hence, we turn to the analysis of short-term relationships between the two markets, without the risk of losing information about their long-term connection. This fact of no-co-integration between the dynamics of the values of the bubbles could support the assumption that the Croatian market is still very young and not well integrated with the centres of the global financial market such as the US market, which additionally gives relevance to our theoretical model aimed at analysing a small, closed and illiquid financial market. The non-co-integration also gives signs that there are other, internal reasons for the occurrence of the equity bubble which we try to explain by our model.

Table 3.2.2.3 Regressing "B_Croatia" to "B_US" – Determining co-integration

Source	SS	df	MS		
Model	4280.42034	1	4280.42034	Number of obs =	185
Residual	322470.157	183	1762.13201	F(1, 183) =	2.43
				Prob > F =	0.1208
				R-squared =	0.0131
				Adj R-squared =	0.0077
Total	326750.578	184	1775.81836	Root MSE =	41.978

B_Croatia	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
B_US	-.2176878	.1396722	-1.56	0.121	-.4932628	.0578872
_cons	-.1782503	3.086547	-0.06	0.954	-6.268044	5.911543

Source: Author, using Stata12

Table 3.2.2.4: Augmented Dickey-Fuller test results on the residuals from the regression of "B_Croatia" to "B_US"

```
ADF Test for series:      Residuals
sample range:             [10, 185], T = 176
lagged differences:       8
no intercept, no time trend

1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -2.5350

intercept, no time trend

1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -2.5277

intercept, time trend

1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -2.5885
```

Source: Author, using JMulti 4.24

Finally, regressing the first differences of the values of the bubbles within the equity indexes on the Croatian and the US markets, we found a positive and significant relation between the "*FD_B_Croatia"* and the first lag of "*FD_B_US"* and also with the second lag of "*FD_B_Croatia"* (Table 3.2.2.5).The one period lagged first difference of the value of the Croatian equity bubble, "*FD_B_Croatia"* and the contemporaneous first difference of the value of the US equity bubble of the US equity market, "*FD_B_US*" both proved to be non-significant and were excluded from the model in first differences (Table 3.2.2.5). We conclude that with a lag of one month, bubbly dynamics, or the value of bubbly innovation on the US equity markets, translates positively into the bubbly dynamics on the Croatian equity market. We expand this relationship in our further research when exploring the domestic factors that could affect the bubbly outcome.

Table 3.2.2.5a: Regressing "FD_B_Croatia" on the first lag of "FD_B_US" and the second lag of "FD_B_Croatia"

```
. reg D.B_Croatia LD.B_US L2.D.B_Croatia

      Source |       SS       df       MS              Number of obs =     182
-------------+------------------------------           F(  2,   179) =   29.75
       Model |   3615.2592     2   1807.6296           Prob > F      =  0.0000
    Residual |  10877.6167   179  60.7688084           R-squared     =  0.2495
-------------+------------------------------           Adj R-squared =  0.2411
       Total |  14492.8759   181  80.0711376           Root MSE      =  7.7954

------------------------------------------------------------------------------
D.B_Croatia |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        B_US |
         LD. |   .9031449   .1247108     7.24   0.000     .6570523    1.149237
             |
   B_Croatia |
        L2D. |   .1797482   .0646471     2.78   0.006     .0521796    .3073167
             |
       _cons |  -.0206692   .5781308    -0.04   0.972    -1.161498    1.120159
------------------------------------------------------------------------------
```

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
4	2.791	4	0.5933

H0: no serial correlation

```
. estat hettest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of D.B_Croatia

         chi2(1)      =    26.93
         Prob > chi2  =   0.0000
```

```
. reg D.B_Croatia LD.B_US L2.D.B_Croatia, vce(robust)

Linear regression                               Number of obs =      182
                                                F(  2,   179) =    13.94
                                                Prob > F      =   0.0000
                                                R-squared     =   0.2495
                                                Root MSE      =   7.7954
```

D.B_Croatia	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf. Interval]	
B_US						
LD.	.9031449	.1754475	5.15	0.000	.5569335	1.249356
B_Croatia						
L2D.	.1797482	.0898089	2.00	0.047	.0025279	.3569684
_cons	-.0206692	.5708516	-0.04	0.971	-1.147134	1.105795

Source: Author, using Stata 12

Because we could not reject the test of no-heteroskedasticity of the residuals in the estimated linear model presented in Table 3.2.2.5a (first regression), we also reported a regression with robust standard errors (second regression). We found that the first month lag of "FD_B_US" variable positively and significantly impacts on the contemporaneous change of the Croatian equity bubbly "FD_B_Croatia" (p-value<0.001). In order to further confirm the exogenous character of the US market in our further modelling, or that the US market is not an endogenous variable in the process of the evolution of the Croatian equity bubble, we perform Granger causality analysis in JMulti presented in Table 3.2.2.5b.

Table 3.2.2.5b: Testing non-causality between "B_Croatia_D1" and "B_US_D1" using Granger-causality test statistics in JMulti 4.24

```
TEST FOR GRANGER-CAUSALITY:
H0: "B_Croatia_d1" do not Granger-cause "B_US_d1"

Test statistic l = 0.4344
pval-F( l; 2, 356) = 0.6480


TEST FOR GRANGER-CAUSALITY:
H0: "B_US_d1" do not Granger-cause "B_Croatia_d1"

Test statistic l = 27.4211
pval-F( l; 2, 356) = 0.0000
```

Source: Author, using JMulti 4.24

Looking at the results from Table 3.2.2.5b, we find supporting evidence on the assumption of an exogenous role of the US markets by rejecting the null hypothesis that the "B_US_D1" do not Granger-cause "B_Croatia_d1" at 99.9% level of confidence.

Now, as we have defined and analysed the variables measuring the value of the bubble within the equity market index in Croatia and in the US, we turn to the explanatory variables derived from our theoretical model in Chapter 2, which should explain the necessary and sufficient conditions for the occurrence of the equity asset bubble on the Croatian financial market.

3.3. Estimating the Dynamic Efficiency of the Croatian financial market "*DYNEF*"

The next important explanatory variable in our model is the one measuring the Dynamic Efficiency of the Croatian financial market. The Dynamic Efficiency of the financial market is not explicitly defined in our theoretical model but, based on the intuition from our theoretical model; it defines the necessary condition for the occurrence of the bubble and even more importantly the sufficient condition for the crash of rational asset bubbles. When the financial market is dynamically inefficient it then represents a fertile platform for the occurrence of a rational asset bubble, while if there is a bubble present on an already dynamically inefficient market and the market suddenly becomes dynamically efficient this transition would be a sufficient condition causing the bubble to collapse.

Dynamic efficiency represents a variable measuring the existence of the fundamental dynamic balance on the financial market, between the productive capital investment opportunities on the one side and the saving/investment needs on the other side. If there exists a misbalance between the two, characterizing the market as dynamically inefficient - represented by, for example, having much greater need for savings compared to the available productive capital investment opportunities - there will be an incentive creating a fertile environment for a positive bubbly asset to rationally occur and increase in value. On the other side, asset bubbles would collapse by becoming irrational in the situation when the financial market becomes dynamically efficient.

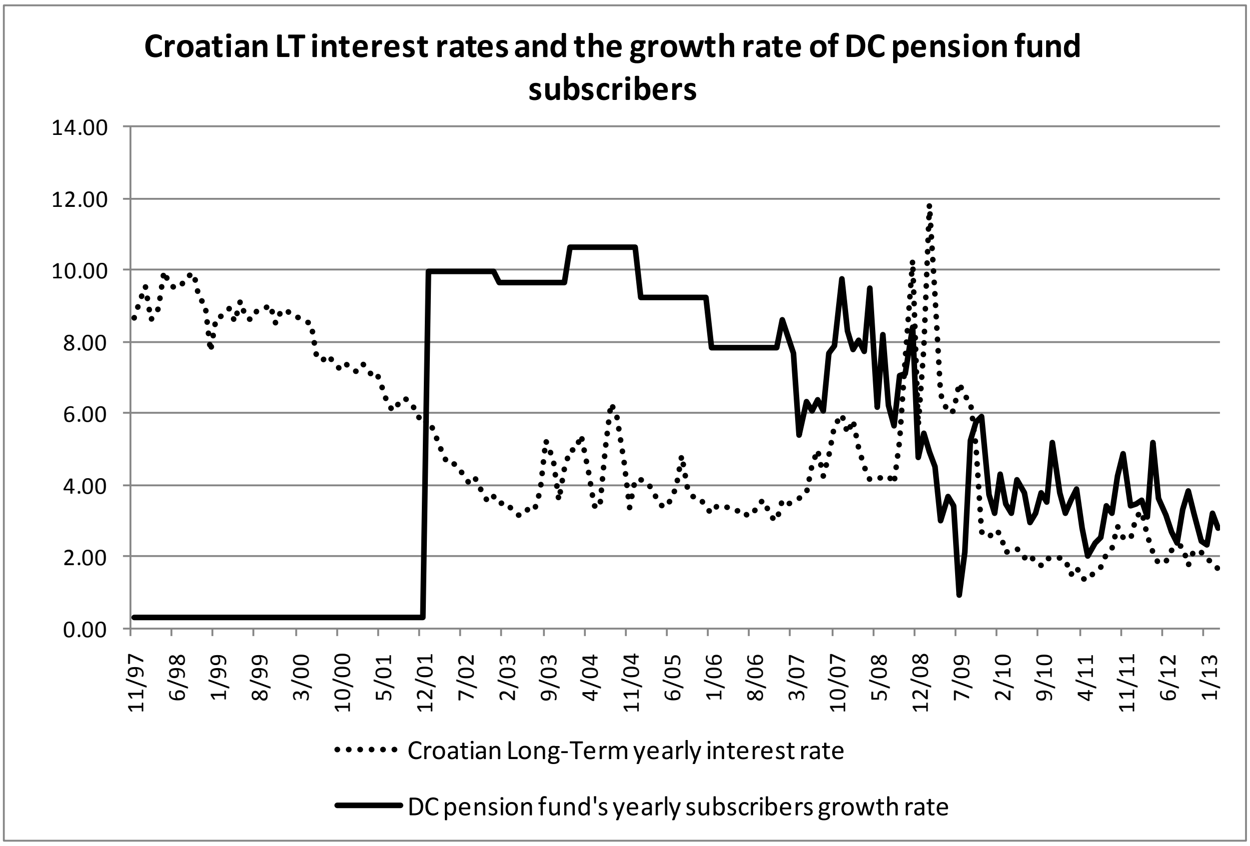
Based on the Tirole (1985) OLG model of Rational Asset Bubbles, the point when financial markets become a fertile environment for rational asset bubbles to occur is the moment when productive capital accumulates over the accumulation point at which it provides a rate of return lower or equal to the population growth rate *n*. This point is called the dynamic efficiency threshold of the financial market. So there are two components defining the measure of dynamic efficiency: first, is the growth rate of the demand for investments (savings to be invested on the financial market); and, second, the rate of return of the productive investments showing the supply capacity of the financial market. By comparing the two, we calculate the threshold used as a major criterion for assessing the dynamic efficiency of the financial market. As previously mentioned, Abel et al. (1989) criticizing the use of short-term government interest rates, suggested that the value of repurchase of equity plus dividends or some comparable rates of return on capital, such as long-term government bond interest rates, should be used to calculate the dynamic efficiency of the financial market by comparing them with the growth rate of the economy. But why is the growth rate of the economy or the population growth used for the calculation of the threshold criterion? Based in the theoretical model of rational asset bubbles made in the OLG framework, it determines the growth rate of investment demand (demand for saving), or the demand of investment assets used to transfer wealth from period one to period two in the model. The growth rate of investment demand is directly connected to the growth rate of the young population in the OLG structure. If this rate is higher than the rate of return provided by the productive capital investments, this is used as a sign of a suboptimal level of productive capital accumulation per young agent, which makes the occurrence of asset bubbles a rational outcome in order to avoid Pareto sub-optimal inter-temporal consumption outcomes in such a financial market economy.

In this respect, for the economy with a significantly underdeveloped financial market and DC pension funds, such as an early transition economy where the population lacks almost any tradition and experience of using the equity market for investments, the rate of growth of the new participants in the mandatory Defined Contribution Pension Fund could be the best estimator for the rate of growth of investment demand in the economy, especially for the part of the investment demand channelled through the equity market. This indicator plays a much better role in the context of the underdeveloped equity markets for the estimation of their dynamic efficiency than would the population growth rate, because of the low general participation of the population on the equity market. This is why we define our crucially important dynamic efficiency variable as a linear relationship between the Long-Term (LT) interest rates of the Croatian government bonds being a proxy of the risk-free rate of return of the productive capital in the economy on the one side, and the rate of growth of the subscribers to the DC pension funds on the other side. As Abel et al. (1989) discussed in their critique of the use of different rates of return to measure dynamic efficiency, long-term interest rates relate much more closely to the return on productive capital than do the short-term interest rates. For this reason, we used the long-term interest rates when measuring the dynamic efficiency of the Croatian financial market.

We collected data from the National Central Bank of Croatia on the dynamics of the Long-Term interest rate and compared it with the data representing the rate of growth of the new subscribers to the DC pension funds, which we collected from the Croatian pension funds reports and the Capital Market Supervision Agency HANFA (Data available in Appendix 3.1). For the period before the introduction of the DC pension funds, we extrapolated the data set using the 1997-2002 yearly average growth rate of population (0.3% p.a.). The near zero growth rate of the Croatian population in the years before the introduction of the DC pension funds, translates into a similar effect on the growth rate of investment demand, as if the participation rate prior to the introduction of the pension funds was zero. After the introduction of the pension funds, there was a real equity market shock with the rapid increase of investments demand placed on the financial market caused by the introduction of the DC pension funds.

On the other side, it could be seen that the Long-Term interest rates fall gradually after the introduction of the DC pension funds, especially in the first years after their introduction. This is a normal consequence of the increase of the amount of investable assets on the financial market, causing a gradual decrease in the financial market dynamic efficiency. We graphically present these two data series and their relationship in Figure 3.3.1.

Figure 3.3.1 Dynamics of the Croatian government Long-Term interest rates and the subscriber growth rate to the DC pension funds in the Republic of Croatia (in annual percentage points)



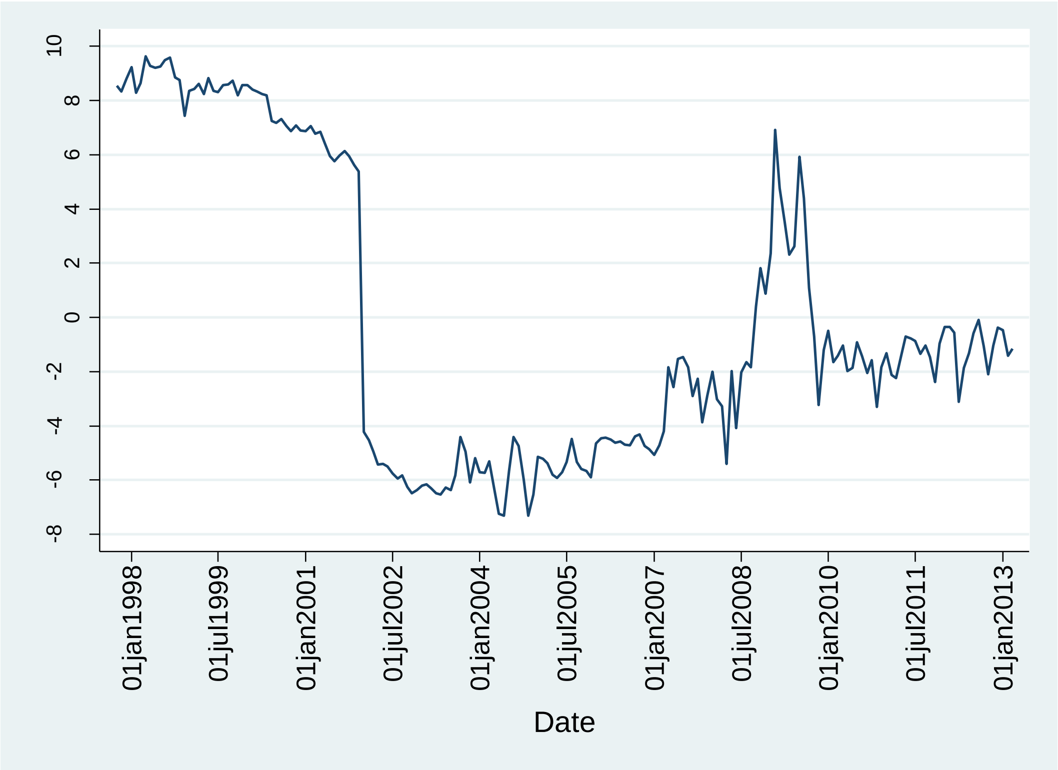
Source: Croatian Central Bank and HANFA – Croatian Capital Markets Regulation Agency

Using the two data sets presented in Figure 3.3.1, we created a new variable called "*DYNEF$_m$*" which measures the dynamic efficiency of the financial market at month *m*. We set "*DYNEF*" to take the values equal to the difference between the Republic of Croatia Long-Term interest rates and the rate of growth of the number of new members of the Croatian DC pension funds. When "*DYNEF*" takes positive values, it means that the financial market is dynamically efficient; and in the case when "*DYNEF*" takes negative values, the financial market becomes dynamically inefficient. We used the following formula to generate the values of the "*DYNEF*" time series variable (Formula 3.5).

$$DYNEF_m = LT\,interest\%_m - DCPensionFundSubscriberGrowth\%_m \tag{3.5}$$

The result from using Long Term interest rates data from the one side, and the DC pension funds subscription growth rates data from the other, plugged in our "*DYNEF*" *f*ormula, gave us the following Dynamic Efficiency variable (Figure 3.3.2).

Figure 3.3.2 Dynamic efficiency variable "*DYNEF*" for the financial market in Croatia



Source: Computed from data sourced by the National Bank of Croatia and HANFA, data on this graph in monthly data from the period 1997-2013

Due to the high level of interest rates of the Long-Term government debt in the period 1997-2001 (Figure 3.3.1), together with the low growth rate of population at an yearly average of 0.3% (the first 50 monthly observations), the resulting value of "*DYNEF*" was highly positive for this period (Figure 3.3.2), which is associated with a dynamically efficient financial market not favourable to the development of rational asset bubbles. Later, in 2002, when the DC pension funds were set in place, thereby rapidly changing the aggregate savings/investment attitude on the financial market, the growth rates of the pension fund subscriptions were especially high, which created a shock to the demand for investment assets required to be invested on the financial market. During the first period of the introduction of the DC pension funds (2002-2005), subscriber assets were predominantly invested into domestic government debt instruments. Decreased LT interest rates combined with the shocking increase of the rate of growth of investible assets (Figure 3.3.1), made the financial market dynamically inefficient in the first years after the introduction of the DC pension funds (Figure 3.3.2). DC pension fund subscription rates started a slow decrease after its peak rates at the introduction of the funds, which was normal as the subscription rates are expected to approach the long-term population growth rate. Besides the falling subscription rates in the DC pension funds after initiation, they were still very high in the first years (2002-2007), and because of significantly decreased LT interest rates in the same period (reaching near 3% in 2008 as compared to near 10% in 1997), caused the financial market to stay dynamically inefficient and the "*DYNEF*" variable to have negative values. This suggested a bubble-fertile financial market environment on the Croatian market.

In 2008 there was another external shock event in terms of the dynamic efficiency of the financial market caused by the global financial crisis. The trend of the decrease of the DC pension fund subscription rate continued, reaching a 3% annual growth rate compared to the 10% rates at the introduction of the system, but there was a significant shocking growth of the LT Government interest rates, caused as a consequence of the global financial crisis hitting the debt markets of the Republic of Croatia. This caused the financial market to sharply switch to being dynamically efficient once again in 2008, a state in which the necessary condition for the rational sustainability of asset bubbles no longer exists because, once again, productive capital investments provide a satisfactory return. Long-Term government interest rates are used as the best proxy for the return of the productive capital investments inspired by the research of Abel at al. (1989), as they represent a benchmark for the required premium of productive capital investments.

Based on the dynamic efficiency criteria, we assume that this rapid switch to the state of the financial market being dynamically efficient was one of the major causes for the crash of the equity asset bubble present on the Croatian equity market. The mechanism of the crash, following the logic of the dynamic efficiency of the financial market, was actually brought about as the major financial market actors, the Croatian DC pension funds, abandon their bubbly asset investments and turn to the LT debt assets as their main investment instrument.

Based on our augmented OLG model with rational asset bubbles, and based on the actions of the DC pension funds which allocated all their further investments into LT government bonds, increased dynamic efficiency of the market and the fear among individual investors caused by the Global Financial Crisis, crashed the Croatian equity bubble. The LT government interest rates stabilized once again but due to the decreased rate of growth of the DC pension funds subscribers rate, falling gradually from the initial near 10% to near 3% in 2010, the financial market stayed near the threshold value in terms of its dynamic efficiency from 2010 on (Figure 3.3.2).

In order to capture the effect of the turning point in the state of the dynamic efficiency of the financial market, we also generated a dummy variable named "dummy_DYNEF" which takes a value of 1 when the financial market is dynamically efficient (DYNEF>0) and value of 0 when the financial market becomes dynamically inefficient. We created this dummy variable, to be able to use it to check if the turning point gives sudden impact on the bubbly asset or the effect is smooth and dominated by the influence of the DYNEF variable.

We analyse and report the stationarity of "DYNEF" and its first order difference transformation "DYNEF_D1" in the last section of this Chapter. Now, we continue with the definition and measurement of our vector of variables representing the introduction and investments of the Croatian DC pension funds.

3.4 Vector of ‘τ’ variables describing the introduction of DC pension funds and their investments

Based on our theoretical model developed in Chapter 2, we determined that the necessary condition for the rational asset bubbles to occur is the presence of dynamic inefficiency on the financial market of the closed economy. This is also suggested by the basic model defined by Tirole (1985). The difference between Tirole’s model and our DC pension fund augmented version of the Rational Asset bubbles model is the inclusion of the DC pension funds, which subtracts part of each individual agent’s wealth and invests it into the equity market and in the Open-end Mutual Funds market, both identified as being potentially bubbly investment instruments. The dynamic outcome identified by our theoretical model of DC pension fund investment in the bubbly assets is a sufficient condition for a rational asset bubble to occur and increase in value.

Once again we stress that there is only one bubbly asset in our model and it is attached to the units of Croatian equity. Its value is represented by the value of Croatian equity over its intrinsic value as measured in the previous Section by the “B_Croatia” variable.

Because by investing in Croatian equity we could invest both in its intrinsic and in the bubbly value attached to the unit of equity, we can identify two potential investment vehicles providing investors with a market approach to investment in the bubbly asset: namely, direct equity investments; and indirect equity investments through the Open End Mutual Funds, which focus their investments in the Croatian Equity market. Later in this Chapter, we will explain in detail the measurement of the DC pension fund investments via those two investment vehicles. The distinction between the two, caused by different agency problems and asymmetry of information, creates the perspective among investors that the indirect investment represents the bubbly or speculative part of Croatian equity. In this case, we would observe different investment attitudes of the DC pension fund investors towards those two investment vehicles connected to the same underlying asset, Croatian equity, such that direct investments correspond to investment into productive assets, while indirect investments through the Open End Mutual Funds correspond to bubbly equity investments.

In the theoretical model we defined the introduction and investments of the DC pension funds as a vector of variables ‘τ’, which we now divide into two groups based on their character.

- **The first group** is represented by a set of four discrete dummy variables (“Dummy1”-“Dummy4”), which signify important temporal moments of the introduction of the legal acts important for the investments and introduction of the DC pension funds in Croatia.

- **The second group** represents continuous variables measuring the absolute monthly investments of the DC pension funds into the bubbly assets,
    - Croatian Equity and
    - the Croatian and SEE equity focused Open-end Mutual Funds.

In the following section we explain the collection process and the measurement of both sets of variables representing DC pension fund investments.

3.4.1. Dummy variables explaining the effect of the introduction of Legal Acts regulating the introduction and investments of DC Pension Funds in Croatia

In order to investigate different moments in time, when the DC pension funds were introduced, and when they were allowed to accept the bubbly assets such as Croatian equity or local equity related Open-end Investment Funds in their portfolios, we conducted detailed chronological research into the legal acts defining the pension reform and introduction of the second pillar of capital based DC pension funds in the Republic of Croatia.

There are two levels of legal acts, based on their judicial and practical importance, which govern the investment policy of the pension reform introducing DC pension funds in the Republic of Croatia. First, are the Legal Acts introduced by the Parliament; and the second are the Bi-Laws introduced by the DC Pension Fund’s regulatory Agency (HANFA).

Both those sets of acts have a determining importance for the introduction and for setting the investment scope of the DC pension funds, defining the general strategic allocation directions at the highest asset-class level and also the ones with respect of its acceptance of the potentially bubbly assets. Those legal acts are a prerequisite for the actual investment policies and investments of the DC pension funds, which are further decided within the DC pension fund management companies themselves and which will be discussed in the section on pension funds' investments.

As we previously discussed, Regulatory Acts are set at two levels. At the highest level stand the Legal Acts regulating the operation and investments of the DC pension funds and they set the broadest investment policy rules and qualitative and quantitative bounds for the acceptance of, and investment in, certain types of financial instruments. Compared to the Legal Acts, Bi-Laws are always more detailed and more strict, and are introduced by the pension fund regulatory agency. Bi-Laws incorporate more restrictive legal definitions for investment policy than the ones included in the Legal Acts. In the following Tables (Table 3.4.1.1a-b and Table 3.4.1.3a-b) we present a chronological account of the development and implementation of the two types of legal regulation with concise explanation of how each legal change affects the investment policies and the actual investments of the DC pension funds. We placed the mark "/" when there was no important legal change made affecting the bubbly asset investments. We mark with grey shading the potentially important changes in the definitions of the legal acts, which might significantly affect the investment policy of the DC pension funds toward the bubbly assets such as equity and open-end equity mutual funds.

Table 3.4.1.1a: Legal Acts governing the DC Pension Funds and their bubbly asset investment treatment

LEGAL ACTS			
Quarter	2Q1999	2Q2000	2Q2003
Reference	49/99	63/00	103/03
Articles regulating direct investments in Croatian Equity	Maximum 30% investment (5% per issuer) in Croatian Equity allowed; but the allowed issuer has to be quoted on the "first" official market	/	/
Articles regulating investments into Croatian Open-end Mutual Funds	Articles 69 and 72 required that Open-end Mutual Funds are quoted on the official Croatian market, which is not a common practice and it practically banned investments in the Croatian Open-end Mutual Funds.	/	Open-end investment funds moved to Article 69.9 and were **allowed** by Article 72.1, if they predominantly invest in Croatian Equity. Max 15% per fund investment allowed
Other important Articles for the investments into the bubbly assets	DC Pension funds, and the system was set to begin on 1.7.2000; Max 15% investments were allowed in foreign currency which practically focused most investments at home	The start of the DC pension funds and the reformed pension system postponed for 1.1.2002	/
exact date	7.5.1999	1.7.2000	12.6.2003

Source: (National Gazette of the Republic of Croatia, 1999/49; 2000/63; 2003/103; 2004/177) (National Gazette of the Republic of Croatia, 2007/71; 2010/124; 2011/114)

The Law regulating the introduction of the Pension Reform in the Republic of Croatia was introduced at the beginning of 1999. Although the Law assumed the middle of 2000 as a starting date for the new reformed pension system, this starting date was postponed until 1.1.2002, with the Revision of the Legal Act only a couple of days before 1.7.2000 when the system originally was set to start. Because this new starting date (1.1.2002) is the date signifying the actual start of the capital based DC pension system in the Republic of Croatia, we assigned the “Dummy1” variable a value of 0 up to this date and a value of 1 after this date. This is represented by the first green highlighted box in Table 3.4.1.1a.

The system started operating in January 2002, and the DC pension fund companies started collecting pension fund subscription amounts as a percentage of subscribers’ gross salaries and began investing them mostly into government bonds, which are a proxy for the productive capital instruments in our model, thereby affecting interest rates and the dynamic efficiency of the financial market. However, it was not until the middle of 2003 that potentially bubbly domestic instruments were for the first time allowed as an investment destination in the Legal Acts. At the beginning of the pension fund reform in 2002, the regulator allowed domestic equity and domestic open-end mutual fund investments only in Equity and Mutual Funds quoted on the “First official market” tier of the Croatian Stock Exchange, and those were only a few Companies and none of the Croatian Open-end Mutual Funds. For the part of the Open-end Mutual Funds, this practical ban on investments by the unfortunate legal definition was because for the Open-end investment funds it is technically unpractical to quote shares on the official market.

So investment into potentially bubbly Croatian equity focused Open-end Mutual Funds did not coincide with the introduction of the pension funds in Croatia in 2002. It was not until 7.2003 that the revision of the Legal Act regulating pension funds’ investments correcting the previous illogical condition, allowed for the first time investments into Croatian equity focused Open-end Mutual Funds, which fully invest into Croatian Equity. This was one of the major moments in terms of the Legal Acts, which concerned DC pension funds’ investments into this potentially bubbly asset. This is why we denote as “Dummy2” a variable with value of 0 until 7.2003 and 1 after this date, in order to capture the information about this important date for the bubbly investments in the model. We see further, when inspecting the investment decisions of the DC pension fund companies themselves from their balance sheets, that it was not until 2005 that the first investments into the Domestic Croatian Equity-focused Open-end Investment funds were actually made. This was because of the fact that, although the Domestic Open-end Mutual funds were allowed as an investment instrument in 2003, there was another qualitative requirement about each individual open-end investment fund set down by the Regulatory Agency’s Bi-Laws, which made none of the actual Croatian Open-end Investment funds an allowable investment instrument until 2005.

Table 3.4.1.1.b: Legal Acts governing the DC Pension Funds and their bubbly asset investment treatment

LEGAL ACTS				
Quarter	4Q2004	3Q2007	3Q2010	3Q2011
Reference	177/04	71/07	124/10	114/11
Articles regulating direct investments in Croatian Equity	/	Restriction for the required quotation on the "first tier" market was changed to the broad "official" Croatian market	/	/
Articles regulating investments into Croatian Open-end Mutual Funds	/	Maximum increased to 30% (5% per fund), with no restriction about the investments as long as registered in Croatia or OEDC	/	/
Other important Articles for the investments into the bubbly assets	/	Max 30% in foreign currency assets was allowed, an increase from previous 15%	Max. 70% equity and corporate bonds in the fund. Also venture capital investments allowed.	/
exact date	10.12.2004	21.6.2007	22.10.2010	30.09.2011

Source: (National Gazette of the Republic of Croatia, 1999/49; 2000/63; 2003/103; 2004/177) (National Gazette of the Republic of Croatia, 2007/71; 2010/124; 2011/114)

The Bi-Laws defining more strict rules on investments in the Croatian Equity-Focused Open-end Mutual funds, required for the funds' Net Asset Value (NAV) to be a minimum of 10 million EUR, in order for the Open-end Mutual Funds to be allowed as an acceptable investment instrument forthe DC pension funds' portfolios.

In 2003, when Open-end investment funds were practically allowed for the first time by the Legal Acts, there were only 11 Domestic Open-end Mutual funds satisfying this minimum of 10 million EUR NAV condition, and all of them were non-equity related (i.e. money market and bond funds, which are not suitable instruments for indirectly reaching Croatian and SEE bubbly equity assets). In 2004, the number of allowed Open-end Investment Funds based on the NAV condition increased to 17, but only 2 of them were investing in Croatian Equity. By 2005, the number of Croatian Equity-focused Domestic Open-end Mutual Funds increased to 3 but the number of such funds surged in 2006 to 13. So in the period 2005/2006 there was a significant increase in number of Domestic Croatian Equity-focused Open-end Investment funds, which were allowed as an investment destination for the Croatian DC Pension Funds, and consequently DC pension funds poured a significant amount of assets into those instruments (Table 3.4.1.2 and Figure 3.4.1.1).

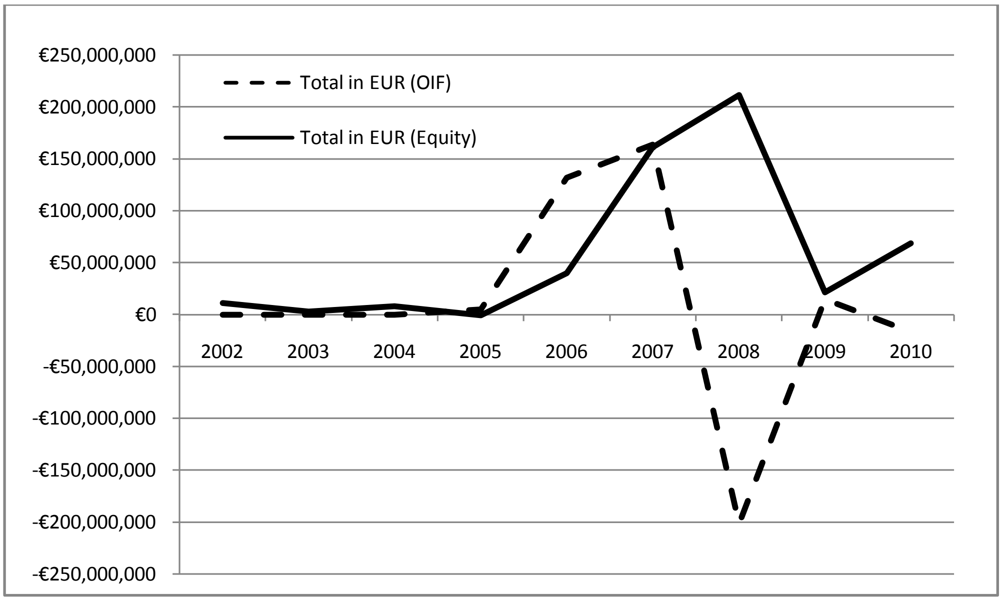
Table 3.4.1.2 Estimated DC Pension Fund yearly new investment/disinvestment into Croatian Equity and Croatian/SEE Equity Open-end Mutual funds in the period 2002-2010

Year	2002	2003	2004	2005	2006
Total in EUR	**€11,204,267**	**€3,099,600**	**€8,201,733**	**€4,436,133**	**€171,997,800**
Croatian Equity	€11,204,267	€3,099,600	€8,201,733	-€744,933	€39,914,133
Croatian and SEE equity focused open-end mutual funds	€0	€0	€0	€5,181,067	€132,083,667
* Based on the Financial Statements of the four DC pension Funds					
** 2009 data of AZ OMF were interpolated based on the average change of the rest of 3 funds					

Year	2007	2008	2009	2010
Total in EUR	**€324,949,200**	**€10,051,413**	**€36,009,331**	**€52,218,711**
Croatian Equity	€161,223,600	€211,557,643	€21,254,457	€68,879,628
Croatian and SEE equity focused open-end mutual funds	€163,725,600	-€201,506,230	€14,754,875	-€16,660,917
* Based on the Financial Statements of the four DC pension Funds				
** 2009 data of AZ OMF were interpolated based on the average change of the rest of 3 funds				

Source: Data derived from the Audited Yearly Financial Statements of the four DC pension funds

Figure 3.4.1.1 DC Pension Fund yearly new investment/disinvestment into Croatian Equity and Croatian and SEE Equity focused Open-end Mutual funds (OIF) in the period 2002-2010



Source:Data derived from the Audited Yearly Financial Statements of the four DC pension funds

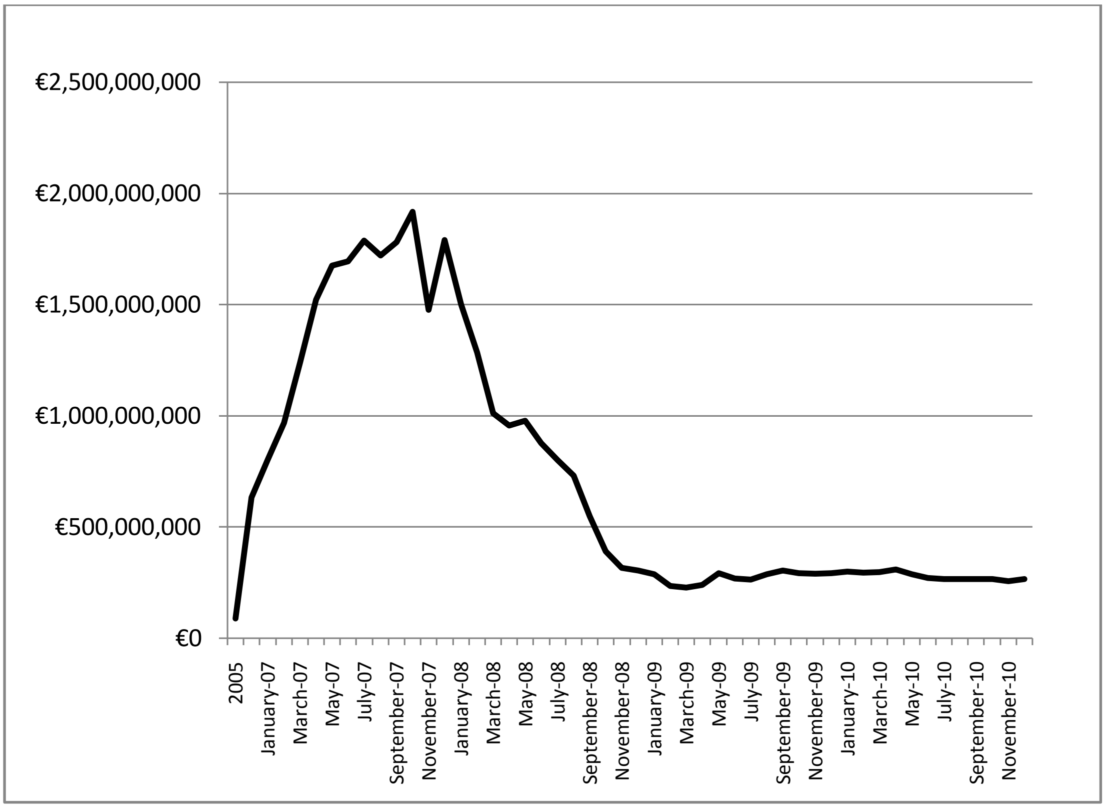
The peak of the popularity of the SEE and Croatian Equity-focused Open-end Mutual Funds, satisfying the condition of having NAV over 10 million EUR, reached over 20 such funds during the period 2007-2008, and suddenly dropped to 11 in 2009 with the crash of the bubble. So, practically, although the middle of 2003 was the year when Legal Acts for the first time allowed the acceptance of the Croatian and SEE equity focused Open-end Mutual funds, because of the Bi-Laws quantitative condition requiring those funds to have a minimum of 10 million EUR AUM in order for them to be allowed as an investment destination, there were no investments in such instruments until the end of the year 2005. The same trend is presented also by the assets under management of the portfolio of Croatian and SEE-equity focused Open-end Investment funds which were used by the Croatian DC pension funds as an investment destination presented in Table 3.4.1.3 and Figure 3.4.1.2. This group of Open-end Mutual Funds managed barely 0.1 million EUR in 1999 increasing to 23.8 million in 2004 and experienced their boom reaching 1788 million EUR assets under management at the peak of the asset bubble at the end of 2007 (Table 3.4.1.3 and Figure 3.4.1.2). After the crash of the bubble, their assets under management dropped from nearly 2 billion EUR to only 267 million EUR in 2010.

Table 3.4.1.3: End of year assets under management (AUM) of the Croatian Open-end and SEE Equity-focused Mutual Funds used as an investment destination by the Croatian DC pension funds in the period 1999-2010 (in EURO)

Year	1999	2000	2001	2002	2003	2004
End of Year AUM (EUR)	105,178	492,052	1,283,321	7,534,705	12,055,974	23,886,065
CROBEX corrected End of Year AUM (EUR)	105,178	395,467	887,170	4,596,403	7,276,635	10,911,887
inflow/outflow based on CROBEX corrected AUM (EUR)	105,178	290,289	491,702	3,709,233	2,680,232	3,635,252
Year	2005	2006	2007	2008	2009	2010
End of Year AUM (EUR)	89,138,695	633,688,951	1,788,119,192	303,937,532	292,215,537	267,450,797
CROBEX corrected End of Year AUM (EUR)	31,920,322	141,232,861	244,140,526	126,235,771	104,300,618	90,628,410
inflow/outflow based on CROBEX corrected AUM (EUR)	21,008,435	109,312,540	102,907,665	-117,904,755	-21,935,153	-13,672,208

Source: HANFA – Croatian financial market regulatory agency and Audited Reports of the four DC pension funds for the period 2002-2010.

Figure 3.4.1.2: Graphical presentation of the end of year assets under management (AUM) of the Croatian and SEE Open-end Equity-focused Mutual Funds used as an investment destination of the Croatian DC pension funds (1999-2010)



Source: HANFA – Croatian financial market regulatory agency and Audited Reports of the four DC pension funds for the period 2002-2010

The second tier of the legal acts defining the DC pension fund operations and investments, having an equal and even more defining effect on investment into potentially bubbly assets for the Croatian DC pension funds, are the Bi-Laws. These are set and introduced by the Croatian Financial Markets Regulatory Authority (HANFA). The Bi-Laws have a more detailed and more restrictive role in defining the quantitative and qualitative criteria for the allowable investments of the DC pension funds. Table 3.4.1.4a-b, gives a detailed description of the legal conditions included in the Bi-Laws, which impact on the investments of the DC Pension Funds.

Table 3.4.1.4a-b: Bi-Laws governing the DC Pension Funds and their bubbly asset investment treatment (important changes are highlighted in grey)

Bi-Laws					
Quarter	1Q2002	4Q2002	3Q2003	4Q2003	4Q2007
Reference	35/02	128/02	144/03	200/03	129/07
Articles regulating direct investments in Croatian Equity	Croatian Equity investments must be lower than 5% per issuer; Equity must be quoted on the "prva sluzbena kotacija" and have minimum 10 million EUR market cap.	\	\	\	max 30%; 5% per issuer; Broader "official market" Croatian equity allowed; minimum 750 mio HRK market cap. or min 1000 mio HRK market cap. for shares with less than 20% free float at the other market segments. Also max 15% investments into the other than the official market segment.
Articles regulating investments into Croatian Open-end Mutual Funds	Maximum 5% into individual fund issue; Net Asset Value of the fund must be over 10 mio EUR and min 80% of funds' invested into OECD investment grade or Croatian investments without quality restriction	\	\	\	max 5% per issuer; max. 20% of the mutual fund; no allocation restrictions; fund must have min 100 mio HRK AUM
Other important Articles for the investments into the bubbly assets	\	Only regulating investments into mutually connected entities	\	\	\
exact date	22.3.2002	23.10.2002	28.8.2003	15.12.2003	13.12.2007

Bi-Laws					
Quarter	1Q2008	3Q2008	4Q2008	1Q2009	4Q2009
Reference	32/08	112/08	121/08; 02/09	09/09	141/09
Articles regulating direct investments in Croatian Equity	Further relaxed investment requirements for the Croatian Equity. Allowed up to 10% of the issue from the "official quotation" market; min 650 mio HRK market cap. Required for the other market quoted equity	Minor change: Croatian companies with less than 10% free float of their equity, must have over 100 mio EUR market cap	\	Further Relaxed direct investments in the domestic stocks	Relaxed investment in domestic equity on the secondary market. Market cap requirement lowered from min 650 to min 300 mio HRK.
Articles regulating investments into Croatian Open-end Mutual Funds	DC Pension Fund must publish every significant exit/entry transactions into Open-end Mutual Funds defined as every over 10% transaction when the mutual fund has over 400 mio HRK AUM; or when buying units of the mutual fund valued over their last 10 day average price.	\	\	\	\
Other important Articles for the investments into the bubbly assets	Pension Fund must publish every transaction higher than 25% of the daily stock market trade volume	\	Every foreign transaction amounting over 5mio EUR be pre-announced to the Regulatory Agency	\	\
exact date	13.03.2008	9.2008	10.2008	1.2009	26.11.2009

Source: (National Gazette of the Republic of Croatia, 2002/35; 2002/128; 2003/144; 2003/200) (National Gazette of the Republic of Croatia, 2007/129; 2008/32; 2008/112; 2008/121) (National Gazette of the Republic of Croatia, 2009/02; 2009/09; 2009/141)

Based on Table 3.4.1.4a-b, representing the changes of the Bi-Laws regulating the investments of the DC pension funds, there are two additional dates with potential importance for the investments of the DC pension funds in the potentially bubbly assets on the financial market in the Republic of Croatia (highlighted in gray in Table 3.4.1.4a-b). We take account of their potential effects, by the following two dummy variables indicating the introduction dates of those two Bi-laws:

- "Dummy3", representing the change introduced at the end of 2007 (129/07) which, for the first time, based on the previous change of the Legal Act just few months earlier, allowed a broad direct investment into Croatian Equity. Besides permission to invest in the small number of equities from the "first official" tier of the financial market and to approach equity investment indirectly via open-end mutual funds, now DC pension funds were able to invest into a broad range of Croatian equity even from the “secondary tier” of the market. Based on this regulatory change we define “dummy3” such that it takes the value of 1 after 12.2007 and 0 up to and including this month;

- “Dummy 4”, is the final dummy variable, indicating a more minor legal change that might have a negative impact on the already large investments into the Croatian Open-end mutual funds. It represents the change made in the March 2008 Bi-laws revisions, when for the first time DC pension funds were obligated to publish all market-significant transactions they made (valued at more than 25% of the average market value of the Croatian market). This could have triggered significant avoidance of further investments on the Croatian equity market and even exit from the open-end investment funds registered in Croatia. We define “Dummy4” with the value of 1 after this month and 0 before and at this month. In our opinion, based on the arguments of the agency-theory school on asset bubbles, the Croatian mutual funds industry, especially the part of it representing Open-end Mutual funds focused on Croatian and regional SEE equity (financial markets such as Macedonia, Serbia, BiH and Slovenia), could have presented a bridge for the DC pension fund industry to the local market bubbly equity, which was practically banned from direct investment until the end of 2007. So this legal change represented by “Dummy4”could capture an important change in the opposite direction for investments in the Croatian open-end mutual fund industry.

Finally, we should note that the dummy variables representing changes in the legal acts and bi-laws, should be used with a lag of one month in the empirical modelling, because some of the regulations were published just before the end of the month and there is a legal period of time, usually 7-8 business days, which has to pass for the legal act to become enforceable after being published in the National Gazette. A one month lag would also allow for the DC pension funds to adapt their investment attitude based on the new regulation.

3.4.2. Continuous variables explaining the actual investments of the Croatian DC pension funds into Croatian Equity and SEE equity focused Open-end Mutual Funds

We determined that the **"τ"** vector of variables consists of two subsets. One, which we explained in the previous subsection, consisting of four dummy variables describing certain periods, at end-of-month frequency, during which important legislative acts were introduced connected to investments of the DC pension funds in potentially bubbly assets. The next important set of variables comprising the remainder of **"τ"**, measuring the investment of the DC pension funds in the bubbly assets, are the actual net investment amounts of the DC pension funds in Croatian Equity and in the Croatian and SEE equity-focused Open-end mutual funds. This is very important data for our empirical analysis, which is not freely available. Accordingly, we had to make an in-depth analysis of every audited yearly report of each of the four DC pension funds in the Republic of Croatia for the period 2002-2010 in order to extract this data.

We encountered several problematic issues when collecting this data set. The first was to determine the net investment amount such that it is free from the period change of the price of the CROBEX equity market index, which on the other side enters our B_m(Croatia) variable representing the value of the equity bubble. When looking at the data about the investments in Croatian Equity and about the investments in Croatian SEE equity-focused Open-end investment funds available in the DC pension funds yearly audited reports, we find three figures in the balance sheet and the profit and loss report (P&L Report) that contain information about the investment positions in the potentially bubbly assets: the beginning of the year market value of each asset held in the portfolio; the end of the year market value of each asset held in the portfolio; and the purchasing costs of investments in the specific asset during the accounting year. Both the beginning of the year and the end of the year market values of investment are subject to direct influence by changes in the market prices of the asset (equity or open-end mutual fund), which is linearly affected by the market value of the bubble. This linear relation would lead to spurious regression results were we to use the change of the market value between the beginning and the end of the accounting period as an explanatory variable for the value of the bubble. In this case, the pure calculation of the absolute change of the value of a certain investment position based on the difference between the beginning and end of period asset market values would contain in itself the value of the bubble and, hence, could lead to a biased conclusion about the impact of DC pension fund investments when we include this variable in a regression explaining the value of the equity asset bubble as the dependant variable.

The way to overcome this problem is based on understanding the accounting rules. At the beginning of each accounting year, the accounting position named "purchasing costs of investments" is set at the amount of the market value of the specific asset established at the last day of the previous accounting year and then adds new investments during the current year according to their purchasing costs. So, if there are no new investments, then the difference of the "purchasing costs of investment" and the previous end of the year market value of investments would equal zero. Any positive difference would signify the new amounts of net investment into a certain asset, while any negative difference would suggest that there was a net disinvestment from that particular asset. So, by subtracting the end of the previous period market value of the investment figure in a certain asset from the end of the current year purchasing value of that asset, we get the net amount of investment value in that particular asset during the current accounting year, free from the price effect[11]. Consequently, the method that we use to establish the net investment amounts in a particular potentially bubbly asset by the DC pension funds is presented by Equation 3.6.

$$NewInvestment(A)_t = Purcha\sin gCost(A)_t - EndOfYearMarketValue(A)_{t-1} \quad (3.6)$$

We applied this approach and derived a yearly time series of the net investment amounts in Croatian Equity and in the Croatian and SEE equity-focused Open-end mutual funds for all four DC pension funds.

[11] This could be explained by a short example of a Pension Fund equity transaction. Assume a Pension Fund owns 1 million stocks that have a market price of £10 p.s on 31.12.2000, and assume the fund purchase 100.000 shares at a price of £14 p.s. during the year 2001 (Spending total of £ 1.400.000). Assume the price at the end of the year on 31.12.2001 reaches £13 and the new value of stocks held by the Pension Fund is 13*1100000=£14300000. In the Balance Sheet for 2000 we can only see the value of the end-of-year assets equalling £ 10.000.000, and at the end of 2001 balance sheet this value is £14.300.000. In the Income statement of the 2001 Report, we can't find the 1.400.000 spent as a separate position, but only the position Cost of Purchase, which represents the value of the equity at the end of the previous accounting year at the last day of the year plus all cash expenses for stock purchase in the current year. This position in our example will amount to £11.400.000 in the Income statement of 2001. If we subtract from it 10.000.000, the end-of-year assets value from the Balance Sheet of the previous year (2000), we are left with the £ 1.400.000, the exact amount spent on purchasing stocks free of asset price appreciation.

The next problem we faced had to do with transparency. Namely, for unknown reasons, one of the DC pension funds did not published the detailed structure of its investments at the end of 2009 (Croatian DC pension funds presented in Table 3.4.2.1). This was the Allianz managed AZ OMF fund, which had around a 30% market share in 2009. This was also one of the more aggressive funds in terms of the amount invested in, and later disinvested from, Croatian Equity and the Croatian and SEE equity-focused Open-end mutual funds. Actually in the spectrum of aggressiveness, only the RZB OMF invested more in the Croatian and SEE Equity-focused Open-end Investment funds than did AZ.

Table 3.4.2.1 Names of the Croatian DC pension funds and their pension fund managing companies

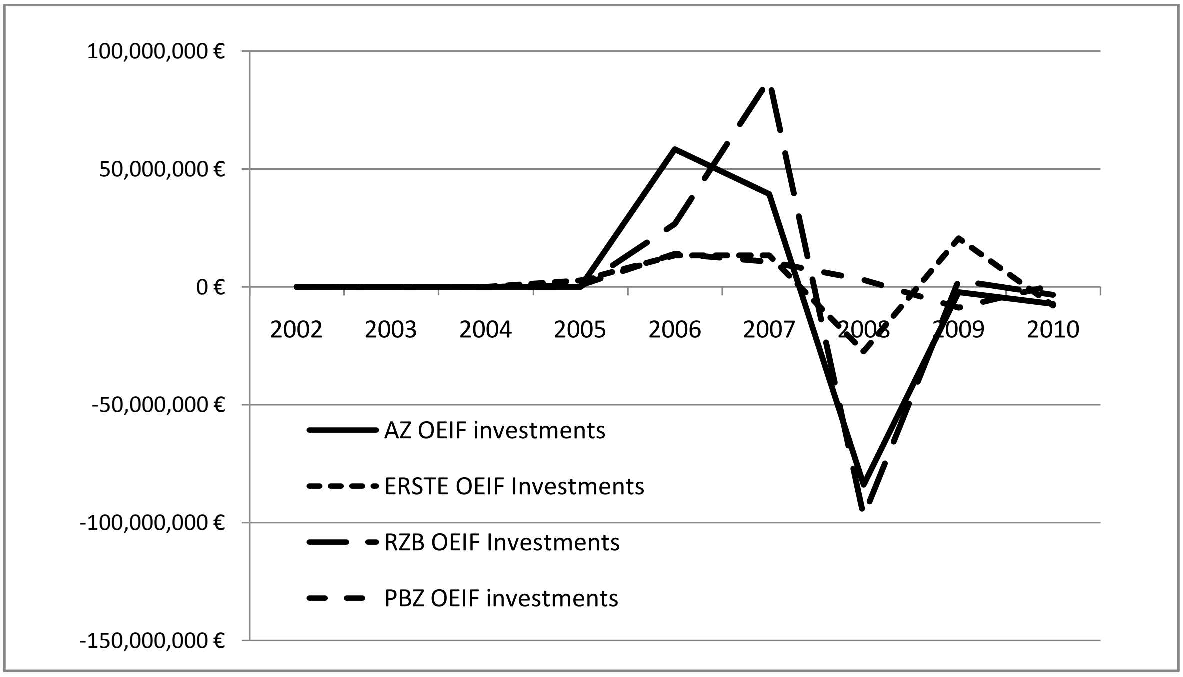
Number	1	2	3	4
Name of the DC pension Fund	AZ OMF	RZB OMF	ERSTE PLAVI OMF	PBZ Croatia Osiguranje OMF
Name of the pension fund management company	AZ Mirovinski Fondovi d.d.	Raiffeisen Mirovinsko Drustvo d.d.	Erste d.o.o.	PBZ Croatia Osiguranje d.d
Owners of the pension fund management company*	Allianz New Europe Holding Gmbh and Zagrebacka Banka	Raiffeisenbank Austria d.d. Zagreb	Erste and Seiermarkische Bank d.d.; Erste Group Bank AG; Steiermarkishe Bank und Sparkassen AG; Zavarovalnica Triglav; TBIH Financial Services Group NV; Istarska Kreditna Banka d.d.	Privredna Banka Zagreb and Croatia Osiguranje d.d.

* Based on the company data from December 2013
Source: Data taken from the Croatian DC pension funds web pages.

We tried contacting the AZ fund management company by e-mail, requesting the investment structure for the year 2009, but received no answer. There was only a figure available for both the total value of equity and total value of open-end investment fund positions at the end of the year in the audited report for 2009, which was not a common practice until that date, and so we had to use a derivation method to estimate the net purchasing value of Croatian Equity and SEE equity-focused Open-end Mutual funds; namely, by interpolating data for 2009, based on average ratios of end of year market value of asset class positions to the value of year purchasing costs taken from the other three DC pension funds representing the rest, cca. 70%, of the market share, and knowing that DC pension funds in Croatia including the AZ Fund had a very similar yearly attitude toward investments in those two asset classes in the other periods (Figure 3.4.2.1).

AZ OMF published the detailed structure once again in 2010, which showed that it decreased its 2007 end of year investment position by nearly EUR 113 million into Croatian and SEE equity-focused Open-end investment funds (about 35mio at the end of 2008) to zero in 2010. Based on the interpolation, using information from the other three funds and the value of the AZ OMF aggregated Open-end investment fund “purchasing costs” position for the end of 2009, published as 27.1 mio EUR, using our equation 3.6 we calculated the position of the new net investment into Croatian and SEE equity related Open-end Mutual Funds of AZ pension fund at negative 2.12 mio EUR in 2009, a continuation of disinvestment after selling over 77 mio in 2008.

Figure 3.4.2.1: Individual DC pension fund’s net yearly amounts invested in Croatian and SEE equity focused Open-end Investment Funds

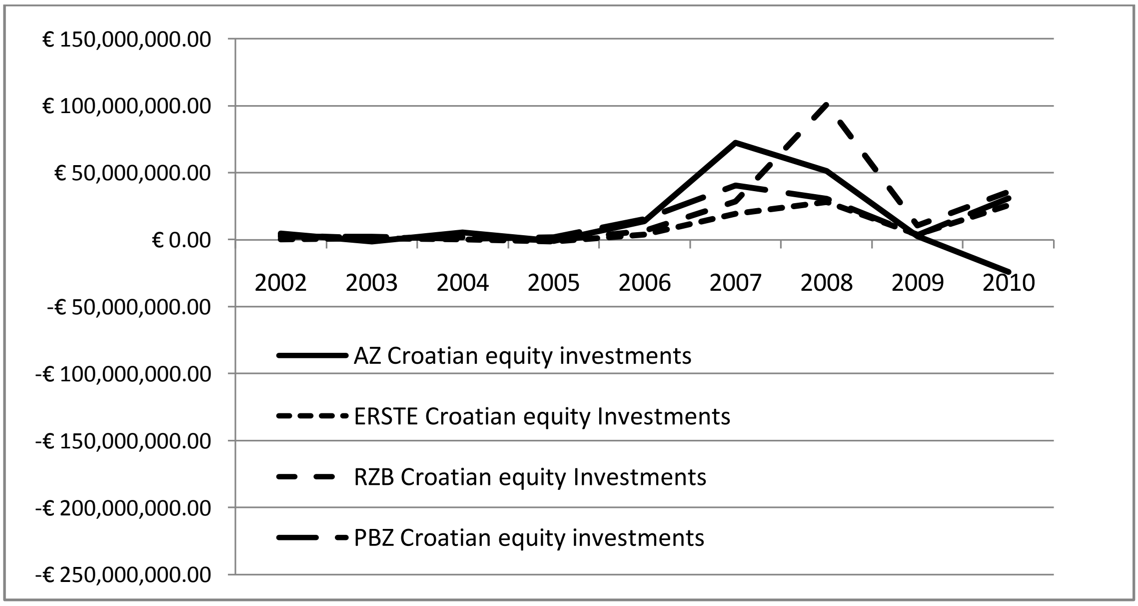


Source: Data derived from fund’s audited yearly reports for the period 2002-2010

We faced the same problem with the end of 2009 data in the audited report of AZ OMF when measuring the variable representing the direct investment into Croatian Equity. Again, we overcame the problem of having only the year end figure for the market value of Croatian equity at the end of 2009, and not the net purchasing cost during the year, by estimating the net purchasing value in that year by using the average ratio of net purchasing value to year end value from the rest of the three funds and applying this ratio to the end of the year value of Croatian Equity of the AZ OMF for 2009.

We have also corrected the value of the direct equity investments into Croatian equity by subtracting the net investments made in the HT (Croatia Telecom) from the data series. We did this because the Croatian Telekom (HT) had its Initial Public Offering (IPO) in October 2007 at the peak of the equity bubble, and since our model is based on the assumption of rigid supply of the bubbly asset, we subtracted the this company from the pool of potentially bubbly equity instruments on the Croatian Market. We also subtracted the minor investments that the DC pension funds had in the Global Depository Receipts (GDR's) of domestic companies in foreign markets. As a result, we established the following structure and dynamics of absolute direct investment into Croatian Equity made by the four Croatian DC pension funds (Figure 3.4.2.2).

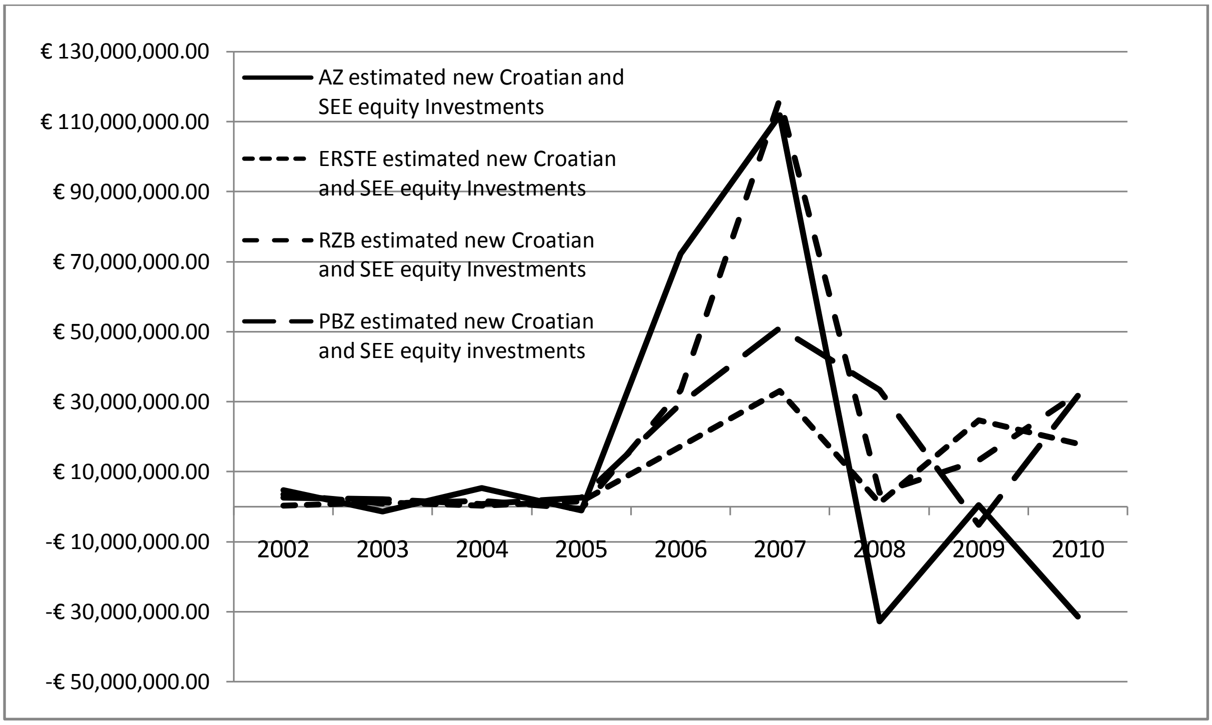
Figure 3.4.2.2: Individual DC pension fund's net yearly amounts invested directly into Croatian equity (excluding GDR's and the stocks of Croatian Telecom – HT)



Source: Data derived from fund's audited yearly reports for the period 2002-2010

Finally, we were interested also to estimate the value of the total from the direct and indirect investments (through Croatian and SEE equity focused open-end investment funds) into the Croatian and SEE equity market made by the Croatian DC pension funds. For this purpose, we summed our two series and arrived at our final series estimating the amount of total direct and indirect investment into the Croatian and SEE equity markets, made by the Croatian DC pension funds in the period between 2002 and 2010 (Figure 3.4.2.3). What could be noticed by looking at Figure 3.4.2.3 is that, starting from 2005 onward, the DC pension funds were rapidly increasing their yearly new net exposure to Croatian and SEE equity-focused open-end mutual funds. This was initiated first in 2005 by investing indirectly over 3.4 million EUR through the Croatian and SEE equity-focused Open-end Mutual funds. Interestingly, in the same year direct investment of the DC pension funds in Croatian equity was negative (a disinvestment of 0.7 million EUR). In 2006, DC pension funds rapidly increased their net new investment in Croatian and SEE equity focused open-end mutual funds by 112 million EUR. During the same year, they also increased their direct exposure to Croatian equity by investing close to 40 additional million EUR, or less than half of what they invested in the Croatian and SEE equity-focused open-end investment funds. 2006 was the year of a wide acceptance of the bubbly asset into their portfolios, increasing the total new investment into the Croatian and SEE equity markets from 2.7 to over 150 million of net new investments, or more than 56 times. Finally, at the peak of the Croatian bubble in 2007, their investments peaked at over 150 million EUR in each of the two observed asset classes, jointly reaching over 300 million EUR of new investments into Croatian and SEE equity related instruments (ex. HT and GDR's) or almost two fold increase compared to the previous year (Figure 3.4.2.3).

Figure 3.4.2.3: Individual DC pension fund's net yearly new amounts invested directly and indirectly into the Croatian and SEE equity markets (excluding GDR's and the stocks of Croatian Telecom – HT)



Source: Data derived from fund's audited yearly reports of the DC pension funds for the period 2002-2010

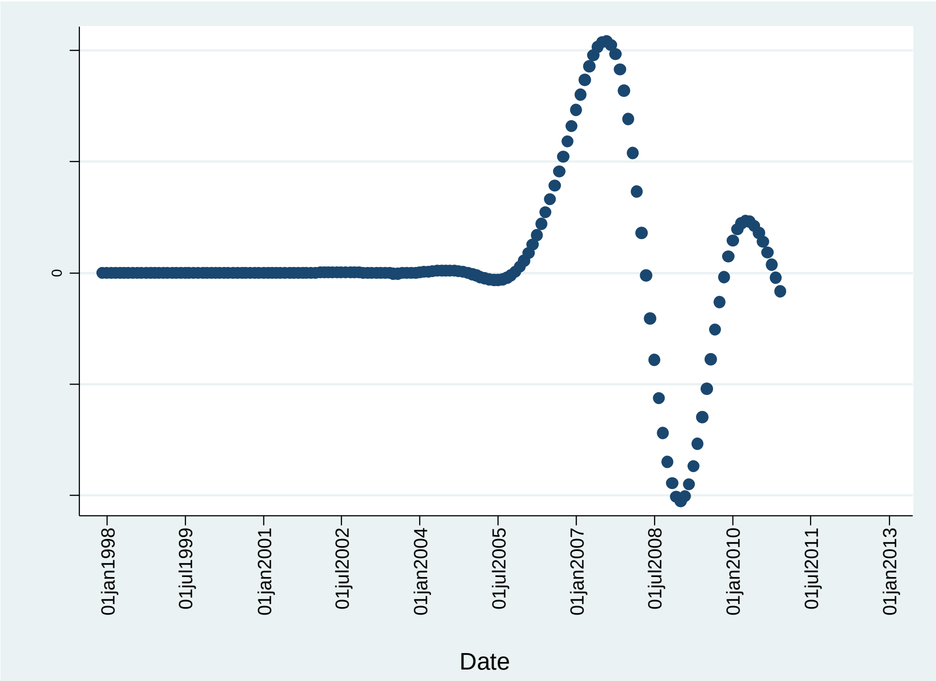
However, in 2008, the situation rapidly changed, but much more rapidly in terms of DC pension fund exposure in the Croatian and SEE equity-focused Open-end investment funds than in terms of their direct exposure to Croatian equity. Croatian DC pension funds suddenly decided to exit Croatian and SEE equity-focused Open-end mutual funds (with over 200 million EUR of net sales), while at the same time continuing their positive direct investment into Croatian Equity by almost the same amount of 200 million EUR. (This was actually the year when they for the first time got legal permission for broad equity investment in most stocks onthe Croatian market.) Their exit from the Croatian and SEE equity-focused Open-end mutual funds investment fully compensated their increased direct investment in Croatian equities. So their net new total investment on the Croatian and SEE equity markets (excluding HT and GDR's) in 2008 was close to zero. After 2008, DC Pension funds continued only with their direct investment into the Croatian Equity market, but with much slower and decreasing amounts of new investments, while continuously exiting their Croatian and SEE equity-focused Open-end Investment fund's positions. The new total SEE and Croatian direct and indirect equity investment achieved only marginal importance after 2008.

Based on our analysis of the audited balance sheets of the DC pension funds in Croatia in the period 2002-2010, we derived two important time series for our empirical analysis. First, concerning the direct investment of the DC pension funds into Croatian equity, a variable which we named "$\text{PFInvDirect}_{\mathbf{y}}$"; and the second, concerning the indirect investments through the Croatian and SEE equity-focused Open-end Mutual funds, which we named "$\text{PFInvIndirect}_{\mathbf{y}}$". Both variables represented new investment amounts made by the DC pension funds in those two connected potentially bubbly asset classes measured at a yearly frequency. There was one large problem, and that was the fact that all our other data are measured at higher monthly frequencies, in contrast to these two important variables. This mismatch is due to the fact that detailed investment data are presented and made available only in the audited end-of-year reports of the DC pension funds.

To solve this issue, we had to find a way to interpolate monthly frequency data out of this yearly frequency data set. For this purpose, we used a method called "Cubic-spline" interpolation (Jianquing & Qiwei, 2005). The cubic-spline method, interpolates the higher frequency values using the stochastic lower frequency time series by defining a cubic function passing through the lower frequency data points. This polynomial (cubic) function, which fits the lower frequency data, enables us to impute the missing higher frequency values, and create monthly frequency investment variables.

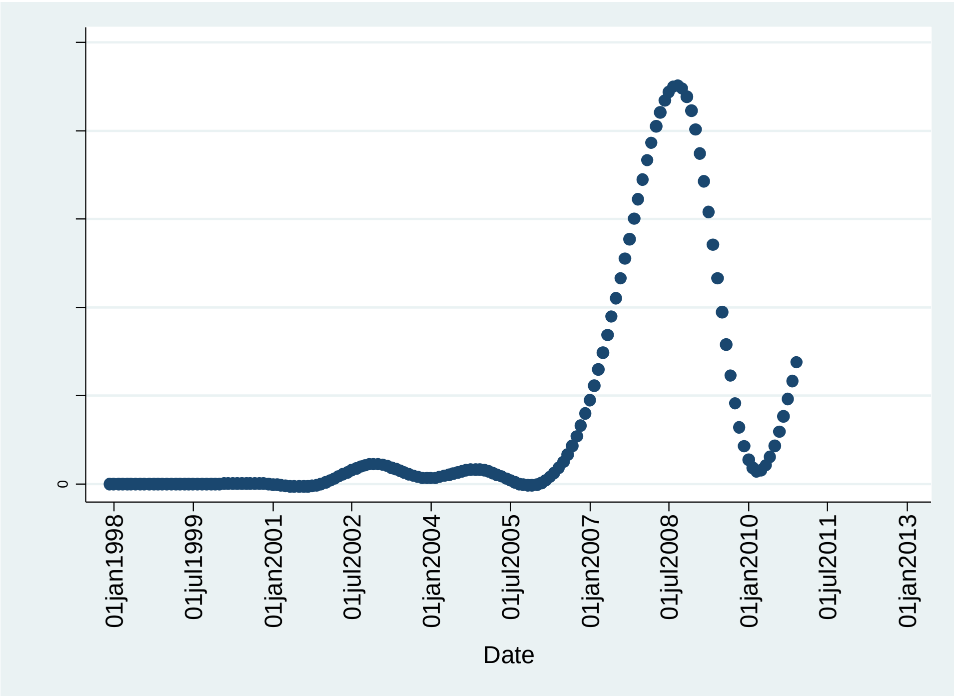
We used a STATA program available on-line, which we adjusted to our frequency and matrix dimensions, and transformed our yearly stochastic data series into a monthly series. We present the sample STATA program in Appendix III.2. As a result of the transformation we created the following monthly series estimating the two investment variables of the DC pension funds' direct and indirect investment into Croatian and SEE equity markets (Figure 3.4.2.4a-b).

Figure 3.4.2.4a: Monthly frequency data of the DC Pension funds net new investments into Croatian and SEE equity-focused open-end mutual funds ($PFInvIndirect_m$) derived using cubic-spline interpolation on annual series ($PFInvIndirect_y$)



Source: Data derived using cubic-spline interpolation program in STATA

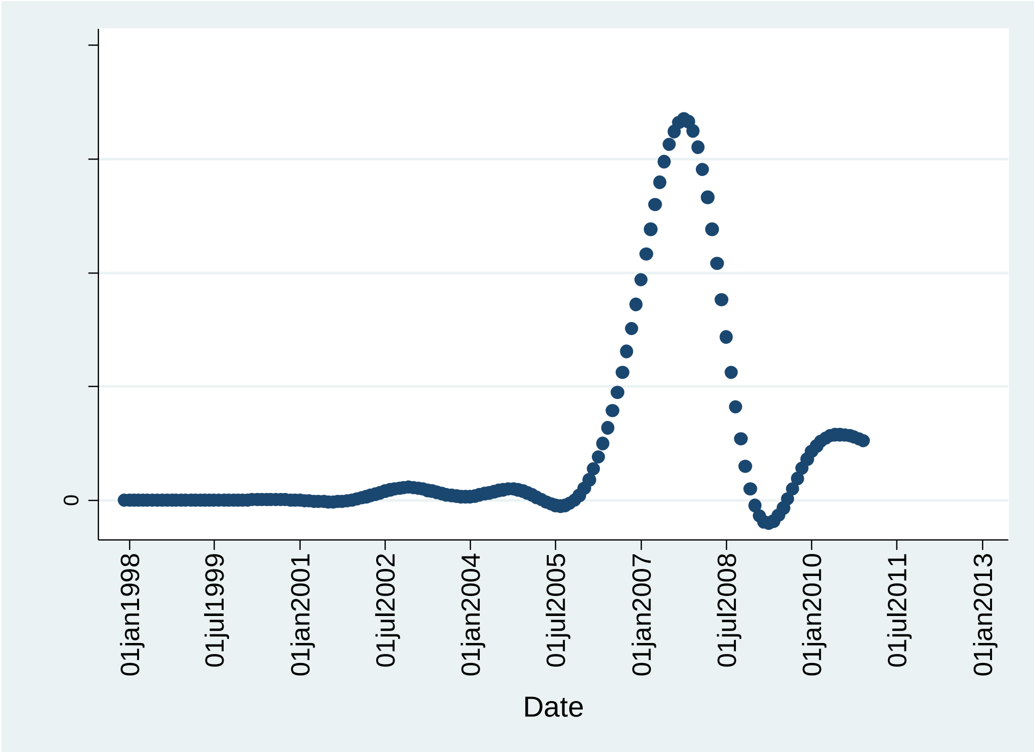
Figure 3.4.2.4b: Monthly frequency data of the DC Pension funds net new direct investments into Croatian equity, excluding GDRs and HT stocks ($PFInvDirect_m$) derived using cubic-spline interpolation on annual series ($PFInvDirect_y$)



Source: Data derived using cubic-spline interpolation program in STATA

We also derive the joint total new investment variable as a sum of the two, direct and indirect new investment. We present the monthly series of the total new investment into Croatian and SEE equity markets in Figure 3.4.2.4c. We note that, based on our theoretical OLG model of rational asset bubbles with DC pension funds, we attribute a value of zero to the variables presenting the net investments of the DC pension funds in the bubbly assets throughout the period of non-existence of the DC pension funds prior to 2002. This follows the implied logic of our theoretical model; namely, that the states of the economy when there is no DC pension fund, and when the pension funds exist but do not invest in the bubbly asset, have equivalent implications for the dynamics of the bubbly asset.

Figure 3.4.2.4c: Monthly frequency data of the DC Pension funds net new direct and indirect investments into Croatian and SEE equity excluding GDRs and HT stocks ($PFInvTotalMonthly_m$)



Source: Data derived using cubic-spline interpolation program in STATA

We will examine the results of the unit root tests of our two DC pension fund investment variables "PFInvDirect" and "PFInvIndirect" in the last section of this Chapter.

3.4.3. Continuous variable explaining the monthly dynamics of the new investments into the Croatian and SEE equity-focused Open End Mutual Funds – "OMF_AUMIndex"

In this Section we explain how we construct a variable capturing DC pension fund investments into the bubbly asset expressed by a genuine monthly data set. This is accomplished by estimating the monthly inflow/outflow of the value of assets under management (AUM) of the portfolio of Croatian and SEE equity-focused Open-end mutual funds, which were particularly used as a bubbly investment instrument by the Croatian DC Pension Funds indirectly approaching the Croatian and the SEE equity markets.

The Croatian financial market regulating agency (HANFA) publishes data on the monthly dynamics of the assets under management (AUM) of all registered Open-end Mutual funds in the Republic of Croatia individually. Unfortunately, they do not publish data about the change of the number of the accounting units, which would be a perfect measure of the inflow/outflow of investments in and out of the fund. Instead, the only figures that HANFA publishes are the Beginning Period AUM, the End Period AUM and their absolute and relative difference. This measure is far from a perfect measure of the inflow/outflow of new investments into each Open-end Mutual Fund, because it is directly and positively affected by the change of the market price of the underlying assets, in our case the bubbly equity asset. Therefore, we had to correct the relative difference between the end period AUM and the beginning period AUM by dividing it with the relative change of the CROBEX equity market index for each monthly period. This way we factored-out the price change information component of the dynamics of the CROBEX index from the change of the Assets Under Management (AUM) index of the Open End Mutual Funds, to get a closer estimate of the actual change of AUM due to the decisions of investors to increase or decrease their positions in the Open-end Investment funds rather than to the change of the market value of the bubble.

In order to relate this derived index of the AUM of the Open-end investment Funds investing in Croatian and SEE equity to the actual investment portfolio of the DC pension funds, we only used those funds actually used by the DC Pension Funds themselves by referring to the yearly portfolio structure of the DC pension funds. We used the following methodology when establishing our monthly index of the change in the CROBEX corrected

AUM of the Open-end Investment Funds focused in SEE and Croatian Equity markets used by the DC Pension Funds.

First, we had to divide the Croatian Open-end investment funds used by the DC pension fund into three groups. The first group, which invested up to 100% of its assets into SEE and Croatian equity markets, we call "fully exposed". The Second group, which invested up to 50% in the Croatian and SEE equity markets, we call "partially exposed". And the Third group, which did not invest any assets into Croatian and SEE markets, we call the "non-exposed" group of Open-end Investment Funds. We made the categorization by checking the portfolio exposure around the peak of the asset bubble 2007-2008, based on the availability of data. We present the three group categorization with its members in Table 3.4.3.1.

Table 3.4.3.1 Croatian Open-end Investment Funds used by the DC Pension funds in the period 2005-2010 divided by their exposure: fully, partially and non-exposed to Croatian and SEE equity markets

1) FULLY EXPOSED	2) PARTIALLY EXPOSED	3) NON-EXPOSED
RBAI-U-RCEU	RBAI-U-RBAL	PBZI-U-HRKN
PBZI-U-EQTF9	ERSI-U-ERIN	PBZI-U-NVCF
ERSI-U-EADE	ICF BALANCED	NEXP-U-ALPH
KD VICTORIA	HI-BALANCED	ICAM-U-FOST
HI-GROWTH	OTP URAVNOTEZENI	ZB EUROAKTIV
ILIRIKA JUGOISTOCNA EVROPA	ILIRIKA JIE BALANCED	VB CASH
SELECT EUROPE	ZB GLOBAL	ZB PLUS
OTP INDEKSNI	HPB GLOBAL	HI-CASH
ZB AKTIV		PBZI-U-EURN
ST GLOBAL EQUITY		RBAI-U-RCAS
HPB DYNAMIC		AZIN-U-AFCA
PBZI-U-GLBF		AGRAM CASH
		HPB NOVCANI
		PBZI-U-STKF
		ALLIANZ CASH
		ERSI-U-ERMO
		OTP NOVCANI
		SELECT NOVCANI
		ERSI-U-ERBO
		ICF MONEY MARKET

Source: Annual audited balance sheets of the Croatian DC pension funds 2002-2010

Second, we had to estimate the portfolio weights of each open-end investment fund in its specific group characterized by its investment focus (Table 3.4.3.2) as a ratio between the absolute amounts invested by the DC pension funds in each fund and the total amount invested in the group of funds to which each fund belongs. As an example in order to demonstrate the logic we refer to the year 2005 when there were only two Open End Investment Funds from the group “partially exposed” used by the four DC pension funds. Those were the “RBAI-U-RBAL” and the “ZB GLOBAL” funds. Based on the year end values of the DC pension fund portfolios, those two Open-end mutual funds participated with 81.5% and 18.5% respectively in the portfolio of the total DC pension fund investments being part of the “partially exposed” Open End Investment Funds portfolio during the year 2005 (Table 3.4.3.2). Consequently, we built the index of the dynamics of AUM of the “partially exposed” Open End Mutual Funds for the year of 2005 by including 81.5% of the AUM index of the “RBAI-U-RBAL” and 18.5% of the “ZB GLOBAL” fund. In other words, the index is composed of 81.5% of the dynamics of “RBAI-U-RBAL” and 18.5% of the dynamics of “ZB GLOBAL” in 2005. This way we capture the dynamics of the AUM of the “partially exposed” Open End Mutual Funds, taking into account only those funds receiving investments as part of the DC pension funds’ portfolio in that year. We change the weights on the index every year, based on the actual year-end structure of the Open End Mutual Funds’ Investments. We did this only for the funds “fully exposed” and “partially exposed” to SEE and Croatian Equity market, because only those funds were of interest. We present the weights in Table 3.4.3.2.

Table 3.4.3.2 Weights of the individual "Fully" and "Partially" SEE and Croatian Equity exposed Open-end Mutual Funds in their group for each year from 2002-2010 for all four funds together

Matrix of weights (calculates the weights of each Open-end Mutual Fund within its group)									
FULLY EXPOSED	2002	2003	2004	2005	2006	2007	2008	2009	2010
PBZI-U-GLBF	0.0%	0.0%	0.0%	**100.0%**	**30.9%**	18.3%	12.8%	5.5%	2.4%
RBAI-U-RCEU	0.0%	0.0%	0.0%	0.0%	21.6%	9.5%	4.1%	9.0%	12.8%
PBZI-U-EQTF9	0.0%	0.0%	0.0%	0.0%	27.3%	**33.0%**	**35.8%**	**40.5%**	**36.6%**
ERSI-U-EADE	0.0%	0.0%	0.0%	0.0%	14.4%	23.0%	26.3%	14.0%	5.9%
KD VICTORIA	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	4.3%	2.4%	0.5%
HI-GROWTH	0.0%	0.0%	0.0%	0.0%	0.0%	1.7%	2.3%	1.0%	0.4%
ILIRIKA JUGOISTOCNA EVROPA	0.0%	0.0%	0.0%	0.0%	0.0%	4.3%	6.0%	3.6%	3.0%
SELECT EUROPE	0.0%	0.0%	0.0%	0.0%	1.8%	5.8%	7.3%	2.4%	0.0%
OTP INDEKSNI	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.4%	24.2%
ZB AKTIV	0.0%	0.0%	0.0%	0.0%	4.1%	1.2%	0.8%	9.6%	14.2%
ST GLOBAL EQUITY	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%
HPB DYNAMIC	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.7%	0.0%
PARTIALLY EXPOSED	2002	2003	2004	2005	2006	2007	2008	2009	2010
RBAI-U-RBAL	0.0%	0.0%	0.0%	**81.5%**	**72.1%**	23.2%	0.2%	20.0%	**42.0%**
ERSI-U-ERIN	0.0%	0.0%	0.0%	0.0%	8.6%	16.2%	19.9%	17.6%	10.4%
ICF BALANCED	0.0%	0.0%	0.0%	0.0%	5.4%	6.3%	7.7%	10.0%	0.0%
HI-BALANCED	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	1.5%	0.0%	0.0%
OTP URAVNOTEZENI	0.0%	0.0%	0.0%	0.0%	0.0%	4.7%	6.7%	4.0%	2.4%
ILIRIKA JIE BALANCED	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	2.8%	0.0%	0.0%
ZB GLOBAL	0.0%	0.0%	0.0%	18.5%	13.9%	**36.5%**	**44.9%**	19.8%	12.5%
HPB GLOBAL	0.0%	0.0%	0.0%	0.0%	0.0%	9.8%	16.1%	**28.7%**	32.7%

Source: DC pension funds end of year balance sheets; own calculations

Finally, using the data on the absolute monthly change of assets under management (AUM) of each open-end investment fund in the Republic of Croatia, published by the Croatian Financial Market Authority (HANFA), weighted by the actual yearly weights of each individual fund in the "fully exposed" or the "partially exposed" portfolio structure of the DC pension funds' investments into Croatian and SEE equity-focused Open-end investment funds (Table 3.4.3.2), we built two monthly indices: the first one, representing the monthly dynamics of the AUM of the portfolio from the group of "fully exposed" open end investment funds; and the second representing the AUM dynamics of the group of "partially exposed" open end investment funds. We corrected both monthly indices by the change of the CROBEX stock market index: the "Fully exposed" by the full percentage change of the CROBEX equity index; and the "Partially exposed" by 50% of the change of the CROBEX equity index. This way we eliminated the effect of the change of the equity market prices on the change of assets under management. Finally, we joined the two indices together into one index, based on the weights of each group of “fully exposed” and “partially exposed” open end investment funds in their total portfolio presence in the DC pension fund asset structure for each year (Table 3.4.3.3).

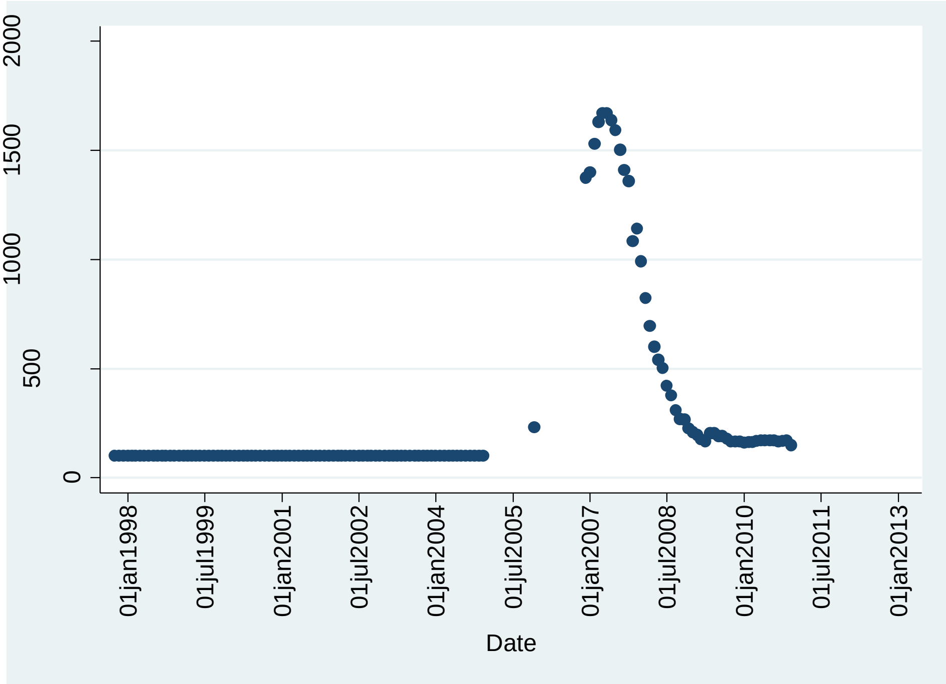
Table 3.4.3.3: Weights of the group of “fully exposed” and “partially exposed” Open-end Investment Funds in the SEE and Croatian equity market, based on the previous end-of-year absolute value of investment

	2002	2003	2004	2005	2006	2007	2008	2009	2010
FULLY EXPOSED				**22.92%**	**64.98%**	**68.85%**	**70.17%**	**77.89%**	**82.77%**
PRATIALLY EXPOSED				**77.08%**	**35.02%**	**31.15%**	**29.83%**	**22.11%**	**17.23%**

Source: Derived from yearly audited balance sheets of the DC pension funds 2002-2010

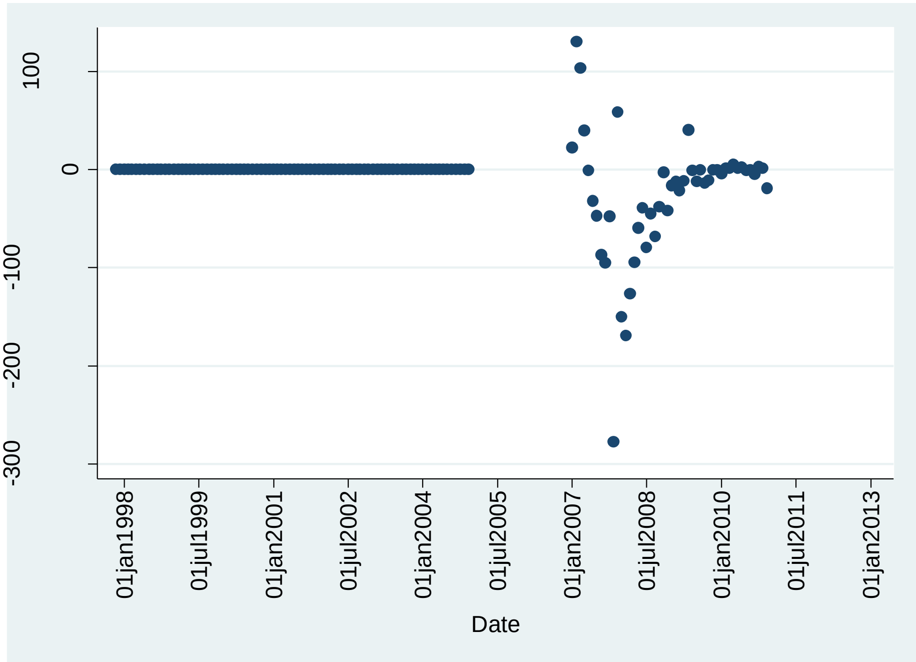
Using, the weights presented in Figure 3.4.3.3 and our two CROBEX-corrected indices on the change of AUM of each group of “fully exposed” and “partially exposed” Open-end Investment Funds investing into SEE and Croatian equity, we established a variable called “OMF_AUMIndex”, which represents an index estimating the relative dynamics of the monthly new net investments of the DC pension funds into the open-end investment funds exposed to the SEE and Croatian equity markets. This variable is presented in Figure 3.4.3.1, and its first difference “FD_OMF_AUMIndex” in is presented in Figure 3.4.3.2.

Figure 3.4.3.1: Dynamics of the "OMF_AUMIndex"



Source: Author, using Stata12

Figure 3.4.3.2: Dynamics of the "FD_OMF_AUMIndex" variable



Source: Author, using Stata12

Since the “OMF_AUMIndex” variable is a stochastic time series variable with a genuine monthly frequency, we had to check whether or not it is stationary and, if not, what is the order of integration of the variable before using it in our empirical modelling. For this purpose we continue with the last part of this Chapter, where we present the test results for the presence of a unit root in the data generating process of each one of our stochastic endogenous variables.

3.5. Unit root tests and the order of integration of our endogenous variables “B_Croatia”, “DYNEF”, “PFInvDirect”, “PFInvIndirect” and “OMF_AUMIndex”

Before we continue with the empirical modelling, in order to determine the proper empirical strategy in our following empirical chapters, we check the order of integration of our set of endogenous variables. We are dealing with variables based on stock market dynamics, for which it is common to have a unit root and to be non-stationary. In the following section we will first test for the presence of unit root in the data generating process of the level forms of our endogenous variables, and then we continue by examining the unit roots for first difference transformations of the variables to establish their level of integration.

3.5.1. Testing if the level forms of the variables have a unit root

In order to determine whether our data has unit roots, we perform several tests. We approach testing for presence of unit roots by both testing the null hypothesis of “non-stationarity”, where rejecting the null confirms that the series is stationary and by performing tests where the null hypothesis is “stationarity” of the data generating process. By applying a combination of tests, we get better inference on the actual order of integration of each variable. In the following lines we give a brief review of the tests we use and the results from testing our endogenous variables in their level form.

We first perform the Augmented Dickey Fuller tests (Dickey & Fuller, 1979) where the null hypothesis is that the Data Generating Process (DGP) of a time series has a unit root. This implies that the variable is non-stationary and integrated at a level higher than zero. The distribution of the test statistic of the ADF test has a negative skew and the design of the test requires a one-sided test. When the test statistic is above the critical values in absolute value (determined at 1%, 5%, or 10% level of significance), we reject the hypothesis of "non-stationarity" and we accept the alternative, that the DGP is stationary (with a probability of making a Type 1 error of 1%, 5% or 10% respectively). If the test statistic is significantly positive on the other side, we could expect that the data generating process is explosive. The original Dickey-Fuller model is augmented by adding lagged differences in the design of the test statistic in order to overcome the potential problem of residual autocorrelation, which if present, could make test statistic biased. On the other side, when we deal with a shorter data sets (at the case with some of our variables with less than 50 observations) we require a parsimonious model, which is unbiased and at the same time has the lowest number of augmented lags. In order to achieve the best balance between the two, we made our decision on the order of lags used in the ADF test based on the p-values of the Portmanteu test (with 12 lags) for residual autocorrelation, accepting the smallest lag in the ADF test, that suggests no-auto-correlated residuals. In all ADF tests applied on our variables in the level form, we reject the residual autocorrelation in the ADF test by augmenting it with 0 to 8 lags depending on the case. Only the interpolated investment variables "PFInvDirect" and "PFInvIndirect", due to their interpolation from annual to monthly frequency with a cubic function, required 28 lags in the test in order to assure no residual autocorrelation. Such a high number of lags consumed too many degrees of freedom for the test applied on our small data set, which was not acceptable, and based on the last significant coefficient included in the ADF test (Sjö, 2008, p. 9), we decided to use a maximum of 4 lags in the ADF test for those variables. We present the critical values of the ADF test for the level variables in Table 3.5.1.

Table 3.5.1a ADF test of non-stationarity (H0: data generating process being non-stationary) performed on our endogenous variables "B_Croatia", "DYNEF", PFInvDirect", "PFInvIndirect" and "OMF_AUMIndex" in level form (and the relevant test statistic critical values)

ADF Test (H0: Non-Stationarity)	B_Croatia	DYNEF	PF_InvDirect	PF_InvIndirect	OMF_AUMIndex
no constant, no trend	-2.1133	-2.0152	-0.6932	-2.1738	-2.2087
constant, no trend	-2.1077	-2.0086	-1.2978	-2.1727	-1.331
constant and trend	-2.2588	-1.7334	-2.3181	-2.1757	-0.6963

critical values	1%	5%	10%
no constant, no trend	-2.56	-1.94	-1.62
constant, no trend	-3.43	-2.86	-2.57
constant and trend	-3.96	-3.41	-3.13

Source: Author, performing tests in JMulti 4.24 (full reports available in Appendix III.3)

In this case, looking at the one per cent level of significance, we fail to reject the non-stationarity hypothesis for all five endogenous variables. When we include only a constant or a constant and a trend, we fail to reject the null at any conventional level of significance. The results thus suggest that our variables are integrated at some higher level of integration than zero. To assure to this we also perform the KPSS test.

The KPSS test (Kwiatkowski, Phillips, Schmidt, & Shin, 1992) is a unit root test where the H_0 hypothesis, opposite to the H_0in the ADF test, states that the DGP of the tested variable is stationary, or the level of integration is zero. The alternative hypothesis on the other side is that the variable is integrated at a level higher than 0. Looking at the test results presented in Table 3.5.1b below, we notice that with the exception of the test for the interpolated variable "PFInvIndirect", the KPSS test of Level stationarity yields test results on all other variables suggesting that the DGP of the variable is non-stationary. Such results correspond to the results of the ADF test results. The only non-rejection is present with the "PFInvIndirect" variable, and could be due to the residuals autocorrelation, which is a product of the used cubic interpolation from a low – annual to a high monthly frequency of the data. This variable in substance is very similar to the "OMF_AUMIndex" variable, which has a genuine monthly frequency, and which strongly rejected the KPSS test of stationarity.

Table 3.5.1b KPSS test of stationarity (H0: data generating process being stationary) performed on our endogenous variables "B_Croatia", "DYNEF", PFInvDirect", "PFInvIndirect" and "OMF_AUMIndex" in level form (and the relevant test statistic critical values)

KPSS Test (H0: Stationaroty)	B_Croatia	DYNEF	PFInvDirect	PFInvIndirect	OMF_AUMIndex
Level stationarity test statistic	0.7656	0.4778	1.3816	0.1407	2.0805
Trend stationarity test statistic	0.3778	0.2501	0.2079	0.1338	0.544

critical values	1%	5%	10%
Level stationarity	0.347	0.463	0.739
Trend stationarity	0.119	0.146	0.216

*we used the same number of lags used in ADF test which showed no autocorrelation of residuals
Source: Author, performing tests in JMulti 4.24 (full reports available in Appendix III.3)

Finally, the DGP might be affected by an outlier or a shift in the series. This means that it is possible that the DGP of the variable, outside this impulse or a shift, has stationary dynamics. This is why we also perform an ADF test augmented by an impulse dummy and by a shift dummy. We used the JMulti search algorithm for finding the most probable impulse/shift date, and applied the test with this date. We present the ADF test results in Table 3.5.1c. We applied similar principle for choosing the lag of the ADF test, as previously explained in relation to the ADF test. All test reports are available in Appendix III.3. Looking at the test results, we fail to reject the Unit Root null in the presence of a structural break on all variables at the highest level of significance (10%). The test results suggest that the variables are non-stationary and integrated at higher level of integration than zero.

Table 3.5.1c Unit Root ADR test with Structural Break (H0: data generating process being non-stationary) performed on our endogenous variables "B_Croatia", "DYNEF", PFInvDirect", "PFInvIndirect" and "OMF_AUMIndex" in level form (and the relevant test statistic critical values)

UR with Structural Break	B_Croatia	DYNEF	PF_InvDirect	PF_InvIndirect	OMF_AUMIndex
Impulse dummy date	Dec-07	Feb-09	Nov-98	Sep-98	Dec-07
Constant, no trend	-2.3645	-2.0988	-1.3254	-2.2097	-2.334
Constant and trend	-2.1903	-1.8894	-1.9431	-2.253	-1.925
Shift dummy date	Aug-98	Feb-09	Sep-98	Sep-98	Oct-08
Constant, no trend	-1.9953	-1.7435	-1.3254	-2.2097	-2.1565
Constant and trend	-2.0289	-2.0812	-1.9431	-2.253	-2.212

Critical values	1%	5%	10%
Test with constant and no trend	-3.48	-2.88	-2.58
Test with constant and trend	-3.55	-3.03	-2.76

Source: Author, performing tests in JMulti 4.24 (full reports available in Appendix III.3)

We conclude, based on the three unit root tests applied, that all our endogenous variables have an order of integration higher than zero, which means they are non-stationary. We believe that although the KPSS test suggested the opposite for the "PFInvIndirect" in the stationarity test in Levels, the ADF test and the ADF test in the presence of a structural break both suggested the opposite for this variable. We continue by first differencing the variables in order to determine whether they are integrated at level one.

3.5.2. Testing if the first differences of the variables have a unit root

Having found evidence that our variables have higher level of integration than 0, in order to find their level of integration, we need to find the order of their first differencing transformation that makes them stationary. For this purpose, we continue by first differencing all our endogenous level variables and performing the unit root test on their first difference transformations. We use the same previously explained approach for determining the correct number of lags in the test.

Table 3.5.2a ADF test of non-stationarity (H0: data generating process being non-stationary) performed on the first difference transformations of our endogenous variables "B_Croatia", "DYNEF", PFInvDirect", "PFInvIndirect" and "OMF_AUMIndex" (and the relevant test statistic critical values)

ADF Test (H0: Non-Stationarity)	B_Croatia_D1	DYNEF_D1	PF_InvDirect_D1	PF_InvIndirect_D1	OMF_AUMIndex_D1
no constant, no trend	-5.3135	-10.4827	-3.8384	-4.5906	-4.118
constant, no trend	-5.3008	-10.4898	-3.8698	-4.5763	-4.5591
Constant and trend	-5.2836	-10.5266	-3.8509	-4.5739	-4.7039

critical values	1%	5%	10%
no constant, no trend	-2.56	-1.94	-1.62
constant, no trend	-3.43	-2.86	-2.57
constant and trend	-3.96	-3.41	-3.13

Source: Author, performing tests in JMulti 4.24 (full reports available in Appendix III.3)

Looking at the ADF test results, testing the null hypothesis suggesting the first difference transformations generate non-stationary series, we reject the unit root null hypothesis for all variables which presents supporting evidence that all our endogenous first differenced transformations represent stationary data generation processes. Looking at Table 3.5.2a, we find that we uniformly reject the null hypothesis even at the 10% level of significance for all variables in all three variants of the test (except for "PFInvDirect" for which we have a borderline result in the least restricted version of the test). In order to search for more evidence on the stationarity of the first differenced transformations, we perform the KPSS test. Looking at Table 3.5.2b which reports the KPSS unit root test results, we fail to find evidence for rejecting the Null hypothesis that the first difference transformations of our endogenous variables are represented by stationary data generating processes.

Table 3.5.2b: KPSS test of stationarity (H0: data generating process being stationary) performed on the first difference transformations of our endogenous variables "B_Croatia", "DYNEF", PFInvDirect", "PFInvIndirect" and "OMF_AUMIndex" (and the relevant test statistic critical values)

KPSS Test (H0: Stationaroty)	B_Croatia_D1	DYNEF_D1	PF_InvDirect_D1	PF_InvIndirect_D1	OMF_AUMIndex_D1
Level stationarity	0.1053	0.1381	0.1382	0.0759	0.000
Trend stationarity	0.1063	0.0542	0.1389	0.0701	0.000

critical values	1%	5%	10%
Level stationarity	0.347	0.463	0.347
Trend stationarity	0.119	0.146	0.216

Source: Author, performing tests in JMulti 4.24 (full reports available in Appendix III.3)

Based on the Unit root test results, we conclude that all our endogenous variables are integrated at order of one I(1). Having described the derivation and the meaning of our variables, together with their order of integration, we continue with the estimation of our empirical models in Chapter 4 and 5.

Conclusion

In this chapter we explained the intuition behind the variables to be used in our econometric analysis (reported in the next chapters) together with the process of their collection and generation. We started first by defining the variables of our implicit functional relationships describing the dynamics of the equity asset bubble derived from our theoretical model developed in Chapter 2.

We defined the variable "B_Croatia" as a stochastic non-stationary variable, which would be used as the dependent variable in our model of asset bubbles. We used the same methodology that we used for generating "B_Croatia" also for generating the "B_US" variable, explaining the bubble US market as representative of global financial markets. We will use this variable as an exogenous variable to isolate the short-term effect of the foreign markets on the Croatian Financial market in the model. We also found that "B_Croatia" and B_USA are not co-integrated, which suggests that the Croatian market is still young and not sufficiently integrated with global financial markets. While influenced in the short-run by global financial markets there is, as yet, no long-run equilibrium relationship with them.

Further, we defined the "DYNEF" variable, which is a stochastic and non-stationary variable measuring the dynamic efficiency of the Croatian financial market. When the financial market is dynamically efficient, this value takes positive values and when the financial market is dynamically inefficient it takes negative values. This variable is integrated of order one, I(1), and should measure the necessary conditions for the occurrence of the bubble or, more precisely, its positive values should identify the necessary conditions for the crash of the bubble. We also generated a dummy variable which signifies the moments when financial market passes over the dynamic efficiency threshold.

Finally, we defined a set of variables explaining the investments of the DC pension funds into the bubbly Croatian equity and Croatian and SEE equity focused open-end mutual funds. First, we defined a set of dummy variables assigned value 1 for periods defined by legal changes affecting the investment policies of the DC pension funds. Next, we also defined a set of level variables describing the new investments of the Croatian DC pension funds. We measured new pension fund investments based on the yearly audited financial reports of the pension funds and we used cubic-spline interpolation to create the monthly frequency of the investment variables based on the cubic function connecting the yearly data. This way we determined three variables: “PFInvIndirect“, “PFInvDirect“ and “PFInvTotalMonthly“, respectively representing the DC pension funds yearly absolute investments into Croatian and SEE equity-focused Open End Investment Funds, Croatian direct equity investments, and their Total expressed at monthly frequency.

In order to identify a genuine monthly variable representing the new net investment into the Croatian and SEE equity-focused Open End Investment funds, we also created a CROBEX-free index variable measuring the monthly Assets Under Management (AUM) of the Croatian Open End investment funds weighted by their acceptance within the DC pension funds portfolios. We named this variable the “OMF_AUMIndex”, whose relative dynamics should represent the monthly investment/disinvestment of the DC pension funds in the Open End Investment Funds investing in Croatian and SEE equity. This variable should add value to our model by containing actual rather than interpolated monthly information about the DC pension fund investments, which was not contained in the “PFInvIndirect” variable

Finally we found supporting evidence that all our endogenous variable are integrated at order one I(1).

Having the complete data set presented in Appendix 3.2, we now pass on to testing our hypotheses in the following chapters.

Chapter 4 – Model1: Explaining the Croatian equity bubble with the DC pension fund investments represented by "PFInvDirect" and "PFInvIndirect"

Introduction

The goal of this chapter is to test our theoretical model of rational asset bubbles with DC pension funds developed in Chapter 2 using the data set from the Croatian financial market presented in Chapter 3.

We focus on two main Vector Error Correction (VEC) Models. This chapter presents the first empirical model, estimated using both Direct ("PFInvDirect") and Indirect ("PFInvIndirect") investment variables as endogenous explanatory variables representing the DC pension funds' investments. Those variables represent investment data extracted from the yearly audited financial statements of the Croatian DC pension funds. The second model presented in the following chapter will use, as an explanatory investment variable, the index of the assets under management of the Croatian and SEE equity focused Open-end Investment Funds weighted by the exact yearly exposure of DC pension funds to those funds "OMF_AUMIndex".

To estimate each VEC Model, we use the JMulTi 4.24 software package, which allows a structured estimation procedure derived from the theoretical and empirical time series analysis literature. Following this procedure, we first check the order of integration of each variable; in Chapter 3, we found that all stochastic variables in our model are integrated of order one. We then test for the optimal lag order of the underlying VAR model in levels, and we examine whether there exists a co-integration relationship among our endogenous variables. Finally, we estimate our VEC Model, check its diagnostic tests and investigate the dynamics suggested by the model long-term and short-term equation components and by the use of Impulse Response Function (IRF) Analysis.

We also introduce a control dummy variable representing a structural shift in the co-integration relationship among our model variables caused by the escalation of the global financial crisis. The reason we take account of this structural break variable is because the significant disturbing effects that the Global Financial Crisis caused on the international financial markets. We find that this structural shift variable plays a significant role in the long-term relationship, especially as a trigger for the crash of the bubble which we quantify and explain.

Finally, we summarize all our empirical results in relation to the hypotheses drawn from our theoretical model in Chapter 2, showing that the suggested theoretical relationships are confirmed by the empirical analysis of the Croatian financial market.

4.1. Summary of the endogenous, exogenous and deterministic variables in our Croatian financial market data set

In the previous chapter we described the data collection and measurement of the variables of interest, which will be used to test our theoretical model of rational asset bubbles with DC pension funds. Before we begin with the structured empirical analysis using our Croatian data set, we present a table containing the order of integration and description of each endogenous and exogenous variable of interest introduced in Chapter 3. The variables with their main characteristics are presented in Table 4.1.1.

Table 4.1.1: Model variables, their type, character, order of integration and frequency

Name	Description	Type	Frequency	Order of Integration
B_Croatia	Relative value of the Croatian equity bubble	Endogenous (continuous)	Monthly	1
DYNEF	Dynamic efficiency of the Croatian equity market	Endogenous (continuous)	Monthly	1
PFInvDirect	Croatian DC Pension funds direct investments into Croatian equity	Endogenous (continuous)	Monthly (interpolated)	1
PFInvIndirect	Croatian DC Pension funds indirect investment in Croatian equity through the SEE and Croatian equity focused Open End Mutual Funds	Endogenous (continuous)	Monthly (interpolated)	1
OMF_AUMIndex	Index (T1=100) describing the dynamics of the assets under management of the portfolio of SEE and Croatian equity focused open end mutual funds (weighted by DC pension funds investment)	Endogenous (continuous)	Monthly	1
B_US	Relative value of the US equity bubble	Exogenous (continuous)	Monthly	1
dummy1	Official start of the DC pension fund system in Croatia	Deterministic (discrete)	Monthly	/
dummy2	Legal Acts allowing investments in the bubbly Croatian and SEE equity focused Open End Mutual	Deterministic (discrete)	Monthly	/

	Funds			
dummy3	Legal Acts allowing broad direct investments in Croatian Equity	Deterministic (discrete)	Monthly	/
dummy4	Legal Acts requiring higher DC pension funds' transaction transparency	Deterministic (discrete)	Monthly	/
dummy_DYNEF	DYNEF passing the 0 threshold	Deterministic (discrete)	Monthly	/
L_shift_M9	Bankruptcy of Lehman Brothers in 09/2008	Deterministic (discrete)	Monthly	/

Source: Author using own calculations

4.2. Vector Error Correction modelling and the order of integration of the variables of interest

There are two common approaches for overcoming the spurious regression outcome when having variables with positive order of integration. First, transforming the continuous non-stationary variables of interest into stationary variables and using them in this form in the regression model. This is done by differencing them until their transformation becomes stationary and then using them in the regression model. In our case, as we saw in Section 3.5.2, this is achieved by first differencing all the variables, since they are all integrated at order of 1.

This approach has one significant issue, and that is the loss of most of the long-term information contained in the level form of the variable each time the variable is first differenced. The other problem associated with such an approach can be the loss of meaningful interpretation of the variables and of the model in which they are used, especially if they have to be more than once differenced before they become stationary (suggesting a higher order of integration). This is why most empirical research done in time series with non-stationary variables is done by the application of the second approach, which is estimation of Error Correction (EC) (Engle & Granger, 1987) and Vector Error Correction (VEC) Models (Johansen S. , 1988).

The advantage of the EC and VEC models is that the long-term information or the equilibrium relationship between the variables of interest is preserved. This relationship is described as a co-integration relationship between the variables, which is the requirement for the use of the EC and VEC Models in modelling time series. This is why the next step is to determine the rank of co-integration between the variables.

But before we move on determining the rank of co-integration, we need to find the order of the lag of the underlying Vector Auto Regression (VAR) model. This is because the Vector Error Correction Model presents a first order transformation of the underlying VAR model. So for the unbiased estimation of the co-integration coefficient matrix of the VECM and for determining the rank of the co-integration, we first need to define the order of the underlying VAR, which represents the optimal lagged model based on the explanatory power and diagnostic properties of the model.

4.3. The lag order of the underlying VAR model

The theory and practice suggests two common approaches for determining the optimal lag order of the underlying VAR model (Lütkepohl & Krätzig, Applied Time Series Econometrics, 2004). One is to find the optimal lag of the underlying VAR model by comparing its explanatory powers defined by certain information criteria. Information criteria compare the value of the determinant of the residual covariance matrix of the model for different lagged versions of the VAR model, corrected by some penalty criteria usually defined by the number of lags and the dimension of the coefficients in the model. This way, the information criteria define the most "informative" model, presented by the one with the smallest information criterion value which corresponds to the amount of un-modelled variance.

The most commonly used information criteria are the Akaike Information Criterion (AIK) (Akaike, 1974), the Hannan-Quinn Information Criterion (HQ) (Hannan & Quinn, 1979) and the Schwarz Information Criterion (SC) also known as Bayesian Information Criterion (Schwarz, 1978) (4.1). Each information criteria tries to minimize the log determinant of the residuals covariance matrix "$\sum_u$", subject to a penalty rule which increases its value with the number of lags of the model "m" times the number of endogenous variables "K". This penalty rule decreases with the increase of the time periods T used for the estimation of the VAR model.

$$\begin{aligned} AIC(m) &= \log\det(\tilde{\Sigma}_u(m)) + \frac{2}{T} mK^2 \\ HQ(m) &= \log\det(\tilde{\Sigma}_u(m)) + \frac{2\log\log T}{T} mK^2 \\ SC(m) &= \log\det(\tilde{\Sigma}_u(m)) + \frac{\log T}{T} mK^2 \end{aligned} \quad (4.1)$$

JMulti finds the optimal number of lags "m" that minimizes each information criterion from a pre-set maximum number of lags of the VAR model. The comparison of different outcomes for different information criteria could serve to choose the most informative VAR model.

Although, the use of an information criterion gives information about the power of descriptiveness and the efficiency of the VAR model at different lag orders, it does not take into account the diagnostic properties of the proposed optimally lagged VAR model. In other words, the information criteria might select a model with poor diagnostics, which can be biased and inappropriate. Moreover, as noted by Lütkepohl and Krätzig, there is inconsistency among different information criterion, where the AIC criterion asymptotically overestimates the optimal order with positive probability, while the HQ and SC estimate the order consistently under quite general conditions if the actual data generation process has a finite order and the maximum order is larger than the true order (Lütkepohl & Krätzig, Applied Time Series Econometrics, 2004). Even in small sample VAR models with I(1) co-integrated variables, Lütkepohl warns of the following relationship (4.2) among the information criterion p-values (Lütkepohl, 1991).

$$\hat{p}(SC) \leq \hat{p}(HQ) \leq \hat{p}(AIC) \quad (4.2)$$

This means that there is likely to be a conflicting outcome from different information criterion used, which is another argument in favour of the usage of an additional criterion when deciding about the order of the underlying levels VAR model. Searching for the optimal lag order of the VAR model of our endogenous variables, we get the following results for the three information criterion previously discussed (Table 4.3.1). Besides the endogenous variables, we also included the shocks from the exogenous and dummy variables, because they also contribute to the description powers of the model. We lagged the dummy variables for one period because of the required enforcement time after the publication of legal acts in the National Gazette, as suggested in Chapter 3.

Table 4.3.1: Optimal lag order of the VAR model of our endogenous variables "B_Croatia", "DYNEF", "PFInvDirect" and "PFInvIndirect" based on different information criterion and estimated in three different model specifications with respect of the deterministic components

a) VAR model without a constant or trend

```
endogenous variables:     B_Croatia DYNEF PFInvDirect PFInvIndirect
exogenous variables:      B_US_d1
exogenous lags (fixed):   5
deterministic variables:  dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1
sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 12 lags of levels):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5
```

b) VAR model with a constant and no trend

```
endogenous variables:     B_Croatia DYNEF PFInvDirect PFInvIndirect
exogenous variables:      B_US_d1
exogenous lags (fixed):   5
deterministic variables:  dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1 CONST
sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 12 lags of levels):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4
```

c) VAR model with a constant and a trend

```
endogenous variables:     B_Croatia DYNEF PFInvDirect PFInvIndirect
exogenous variables:      B_US_d1
exogenous lags (fixed):   5
deterministic variables:  dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1 CONST TREND
sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 12 lags of levels):
Akaike Info Criterion:    7
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4
```

Source: Author, using JMulti 4.24

To avoid the problem of potential bias in the information criteria and especially its exclusion of the suggested VAR model diagnostics, for the purpose of finding the optimal lag of the underlying VAR model, we score different lag VAR models based on the diagnostic test of each model. When checking the residual diagnostics at different lags of the underlying VAR model, we also investigate different model structures depending on the inclusion of a constant and a time trend variable in each model. We mainly check for the following two diagnostic tests on each model: first, is the one checking the existence of an autocorrelation in the residuals, which is of primary importance due to the fact that auto-correlated residuals impose bias on the estimated VAR model coefficient matrices; and, second, we test for the normality and the component distributional characteristics (skeweness and kurtosis) of the VAR model residuals.

We implement the Lagrange Multiplier F (LMF) – Autocorrelation test based on the Breusch-Godfrey Lagrange-Multiplier (LM) test of residual autocorrelation (Godfrey, 1988), which is adjusted for use with small sample data where the test statistic tends to the F-test distribution at its limit (Doornik, 1996). Edgerton and Shukur showed that the original LM test gives biased results in small samples and suggested the use of the LMF test statistic (Edgerton & Shukur, 1999). The LMF test is testing the null hypothesis H_0: $B_1^*=B_2^*=...=B_h^*=0$ against the alternative that at least one of the "B" coefficient matrices is non-zero. Here "B" matrices are the coefficient matrices in the residual autocorrelation model as presented in equation 4.3. The test hypothesis is considering the residual autocorrelation of the "h" lags of the residuals.

$$u_t = B_1^* u_{t-1} + ...B_h^* u_{t-h} + e_t \qquad (4.3)$$

Finally, the LMF(h) test statistic is compared with the critical value from the F distribution; if it gives a value lower than the F critical value, then we fail to reject the null hypothesis that there is no autocorrelation of the error terms in the VAR model. Otherwise we reject the null and accept the alternative hypothesis claiming the presence of autocorrelation in the residuals of the VAR model making the VAR coefficients biased. The LMF test – as opposed to some other tests such as the "Portmanteau" or the LM test – is suitable for smaller data series such as the one that we use (with T<200).

In Table 4.3.2 we present the LMF test statistic for the test of residuals no autocorrelation, and their respective p-value's for different lags of the underlying VAR model inspected in three cases: first without a constant or a trend; second with the constant term only; and third with both a constant and a trend term. We first analyse the underlying VAR where DC Pension fund investments are represented by both direct and indirect investments "PFInvDirect" and "PFInvIndirect" variables, based on the DC pension funds yearly balance sheets. Later, in the following chapter, we will apply the same analysis in a model using the "OMF_AUMIndex" as our investment variable on a smaller dataset.

Based on the values of the LMF test, we conclude that the underlying levels VAR models – i.e. without constant or trend and with only a constant – at 5 lags are optimal in terms of satisfying the regression diagnostics with respect to the residuals no-auto-correlation criterion.

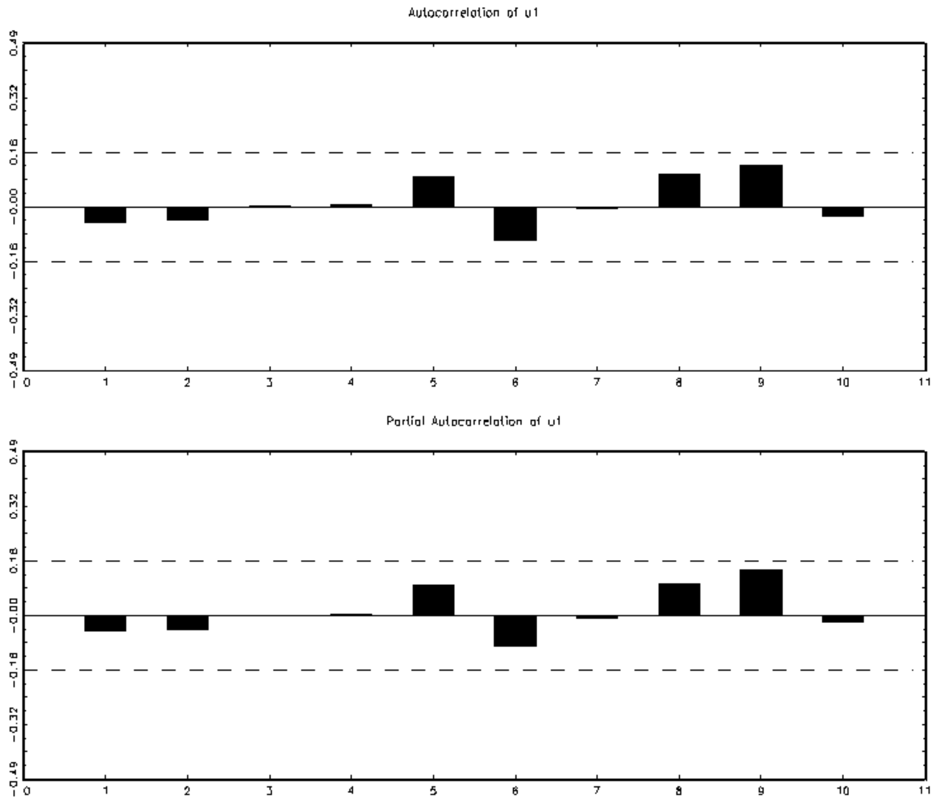
Table 4.3.2: LMF-no-autocorrelation test statistics with 2 and 3 lags applied on different structures of the VAR models of our endogenous variables, T=145

	no constant or trend		constant, no trend		constant and trend	
	LMF(2)	LMF(3)	LMF(2)	LMF(3)	LMF(2)	LMF(3)
VAR(1)	0	0	0	0	0	0
VAR(2)	0	0	0	0	0	0
VAR(3)	0	0	0	0	0	0
VAR(4)	0.0004	0	0.0009	0	0	0
VAR(5)	**0.1966**	**0.0823**	**0.0799**	0.0304	0	0
VAR(6)	**0.1172**	0	0	0	0.0036	0
VAR(7)	**0.0833**	0.0012	**0.0776**	0.0005	**0.1103**	0.0061
VAR(8)	0.0001	0	0	0	0	0

Source: Author, using JMulti4

In addition to our LMF diagnostic test, we also present the autocorrelation graphs of the estimated residuals autocorrelations for each of the four endogenous variable equations "u1" to "u4" in our best candidate VAR(5) model without a constant and trend, for visual inspection. The autocorrelation graphs are presented in Figure 4.3.1a-d, where the autocorrelation of the residual with each of its lagged values up to 10 lags is compared with the 95% confidence interval critical value in order to visually inspect the significance of its intensity.

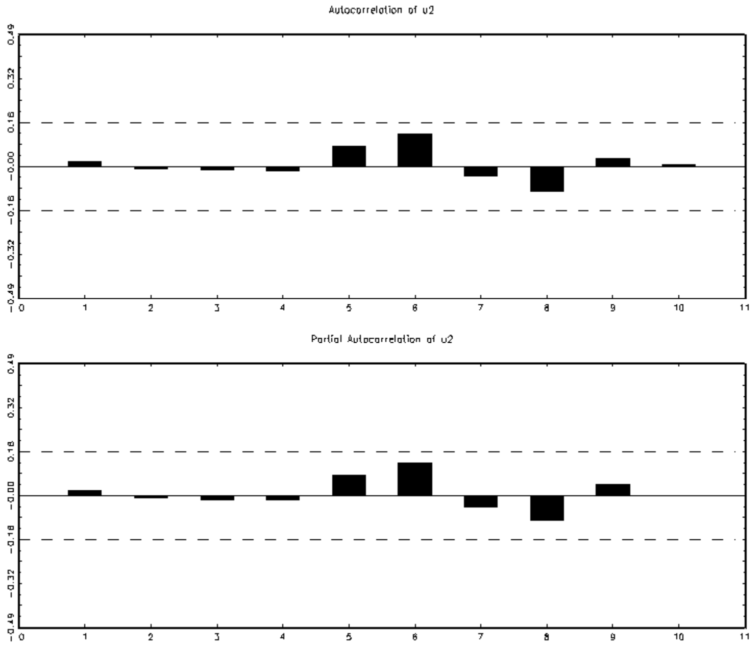
Figure 4.3.1a Autocorrelation of the residuals of the "B_Croatia" equation in the VAR (5) model without a constant or a trend: autocorrelation and partial autocorrelation coefficients



Source: Author, using JMulTi4.24

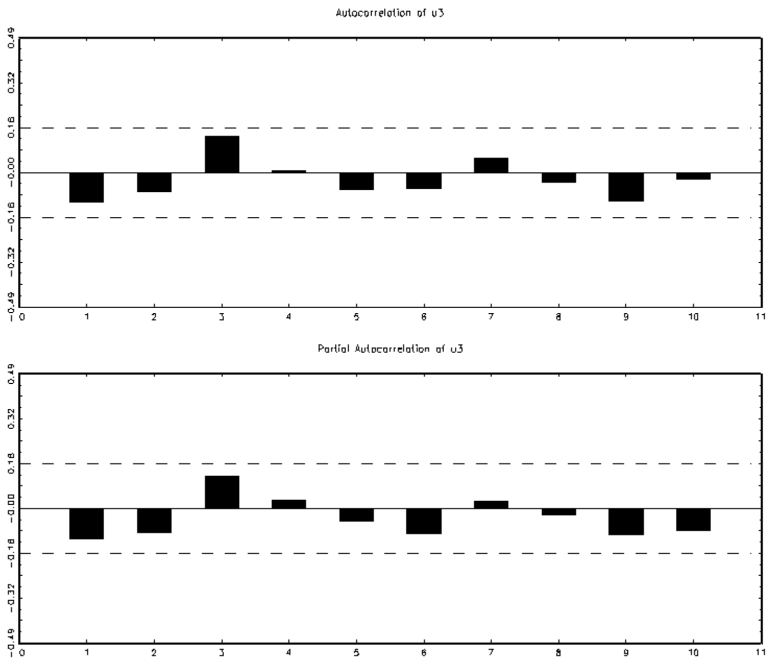
This approach in addition helps to confirm whether the residuals of the underlying level VAR model with 5 lags without a constant or a trend has a problem of regression residuals autocorrelation.

Figure 4.3.1b Autocorrelation of the residuals of the "DYNEF" equation in the VAR (5) model without a constant or a trend: autocorrelation and partial autocorrelation coefficients



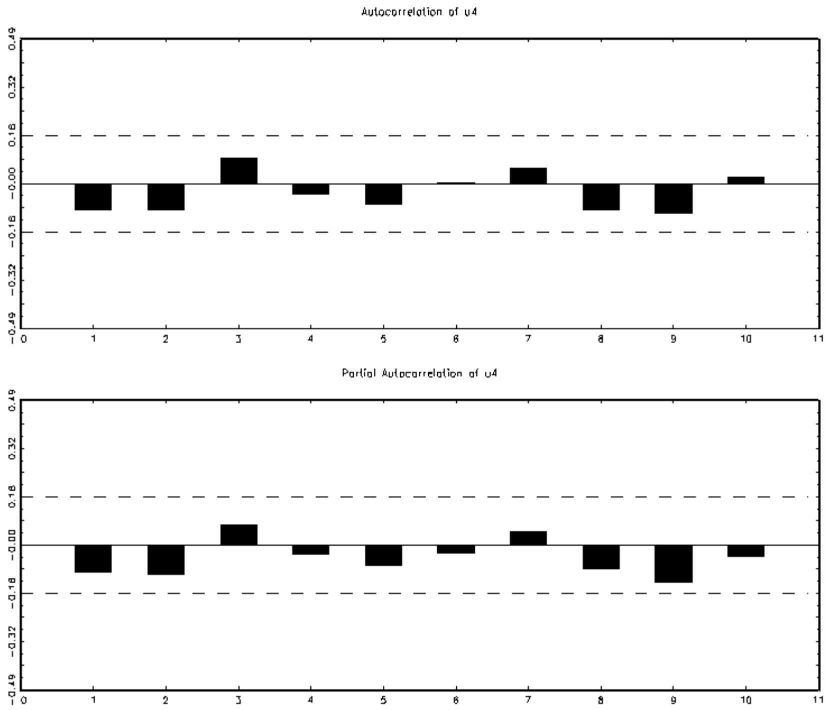
Source: Author, using JMulTi 4.24

Figure 4.3.1c Autocorrelation of the residuals of the "PFInvDirect" equation in the VAR (5) model without a constant or a trend: autocorrelation and partial autocorrelation coefficients



Source: Author, using JMulTi 4.24

Figure 4.3.1d Autocorrelation of the residuals of the "PFInvIndirect" equation in the VAR (5) model without a constant or a trend: autocorrelation and partial autocorrelation coefficients



Source: Author, using JMulTi 4.24

Looking at the auto-correlation plots in Figure 4.3.1a-d, we notice that there is no significant auto correlation of the residuals at any equation in the system. We conclude that the underlying levels VAR model with 5 lags of the endogenous variables has no-significant residual autocorrelation both by the formal test and by the visual test.

We continue by applying the residuals normality tests as part of the VAR diagnostics. Residuals normality tests are applied on the regression residuals from the VAR model at different lag orders, without constant or trend, with only a constant and with both constant and trend. For the testing purpose, we use the generalized version of the Lomnicki-Jarque-Bera (L-J-B) test as given by Lütkepohl and Krätzig as well as the Jarque-Bera tests of normality for the residual of each equation in the VAR (Lütkepohl & Krätzig, 2004). The L-J-B test statistic is based on the principal component analysis of the covariance matrix of the residuals of the VAR system, whose matrix is used to compute the third and the fourth moment, the skewness and kurtosis, of each residual. They are then compared against skewness and kurtosis characteristic for the normal distribution, deriving the L-J-B residuals normality test statistic. The L-J-B test statistic is asymptotically Chi-square distributed and represents the sum of the skewness and kurtosis test statistics (Lütkepohl & Krätzig, 2004). We present the results of the residuals normality tests in the Table 4.3.3.

Table 4.3.3 Residual normality tests for VAR models with different lag orders

normality tests – p values									
		no const or trend	**constant, no trend**	**constant and trend**			**no const or trend**	**constant, no trend**	**constant and trend**
VAR(4)	Jarque-Berra				VAR(6)	Jarque-Berra			
	U1	0.0006	0.0007	0.187		U1	0.0006	0.0003	0.1309
	U2	0	0	0		U2	0	0	0
	U3	0	0	0		U3	0	0	0
	U4	0	0	0		U4	0	0	0
	Joint test LJB	0	0	0		Joint test	0	0	0
VAR(5)	Jarque-Berra				VAR(7)	Jarque-Berra			
	U1	0	0	0.0205		U1	0.0422	0.0301	0.3258
	U2	0	0	0		U2	0	0	0
	U3	0	0	0		U3	0	0	0
	U4	0	0	0		U4	0	0	0
	Joint test LJB	0	0	0		Joint test	0	0	0

Source: Author, using JMulti 4.24

Investigating the residuals normality test results for different lag orders of the underlying level VAR model, we find that the VAR model suffers from the problem of non-normality of the regression residuals at every lag order. Looking at Table 4.3.3, where the results for the residual normality tests of the VAR model without a constant, with a constant and with constant and trend at different lags are displayed, we see that at all level orders of the VAR model, there exists non-normality of the residuals. We only find normality of the residuals in the first equation of the system explaining the "B_Croatia" variable in the case of the VAR model with 4, 6 and 7 lags in the VAR model with a constant and a trend.

Considering the information criteria and diagnostic tests, our choice for the optimal lag order of the underlying level VAR model will be the lag of 5 for the model without a constant and a trend (we will investigate also the model with a constant). This model was preferred by all of the information criteria, and it passed the non-autocorrelation LMF test with one of the highest levels of confidence of the LMF test. The only weakness of the VAR(5) model without a trend or a constant, was that it showed poor results on the normality of the residuals. Still, we give more weight on the residual no-autocorrelation tests when deciding on the lag order of the underlying VAR. Choosing VAR(5) without a constant or a trend as an underlying model means that our Vector Error Correction Model (VECM) would be the one with a restriction on the constant and the trend and with 4 lags. Yet this model could potentially have a constant within the error correction term, representing an otherwise un-modelled growth process. Accordingly, in order to get the proper form of the VECM, we present the transformation of the VAR(5) with a constant and a trend into a VEC(4) Model in the next section.

4.4. Transforming the VAR(5) into a VECM(4) model

We begin with the VAR(5) model with a constant and a trend. In Equation 4.4 "y" represents the vector of endogenous variables (in our case 4x1 vector) which is lagged 5 periods. "A" matrices (4X4) represent the coefficients on each vector of lagged endogenous variables; they are specific for each of the five lags in the model. Finally, we have the vector of constants "c_0" and time-trend coefficients "δ", which we could later restrict giving one or both the value of zero.

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + A_3 y_{t-3} + A_4 y_{t-4} + A_5 y_{t-5} + c_0 + \delta t + e_t \tag{4.4}$$

In order to transform this VAR(5) system into a VECM, we begin by adding and subtracting the coefficient matrix "A" in front of the last level lag of y (A_5 in the first step) multiplied by the previous lag of y (t-4 in the first step). In the first step, this is done by adding and subtracting "$A_5 y_{t-4}$" from the RHS of the equation. By doing this we get to our first transformation 4.4a.

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + A_3 y_{t-3} + A_4 y_{t-4} + A_5 y_{t-5} + A_5 y_{t-4} - A_5 y_{t-4} + c_0 + \delta t + e_t$$

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + A_3 y_{t-3} + (A_4 + A_5) y_{t-4} - A_5 \Delta y_{t-4} + c_0 + \delta t + e_t \tag{4.4a}$$

We continue the same process for all lags, the next addition and subtraction on the RHS of 4.4a is made by adding and subtracting "$(A_4+A_5)y_{t-3}$". By doing this we get to the next transformation represented by 4.4b.

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + (A_3 + A_4 + A_5) y_{t-3} - (A_4 + A_5)\Delta y_{t-3} - A_5 \Delta y_{t-4} + c_0 + \delta t + e_t \tag{4.4b}$$

We continue the same way until there is only the first lagged level form of the endogenous variables left in the transformation of the VAR model, then we subtract the first lag of the level form vector of endogenous variables "y_{t-1}" from both sides of the equation, which subtracts an identity matrix in the first bracket on the RHS, and we get the following equation 4.4c

$$\begin{aligned}\Delta y_t &= -(I - A_1 - A_2 - A_3 - A_4 - A_5) y_{t-1} \\ &- (A_2 + A_3 + A_4 + A_5)\Delta y_{t-1} - \ldots\ldots - A_5 \Delta y_{t-4} + c_0 + \delta t + e_t\end{aligned} \tag{4.4c}$$

Finally, we have transformed our VAR(5) model into a VECM(4) model. The final representation of the VECM model is simplified in equation 4.5. We see that the model consists of two parts: first, the long-term relationship of the level endogenous variables (the "y_{t-1}" vector) represented by the error correction matrix ∏ which generates an I(0) process(the component variables of the "y" vector are all I(1), yet one or more linear combinations of these variables can be I(0)); and second, the VAR model of first differences of the endogenous variables being I(0), representing the short-term effects characterized by the "Γ_i" matrices.

$$\Delta y_t = -\Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_4 \Delta y_{t-4} + c_0 + \delta t + e_t$$

$$\Delta y_t = -\Pi y_{t-1} + \sum_{i=1}^{4} \Gamma_i \Delta y_{t-i} + c_0 + \delta t + e_t \tag{4.5}$$

If the rank of the "∏" error correction matrix equals zero (r=0), then there is no co-integration of the endogenous variables and the model is reduced to its VAR component in first differences, i.e.to a short-term effects VAR model. At the other extreme, when "∏" has a full rank (r=k), then the VECM is equivalent to the VAR model, so we should use the original VAR model (Juselius, 2006). In all other cases, the rank of the ∏ matrix is smaller than its full rank (0<k<r), which means there is at least one independent co-integration vector that transforms the set, or two or more subsets, of the endogenous variables into one or more I(0) process. This co-integration vector represents some long-run relationships among the co-integrated variables. In this case, we should use the VECM and we decompose the (kxk) ∏ matrix into its component matrices: α (kxr) called the "loading matrix"; and β(kxr) containing the co-integration vectors. Applying this transformation we obtain equation 4.5a.

$$\Delta y_t = -\alpha\beta' y_{t-1} + \sum_{i=1}^{4} \Gamma_i \Delta y_{t-i} + c_t + \delta t + e_t \tag{4.5a}$$

This means that if some dis-equilibrium occurs from the long-term relationship of the co-integrated endogenous variables represented by the co-integration matrix "β", this disturbance (error) feeds into each endogenous variable of the system by the coefficients of the loading vector α, which determines the speed of adjustment. This describes the error correction effect toward the long-term relationship among the endogenous variables. This error correction, together with lagged VAR relationships of the first differences of the endogenous variables, other I(0) variables, representing exogenous or deterministic shocks, and a constant, together explain the short term contemporaneous dynamics of the vector of endogenous variables "Δy_t".

Now let us see what happens with the trend and the constant through the transformation from VAR(5) to VECM(4). Let us first assume there is only a constant and no trend in the VECM model as shown in equation 4.5b

$$\Delta y_t = -\alpha\beta' y_{t-1} + \sum_{i=1}^{4} \Gamma_i \Delta y_{t-i} + c_0 + e_t \tag{4.5b}$$

The constant included in the VECM represents both a vector of intercepts within the long-term co-integration relationships β, and a drift outside the co-integration vectors in the short-term VAR represented by the first differences. To see this, apply expectations on the

$$E[\beta' y_{t-1}] = \mu$$

co-integration vectors. Namely, the expected value of the long-term relationships is some vector of constants μ.

On the other side the expected value of the first differences of the endogenous variables is some other value γ .

$$E[\Delta y_t] = \gamma$$

If we transform 4.8b using expectations, then we get the following decomposition of the constant in the VEC Model:

$$\left(I - \sum_{i=1}^{4} \Gamma_i\right)\gamma = -\alpha\mu + c_0 \Rightarrow c_0 = \left(I - \sum_{i=1}^{4} \Gamma_i\right)\gamma + \alpha\mu$$

This shows that the constant in the VECM defined and estimated outside of the co-integration vector, contains both the mean (intercept) of the co-integration relationship represented by $\alpha\mu$, and the growth component or a drift in the first differences part of the model represented by $\left(I - \sum_{i=1}^{4} \Gamma_i\right)\gamma$.

We continue by analysing the trend component in the VECM represented by "δt" in our equation 4.8a. We could also decompose the trend component into two parts, one related to the co-integration relationship $\alpha\rho$ (where ρ represents the coefficient on the time trend within the co-integration relationship), and the other related to the growth rate "τ" outside the co-integration relationship.

We summarize the two decompositions of the constant and the trend (Juselius, 2006):

$c_0 = \alpha\mu + \gamma$, the first representing the (constant, when γ=0) growth rate of y

$\delta = \alpha\rho + \tau$, and the second representing the (constant, when τ=0) trend growth rate

Plugging the two decompositions in equation 4.5a, we get the following form of the VECM represented by equation 4.5c. Here we see that the constant and the trend from the

VAR model are now present in both the co-integration long-term relationship and in the part of the model explaining the impact of the lagged first-differences of the endogenous variables.

$$\Delta y_t = -\alpha\beta' y_{t-1} + \sum_{i=1}^{4}\Gamma_i \Delta y_{t-i} + \alpha\mu + \gamma + \alpha\rho t + \tau t + e_t$$

$$\Delta y_t = -\alpha\beta'\left(y_{t-1} + \mu + \rho t\right) + \sum_{i=1}^{4}\Gamma_i \Delta y_{t-i} + \gamma + \tau t + e_t \qquad (4.5c)$$

This is an important point for understanding the role of the constant and the trend in the VEC Model. We have several possible combinations of the inclusion/exclusion of constant or/and the trend when estimating our VEC Model:

- First is the fully restricted model without a constant and a trend in the co-integration vector or outside the co-integration vector. This model presumes that the long-term drift in the system is determined by the average value of the error in the correction relationship, which could be a non-zero value.

- Second, is the restricted constant case, when we set the restriction $\mu=0$ and estimate only γ, the constant outside the co-integration vector. In this case γ includes both the mean of the co-integration relationships and the drift (an average growth in the model).

- Third, is the unrestricted constant, when we have both a constant in the co-integration vector and a constant outside the co-integration vector. The constant inside the co-integration determines the mean of the co-integration relationships and the constant outside the co-integration vectors determines the pure drift or temporal growth of the system. It is important to note at this point, that if the co-integration relationship exhibits some systemic shift, then there could be a structural shift in the mean within the co-integration relationship at certain point in time. This shift could be modeled by inserting a dummy variable in the co-integration vector that changes the mean at a certain date.

- Fourth, is the case with a constant restricted to the co-integration vector. This implies a non-zero mean within the co-integration relationship, but no drift (growth) in the short-term relationships part of the model.

- Fifth, is the restricted trend, when we add the trend in the co-integration relationship but restrict the trend outside the co-integration relationship by setting a restriction on τ=0. In this case we imply existence of some joint trend within the endogenous variables in the long-term co-integration relationships. To accept this model, we need both theoretical background hypothesizing such a relationship and empirical testing for the presence of such a trend within the co-integration vector.

- Sixth, is the unrestricted trend, when we have no restriction on the trend variable and we add it both in the co-integration vector and outside of the co-integration vector. This implies not only that the co-integration relationship has some linear trend, but also the system grows non-linearly (the trend itself grows with the passage of time). This is a very rare case and we need hard theoretical evidence to use such strong model restrictions.

We could further have other different combinations of restrictions for the constant and the trend.

Finally, we could add a set of exogenous $\sum_{i=0}^{n} X_i \Delta x_{t-1}$ and deterministic variables $\Psi \Delta d_t$ in the VECM(4) as long as they are integrated of order 0. In our case, we use lagged first differences of the "B_US" variable as an exogenous variable and the first differenced dummy variables representing one time shock events on the Croatian financial market. Alternatively, we could also lag the differenced dummy variables if we believe their impact on the system might be prolonged due to the enforcement delay as suggested in Chapter 3. Including the set of exogenous and dummy variables in the model we get to our final reduced form VEC

Model (without applying restrictions on the constant and the trend) represented by Equation 4.6

$$\Delta y_t = -\alpha\beta'\left(y_{t-1} + \mu + dshift_i + \rho t\right) + \sum_{i=1}^{4}\Gamma_i \Delta y_{t-i} + \sum_{i=0}^{n} \mathrm{X}_i \Delta x_{t-i} + \Psi\Delta d_{t-1} + \gamma + \pi t + e_t \tag{4.6}$$

Now that we specified our VECM(4) model, we continue by testing for the existence of co-integration among our endogenous variables.

4.5. Estimating the rank of the co-integration matrix

The Vector Error Correction Model (VECM) has one important requirement placed on the data series used in the modelling process. This requirement as previously discussed, is the existence of one or more co-integration relationship(s) between the variables in levels and, hence, their underlying data generation processes (DGP). Two or a sequence of variables are said to be co-integrated, if there exists a vector of constants which, when multiplied by the matrix consisted of the variables in its columns - each individually integrated at a certain order of integration higher than 0 - generates a new data series integrated at a lower level. In the most common case, co-integration is associated with the existence of at least one vector of constants, which presents a linear combination of the I(1) variables producing a new time series integrated at level 0 (I(0)) .

The cause of the existence of such a linear combination between variables, which produces a stationary linear combination, is the existence of a common stochastic trend which, in the long term, connects the individual variables among themselves. In other words, although a set of variables might contain non-stationary properties when observed individually, when combined together by a certain linear combination with other non-stationary variables, due to some joint long-term relationship they share, they will generate a stationary joint outcome.

When Error Correction (EC) is applied to a single equation model, with K non-stationary variables which are co-integrated, there can be only one (1xK) independent vector of coefficients defining their stationary long-term relationship. This vector could be estimated using OLS regression as shown by Engle and Granger, which produces "super consistent" coefficient estimates (Engle & Granger, 1987), although the asymptotic properties of the estimated co-integration coefficients are not normal, and the corresponding OLS standard errors and t-statistics are biased (Stock, 1987).

In the case when we are analysing a system of equations, such as the one represented by an unrestricted VAR model, when there can be a higher order of independent vectors of constant coefficients defining different independent co-integrating relationships among the variables in the model, then the matrix consisted of independent vectors of co-integrating relationships among the variables in the system used in the VEC Model, is denoted by the Greek letter "∏". This matrix is a square matrix with dimensions equal to the number of equations of the VEC Model or the number of endogenous variables in the system (k). Its rank is most commonly smaller than its dimension and it represents the number of independent co-integration vectors (r). As previously discussed, the co-integration matrix "∏", is decomposed to its two component matrices (4.7). Here "α" is a (Kxr) matrix representing the "loading" or "the speed of adjustment" coefficients, or correction coefficients for each co-integration vector contained in the "β'" matrix to different endogenous variable in the VECM system.

$$\Pi = \alpha\beta' \tag{4.7}$$

Knowing the optimal number of lags in the levels VAR model, we proceed with the next step where we estimate the rank of co-integration matrix "∏" between the endogenous variables. This is the crucial test, whose outcome would decide not only if there exists one or more long-term equilibrating relationship(s) among our endogenous variables in their levels forms (the case when the co-integration rank is higher than zero), but also would tell us if we can use the VEC Model in the further analysis or, rather, that we should restrict our analysis to the VAR model with first differenced transformations of our endogenous variables (the case if the rank of co-integration matrix is equal to zero).

We test for co-integration using the "Johansen Trace" test criteria. The test requires the use of the previously determined optimal order of the VAR in levels model, because when determining its Maximum Likelihood (ML) Reduced Rank (RR) estimator, the Johansen method requires the use of the optimal lag order of the VECM model (Johansen S. , 1988; 1991; 1995).

The test aims at discovering the number of Eigen values of the co-integration matrix, which are statistically different than zero. The test is drawn from the basic laws of linear algebra, that a square matrix would contain R non-zero Eigen values if and only if it consists of R independent vectors which determine its rank. The Johansen Reduced Rank test methodology defines two test statistics that identify the rank of the co-integration matrix. The first is the "Johansen trace statistic"; and second is the "the maximum Eigen value statistic". We briefly explain each test statistic.

4.5.1. Johansen trace statistic

The Johansen likelihood ratio trace statistic H, tests the null hypothesis that the co-integration matrix $\Pi = \alpha\beta'$ is of rank "r" against the unrestricted model where Π is of full rank "k". The procedure first estimates all Eigen values "λ_i" of the co-integration matrix $\prod$, and orders them from the highest to the lowest ($\lambda_1 > \lambda_2 > ... > \lambda_k$). Then we determine the number of characteristic roots that are different from zero, which equals the number of independent co-integration vectors. The simplest case is the one of no-co-integration, when all characteristic roots are not different from zero. In this case $\ln(1-\lambda_i)=0$ for all Eigen values, and all Eigen values are not different from 1. In another case, when the rank of $\prod$ equals 1, which means there exist one independent co-integration vector, only the root of the highest Eigen value will be different from zero or $\ln(1-\lambda_1)<0$.

The Johansen trace statistics looks at the sum of the estimated roots of the Eigen values multiplied by the number of observations T. This test is performed in a sequence of tests controlled by r which is increased from zero to k-1 in each consecutive test, each time estimating the test statistic H (equation 4.8).

$$H = -T \sum_{i=r+1}^{k} \ln(1 - \hat{\lambda}_i) \qquad (4.8)$$

The test is done for a specific rank of the co-integration matrix. The null hypothesis based on the H in equation 4.8 is equivalent to saying that the co-integration matrix $\Pi = \alpha\beta'$ has (k-r) unit roots, where k is the number of endogenous variables or the full rank of $\prod$, and r is the number of co-integrating vectors. This test statistic is often referred as the "Johansen trace" statistic, because it is associated with the sum of diagonal values of the Eigen value matrix, which is the trace of the Eigen value matrix. This statistic is expected to be close to zero, if there are at most "r" linearly independent co-integrating vectors. This is simply because in such a case, the r+1 to k-th Eigen-values, should have values statistically not different from zero.

The test statistic is calculated for every rank restriction of the co-integration matrix in a pre-specified lag order VEC Model. The null hypothesis for each test states that the rank of the matrix is lower or equal to "r", and the alternative is that the rank of the co-integration matrix is higher than "r". The testing procedure usually starts from the restriction of "r"=0. If we reject the null, then we continue testing for each higher rank restriction until we reject the null for the first time. We accept the highest rank restriction for which we fail to reject the null on the corresponding rank of the co-integration matrix.

4.5.2. Johansen maximum Eigen-value statistic

Another similar test statistic, related to the Johansen methodology is the "maximum Eigen value" test or the "$\lambda \max$" test statistic. The idea of the estimator and the test is to discover the highest Eigen value for which the test statistic $-T\ln(1-\hat{\lambda}_{r+1})$ is not statistically different from zero. If the r+1 Eigen value is not statistically different from zero then all lower Eigen values are also not statistically different from zero, which can be used to define the number of non-zero Eigen values and the rank of co-integration

The asymptotic distribution of the LR test statistic is a function of a Brownian motion (Johansen S. , 1991) and is tabulated by Osterwald-Lenum (Osterwald-Lenum, 1992). Most statistics software packages such as STATA or JMulTi, have the tabulated critical values of the Johansen co-integration tests and compare them to the generated test statistic at different reduced rank restrictions.

4.5.3. Johansen "Trace Test" and the rank of co-integration of the Croatian data

We use the predetermined VAR system in levels with 5 lags in the Johansen trace test which transforms to a VECM with 4 lags. The null hypothesis is tested for every reduced rank restriction (RR) starting from zero up to the maximum possible rank determined by the dimension of the co-integration coefficient matrix (k). Each time, the null hypothesis will be that the actual rank of the matrix is lower than or equal to the tested one. In other words, the null hypothesis for the rank restriction of zero would claim that the rank is lower than or equal to zero. If we reject this hypothesis, we continue testing the same null hypothesis for the ML reduced rank estimator of a higher rank, the rank of one. There, the null hypothesis will be that the rank of the coefficient matrix is lower than or equal to one. The alternative hypothesis is that the rank of the co-integration coefficient matrix is higher than one and so on. So, if we fail to reject the null that the rank is lower than or equal to 1, having previously rejected the null that the rank is lower than or equal to zero, we would logically conclude that the rank of the co-integration matrix is one, which is same as saying that there exists one independent vector of coefficients that represents a linear transformation of variables making them stationary.

In Table 4.5.3.1, we present the co-integration tests for the rank of the co-integration matrix between the endogenous variables "$B\_Croatia_m$", "$DYNEF_m$", "$PFInvDirect_m$" and "$PFInvIndirect_m$". We perform the test by including first only a constant, and secondly a constant and trend in the co-integration matrix. We also test allowing for a potential structural break (the Global financial crisis, 2008 Month 9). We will be using the 95% level of confidence as our decision criteria. We note that in the case without the structural break, we have stronger evidence of the existence of one co-integration vector; while when we include the structural break in the co-integration vector, we have strong evidence of the existence of two co-integration relationships.

Table 4.5.3.1 Johansen Trace Test for co-integration

a) Not allowing for a structural break

```
Johansen Trace Test for:  B_Croatia DYNEF PFInvDirect PFInvIndirect
unrestricted dummies:     dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1
restricted dummies:
sample range:             [1998 M5, 2010 M12], T = 152
included lags (levels):   5
dimension of the process: 4
intercept included
response surface computed:
----------------------------------------------
r0  LR       pval     90%      95%      99%
----------------------------------------------
 0   63.03    0.0056   50.50    53.94    60.81
1   32.34    0.0980   32.25    35.07    40.78
 2   9.01     0.7371   17.98    20.16    24.69
 3   1.16     0.9146   7.60     9.14     12.53

Johansen Trace Test for:  B_Croatia DYNEF PFInvDirect PFInvIndirect
unrestricted dummies:     dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1
restricted dummies:
sample range:             [1998 M5, 2010 M12], T = 152
included lags (levels):   5
dimension of the process: 4
trend and intercept included
response surface computed:
----------------------------------------------
r0  LR       pval     90%      95%      99%
----------------------------------------------
 0   86.30    0.0001   60.00    63.66    70.91
 1   40.03    0.0938   39.73    42.77    48.87
 2   18.69    0.3056   23.32    25.73    30.67
 3   5.04     0.5975   10.68    12.45    16.22
```

b) Testing for co-integration in the presence of a structural break (2008 Month 9)

```
Johansen Trace Test for:  B_Croatia DYNEF PFInvDirect PFInvIndirect
unrestricted dummies:     dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1 D[2008 M9] D[2008 M10] D[2008 M11] D[2008 M12] D[2009 M1]
restricted dummies:       S[2008 M9]
sample range:             [1998 M5, 2010 M12], T = 152
included lags (levels):   5
dimension of the process: 4
intercept included
response surface computed:
----------------------------------------------
r0  LR       pval     90%      95%      99%
----------------------------------------------
 0   143.89   0.0000   58.87    62.37    69.31
 1   47.05    0.0118   38.80    41.73    47.61
2   11.67    0.8015   22.60    24.96    29.80
 3   2.68     0.8514   10.35    12.19    16.18

Johansen Trace Test for:  B_Croatia DYNEF PFInvDirect PFInvIndirect
unrestricted dummies:     dummy1_L1_d1 dummy2_L1_d1 dummy3_L1_d1 dummy4_L1_d1
Dummy_DYNEF_L1_d1 D[2008 M9] D[2008 M10] D[2008 M11] D[2008 M12] D[2009 M1]
restricted dummies:       S[2008 M9]
sample range:             [1998 M5, 2010 M12], T = 152
included lags (levels):   5
dimension of the process: 4
trend and intercept included
response surface computed:
```

r0	LR	pval	90%	95%	99%
0	148.80	0.0000	65.92	69.77	77.37
1	60.88	0.0014	44.36	47.58	54.02
2	**22.33**	**0.2668**	**26.62**	**29.20**	**34.46**
3	5.92	0.6450	12.52	14.42	18.45

Source: JMulti co-integration test

This is a very important outcome, which points to the existence of two co-integrating vectors of constants for our endogenous variables in the model with a structural break occurring at the onset of the Global Financial Crisis. We conclude that there exist two long-term co-integrating relationships among the relative value of the Croatian equity bubble "$B\_Croatia_m$", the dynamic efficiency of the Croatian financial market "$DYNEF_m$", direct equity investments of the DC pension funds into the Croatian equity market "$PFInvDirect_m$", and the indirect investment of the DC pension funds into the Croatian and SEE equity market through the Croatian and SEE equity focused open end investment funds "$PFInvIndirect_m$".

Knowing that our non-stationary endogenous variables are co-integrated with a rank of one (excluding the structural break) but with a rank of two when accounting for the structural break, we can accept the use of and correctly specify the VEC Model in order to estimate both the long-run and the short-run relationships in the system.

4.6. Vector Error Correction Model Estimation

Concluding that our endogenous variables are best fitted by a VAR model of order 5 (which transforms into optimal lag of the VECM model of 4), and that there are two co-integration relationships when the model accounts for a structural break caused by the onset of the Global Financial Crisis, we chose to estimate a VECM (4) with two error correction vectors. Although the model with a constant and a trend compared to the model with only a constant restricted to the co-integration vector showed more significant evidence of the existence of two co-integration vectors, we prefer the model with a restricted constant and no trend in the co-integration vector. This choice is made because the theoretical model developed in Chapter 2 does not suggest the existence of a deterministic trend in any of the endogenous variables, especially not in the value of the bubble or in the dynamic efficiency.

The explicit structure of our VECM(4) model with two co-integration vectors, structural break and a restricted constant in the co-integration vector is represented in the following equation 4.9.

$$\Delta y_t = -\alpha\beta'\left(y_{t-1} + \mu + dshift_i\right) + \sum_{i=1}^{4}\Gamma_i \Delta y_{t-i} + \sum_{i=0}^{n}\mathrm{X}_i \Delta x_{t-i} + \Psi\Delta d_{t-1} + e_t \quad (4.9)$$

In this general equation describing the VECM (4) system, the "Γ" matrices are the coefficient matrices of the lagged difference variables in the model, which are squared (4x4) matrices in our case with four endogenous variables. Matrices "X_i" represent the coefficient matrices of the exogenous stochastic variables in our model. We have only one stochastic exogenous variable, which is the first difference of the relative value of the equity bubble on the US equity market "B_US_m". Each matrix of coefficients "X" has a dimension of 4x1 containing the coefficients of the lagged impact of the exogenous variable on our endogenous variables in the system. Then, our "ψ" matrix contains the coefficients measuring the impact of shocks captured by the deterministic variables, including all the dummy variables that are suspected to have an impact on our endogenous variables in the system. They are lagged at one period, due to the enforcement time required after the introduction of the legal acts. Finally, the co-integration matrix ∏ is presented in its decomposed form α[β':η'], where η represents the co-integration coefficients of the deterministic term, in this case the constant (μ) and the structural shift (l_dShift_m9), which are part of the co-integration matrix. All variables in the VEC model are divided into three major groups - endogenous, exogenous and deterministic - and are presented in their vector form below in 4.10.

$$y_m = \begin{bmatrix} B\_Croatia \\ DYNEF \\ PFInvDirect \\ PFInvIndirect \end{bmatrix}_m ; \Delta y_m = \begin{bmatrix} D.B\_Croatia \\ D.DYNEF \\ D.PFInvDirect \\ D.PFInvIndirect \end{bmatrix}_m ;$$

$$x_m = \left[D.B\_US\right]_m ; \qquad D_m = \begin{bmatrix} D.dummy1_{m-1} \\ D.dummy2_{m-1} \\ D.dummy3_{m-1} \\ D.dummy4_{m-1} \\ D.dummy\_DYNEF_{m-1} \\ \mu \\ l\_dShift\_m9 \end{bmatrix}_m \qquad (4.10)$$

When we first difference our optimal five-period lagged levels VAR model, we generate a four period lagged VEC model. This is why we choose an optimal lag of 4 when estimating the VEC Model using JMulti. Because we want to estimate the VECM model in the presence of exogenous variables and with the optimal sub-set model restrictions, we use the two stage estimation procedure. For the estimation of the co-integration matrix in the first stage we performed S2S method. In such a case, JMulti eliminates all exogenous variables in the first stage, when estimating the co-integration matrix. Bruggeman and Lütkepohl proposed the use of S2S as a more efficient method with a small sample for estimation of the co-integration matrix in the first stage compared to the Johansen ML estimator (Bruggemann & Lutkepohl, 2005). The S2S method consists of two estimation steps. In the first step, we estimate the co-integration matrix using Least Square (LS) estimation. We identify the "α" and the "β" component matrices from the first predetermined "r" columns of the ∏ matrix. This requires that we pre-specify the rank of the co-integration matrix and properly structure the data before estimation. We found in the previous step using the Johansen test procedure, that the rank of the co-integration matrix equals two when we include a structural break. JMulti automatically normalizes the two co-integration vectors in the estimation procedure: the first, by restricting the value of the coefficient for the first variable "B_Croatia" to one and by restricting the second coefficient – on "DYNEF" – to zero; and the second, by restricting the value of the coefficient on the first variable "B-Croatia" to zero and normalizing the coefficient on the second variable "DYNEF" to one. This solves the identification problem, since otherwise there is an indefinite number of vectors spanning the vector space of each linearly independent co-integration vector. Given our theoretically determined ordering, this default procedure implemented by JMulti exactly identifies the two co-integrating vectors as economically meaningful long-term equilibrium relationships.

The decision as to which two variables are to be ordered at the beginning of the vector of endogenous variables is determined by theoretical reasoning. In our case, we place the relative value of the Croatian equity bubble "B_Croatia" first and the dynamic efficiency of the Croatian financial market "DYNEF" second. These two variables are then normalized in the two co-integrating equations and related to the two DC pension fund investment variables "PFInvDirect" and "PFInvIndirect", which are placed in the third and the fourth positions respectively in the vector of endogenous variables. This is the most logical order based on the theoretical grounds in Chapter 2. It allows for two economically meaningful co-integrating vectors in the estimation process: first, where the relative value of the Croatian equity bubble “B_Croatia” is related to the two DC pension fund investment variables, while restricting the coefficient of the “DYNEF” to zero; and Second, where the Dynamic efficiency of the Croatian financial market “DYNEF” is related to the two DC pension fund investment variables, restricting the coefficient of “B_Croatia” to zero. Then, the co-integration matrix is estimated using the S2S method, and the “α” and “β” component vectors are estimated in the first step using OLS.

What follows is the second step, which now expands the EC model by estimating the coefficient matrices on the short-term – i.e. first differenced – variables in the VECM system. JMulti uses Generalised Least Square (GLS) to estimate the short-term VAR system of first differences as part of the VECM conditional on the previously determined long-term equilibrium relations among the endogenous variables (Lütkepohl & Krätzig, 2004). Besides the inclusion of the exogenous and deterministic variables in the model, we also include a constant in the VECM. The constant term is restricted to the co-integration vector of the VECM system. The theory suggests that there should not be a trend connected to the development of the asset bubble so, by theory, a constant restricted to the co-integration vector without a trend represents the most suitable model. The estimation results of the unrestricted VEC Model, using the two-stage approach in the JMulti software are presented in Table 4.6.1.

Table 4.6.1 Unrestricted VECM estimation results of a two-stage method [1st:S2S;2nd: EGLS]

$$\begin{bmatrix} \Delta B\_Croatia_m \\ \Delta DYNEF_m \\ \Delta PFInvDirect_m \\ \Delta PFInvIndirect_m \end{bmatrix} = \left[\begin{bmatrix} -0.183 & -0.253 \\ (-3.263) & (-2.939) \\ -0.007 & -0.016 \\ (-0.729) & (-1.160) \\ 240.052 & 1103.898 \\ (0.748) & (2.241) \\ 907.201 & -2207.659 \\ (1.127) & (-1.785) \end{bmatrix} \begin{bmatrix} 1 & / & -0.0000008234 & -0.0000002228 \\ / & / & (-3.6029) & (-0.4126) \\ / & 1 & 0.0000002309 & -0.0000004028 \\ / & / & (1.1925) & (-0.88042) \end{bmatrix} \begin{bmatrix} B\_Croatia_{m-1} \\ DYNEF_{m-1} \\ PFInvDirect_{m-1} \\ PFInvIndirect_{m-1} \end{bmatrix} + \begin{bmatrix} 20.774 & 24.882 \\ (0.642) & (6.151) \\ -7.234 & 0.213 \\ (-0.264) & (0.062) \end{bmatrix} \begin{bmatrix} l\_shift\_m9_{t-1} \\ Const \end{bmatrix} \right]$$

$$+ \begin{bmatrix} -0.114 & -0.131 & -0.000 & -0.000 \\ (-1.356) & (-0.194) & (-0.646) & (-0.731) \\ 0.003 & -0.163 & -0.000 & -0.000 \\ (0.205) & (-1.479) & (-1.533) & (-0.259) \\ -109.615 & -7776.678 & 3.013 & 0.051 \\ (-0.228) & (-2.007) & (25.449) & (1.224) \\ -572.705 & 17833.095 & 0.249 & 2.996 \\ (-0.474) & (1.833) & (0.839) & (28.383) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-1} \\ \Delta DYNEF_{m-1} \\ \Delta PFInvDirect_{m-1} \\ \Delta PFInvIndirect_{m-1} \end{bmatrix} + \begin{bmatrix} 0.085 & 0.217 & 0.000 & 0.000 \\ (1.002) & (0.455) & (0.865) & (0.946) \\ 0.014 & -0.107 & 0.000 & 0.000 \\ (1.037) & (-1.373) & (1.304) & (0.324) \\ 594.423 & -9408.381 & -3.559 & -0.092 \\ (1.221) & (-3.447) & (-10.572) & (-0.740) \\ -1540.349 & 20707.717 & -0.013 & -3.335 \\ (-1.260) & (3.023) & (-0.015) & (-10.670) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-2} \\ \Delta DYNEF_{m-2} \\ \Delta PFInvDirect_{m-2} \\ \Delta PFInvIndirect_{m-2} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.064 & 0.122 & -0.000 & -0.000 \\ (0.755) & (0.252) & (-0.983) & (-0.939) \\ 0.017 & -0.098 & -0.000 & -0.000 \\ (1.200) & (-1.246) & (-1.035) & (-0.325) \\ 25.400 & 2603.889 & 1.980 & 0.025 \\ (0.052) & (0.940) & (5.894) & (0.191) \\ 1154.717 & -4734.186 & -0.536 & 1.673 \\ (0.950) & (-0.681) & (-0.635) & (5.079) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-3} \\ \Delta DYNEF_{m-3} \\ \Delta PFInvDirect_{m-3} \\ \Delta PFInvDirect_{m-3} \end{bmatrix} + \begin{bmatrix} 0.034 & -0.801 & 0.000 & 0.000 \\ (0.460) & (-1.642) & (0.984) & (0.727) \\ -0.014 & -0.121 & 0.000 & 0.000 \\ (-1.175) & (-1.519) & (0.747) & (0.267) \\ -472.741 & -818.449 & -0.436 & 0.026 \\ (-1.125) & (-0.293) & (-3.713) & (0.527) \\ 2317.915 & -12.720 & 0.295 & -0.366 \\ (2.199) & (-0.002) & (1.000) & (-2.942) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-4} \\ \Delta DYNEF_{m-4} \\ \Delta PFInvDirect_{m-4} \\ \Delta PFInvIndirect_{m-4} \end{bmatrix} +$$

$$+ \begin{bmatrix} -0.126 \\ (-0.966) \\ 0.003 \\ (0.122) \\ 958.040 \\ (1.282) \\ -1566.944 \\ (-0.835) \end{bmatrix} \Delta B\_US_m + \begin{bmatrix} 0.656 \\ (5.107) \\ -0.019 \\ (-0.911) \\ 3.573 \\ (-0.005) \\ 1497.527 \\ (0.811) \end{bmatrix} \Delta B\_US_{m-1} + \begin{bmatrix} 0.184 \\ (1.315) \\ -0.011 \\ (-0.491) \\ 829.941 \\ (1.037) \\ -931.653 \\ (-0.464) \end{bmatrix} \Delta B\_US_{m-2} + \begin{bmatrix} 0.105 \\ (0.763) \\ -0.020 \\ (-0.875) \\ 43.526 \\ (0.055) \\ -290.463 \\ (-0.147) \end{bmatrix} \Delta B\_US_{m-3} + \begin{bmatrix} -0.170 \\ (-1.233) \\ -0.002 \\ (-0.078) \\ 220.081 \\ (0.280) \\ -1070.897 \\ (-0.543) \end{bmatrix} \Delta B\_US_{m-4} +$$

$$+ \begin{bmatrix} -0.767 & -2.099 & -4.491 & 16.173 & -5.346 \\ (-0.072) & (-0.299) & (-0.500) & (1.890) & (-0.944) \\ -0.372 & 0.556 & -0.072 & 3.243 & 1.661 \\ (-0.214) & (0.485) & (-0.049) & (2.320) & (1.795) \\ -148244.238 & 23097.647 & -20150.255 & -15384.147 & -54417.050 \\ (-2.434) & (0.574) & (-0.392) & (-0.314) & (-1.677) \\ 230116.815 & -36858.169 & 824972.534 & 143024.249 & 102712.648 \\ (1.506) & (-0.365) & (6.390) & (1.162) & (1.261) \end{bmatrix} \begin{bmatrix} D.dummy1_{m-1} \\ D.dummy2_{m-1} \\ D.dummy3_{m-1} \\ D.dummy4_{m-1} \\ D.dummy\_DYNEF_{m-1} \end{bmatrix}$$

*t-statistic stated in parenthesis

**T=152

$$Corr = \begin{bmatrix} 1 & -0.1277 & 0.0009 & 0.0767 \\ & 1 & 0.2155 & -0.2471 \\ & & 1 & -0.8640 \\ & & & 1 \end{bmatrix}, \det(\mathrm{cov}) = 2.211588e+20$$

Source: Author, using JMulti 4.24

Before we specify a subset model by restricting the "Γ", "X" and "ψ" matrices from our basic VECM model as represented in equation 4.9 and estimated in Table 4.6.1, we test for the significance of the intercept as a deterministic component in the VECM. We did this by estimating the model without the constant term. We present the residual statistics of this restricted model in Table 4.6.2.

Table 4.6.2: Residuals statistics from the restricted model (omitting the constant term)

$$Corr = \begin{bmatrix} 1 & -0.0672 & 0.0143 & -0.0091 \\ & 1 & 0.1870 & -0.2221 \\ & & 1 & -0.8519 \\ & & & 1 \end{bmatrix}, \det(\text{cov}) = 2.394321e+20$$

Source: Author, using JMulTi 4.24

Then we check the significance of the inclusion of the constant term in the model by calculating the Likelihood Ratio (LR) test statistics as presented in the equation 4.11.

$$LR = \left\{ \ln\left|\tilde{\Sigma}_{unrestricted}\right| - \ln\left|\tilde{\Sigma}_{restricted}\right| \right\} \bullet T \tag{4.11}$$

The LR statistic is calculated as the difference between the natural log of the determinant of the VECM residuals covariance matrix of the unrestricted model (with intercept) and the natural log of the determinant of the restricted model (without intercept). The distribution of the LR test statistic asymptotically goes to the chi-square distribution with the degrees of freedom equal to the number of restrictions. In our case, the vector of constants within the co-integration matrix has a dimension of 1x2, which means we placed two restrictions on our restricted model setting the value of the two constants to zero. Based on the determinants of the residuals covariance matrices of the restricted and the unrestricted model, equalling 2.211588E+20 and 2.394321E+20, and their natural logs 46.84541 and 46.92480 respectively, we calculated the LR statistic of 12.07 for our sample of T=152. The value of the test statistic 12.07 is higher than the critical value of the Chi-square distribution at two degrees of freedom of 5.99 for p=0.05, which suggests a strong rejection of the null and thus favours acceptance of the alternative hypothesis that the deterministic constant term in the co-integration matrix of the VECM has significant explanatory power.

We also want to check the subset VEC Model, which is constructed according to different selection criteria by placing zero restrictions on selected (non-significant) regression coefficients. JMulti provides several subset model selection methodologies, such as the "top down approach", the System SER and SER/Testing procedures and the method of elimination by t-statistic criteria. All methods, except for the elimination by t-statistics, use one of the three information criteria to select the optimal model restrictions. The AIC, HQ and SC information criterion are used within the Sequential Elimination of Regressors (SER), aiming at sequentially eliminating regressors that lead to a significant decrease of the underlying criterion used to compare the restricted and unrestricted method at each step. Brüeggemann, Krolzig and Lütkepohl, showed that choosing AIC, HQ or SC decision criteria for a model with 20 regressors based on T=100, gives a restriction results corresponding to a method of regressor elimination based on t- statistics that are not significant with 15-20%, 10% or 2-3% confidence levels respectively (Brüeggemann, Krolzig, & Lütkepohl, 2002).

In order to select the optimal subset model selection criteria, we choose to apply not only all the available filtering approaches but also the diagnostic tests for each resulting model, as well as the LR-test statistic comparing the restricted model with the unrestricted model. We present the results of the selection process in Table 4.6.3.

Table 4.6.3 Subset model selection criteria results for the VECM(4)

Model	Number of variables	Testing residuals autocorrelation		Testing joint residuals joint normality (p-value)	LR-test	Det(Σ(u))
		LM(2) p-value	LM(3) p-value			
Unrestricted	128	0.0046	0.0009	0	1	2.21E+20
Top down -AIC	67	0.1252	0.1189	0	0.8158	3.21E+20
Top down -HQ	51	0.599	0.4904	0	0.3914	3.76E+20
Top Down -SC	40	0.4492	0.4768	0	0.1321	4.29E+20
Restricted by abs(t)>2	55	0.8397	0.9288	0	0.7145	3.52E+20
System SER (AIC)	72	0.2482	0.4438	0	0.999	2.66E+20
System SER (HQ)*	60	0.9817	0.9899	0	0.9627	3.13E+20
System SER (SC)	43	0.4422	0.5235	0	0.2171	4.16E+20
SER/Testing procedure (AIC)	72	0.6683	0.8945	0	0.9906	2.89E+20
SER/Testing procedure (HQ)	57	0.9561	0.6363	0	0.5257	3.59E+20
SER/Testing procedure (SC)	46	0.6647	0.375	0	0.127	4.24E+20

Source: Author, using JMulti 4.24

Based on comparing different filtering criteria for the selection of the optimal subset model, we find that the System SER (HQ) selection criterion has advantages over the other selection criteria. It provides a model with the best balance between the number of explanatory variables kept in the model and the unexplained variance measured by the determinant of the residuals covariance matrix. It also rejects the residuals autocorrelation LM test with very strong confidence. We decide to use System SER based on the HQ information criterion as an optimal subset filtering method. We present the resulting subset regression model in Table 4.6.4 suggested by applying the System SER (HQ) subset selection criterion.

Table 4.6.4 Subset Model with the System SER (HQ) selection method estimated by the two-stage method [1st:S2S;2nd: GLS];T=152

$$\begin{bmatrix} \Delta B\_Croatia_m \\ \Delta DYNEF_m \\ \Delta PFInvDirect_m \\ \Delta PFInvIndirect_m \end{bmatrix} = \left[\begin{bmatrix} -0.175 & -0.261 \\ (-3.733) & (-3.245) \\ ---- & ---- \\ ---- & ---- \\ ---- & 853.994 \\ ---- & (2.403) \\ 947.485 & -2061.934 \\ (2.711) & (-2.137) \end{bmatrix} \begin{bmatrix} 1 & / & -0.0000008234 & -0.0000002228 \\ / & / & (-3.6029) & (-0.4126) \\ / & 1 & 0.0000002309 & -0.0000004028 \\ / & / & (1.1925) & (-0.88042) \end{bmatrix} \begin{bmatrix} B\_Croatia_{m-1} \\ DYNEF_{m-1} \\ PFInvDirect_{m-1} \\ PFInvIndirect_{m-1} \end{bmatrix} + \begin{bmatrix} 20.774 & 24.882 \\ (0.642) & (6.151) \\ -7.234 & 0.213 \\ (-0.264) & (0.062) \end{bmatrix} \begin{bmatrix} l\_shift\_m9_{t-1} \\ Const \end{bmatrix} \right]$$

$$+ \begin{bmatrix} ---- & ---- & ---- & ---- \\ ---- & ---- & ---- & ---- \\ ---- & ---- & -0.000 & ---- \\ ---- & ---- & (-3.146) & ---- \\ ---- & -7514.397 & 3.027 & 0.049 \\ ---- & (-2.591) & (56.616) & (3.252) \\ ---- & 13241.882 & 0.256 & 3.009 \\ ---- & (1.975) & (5.080) & (62.763) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-1} \\ \Delta DYNEF_{m-1} \\ \Delta PFInvDirect_{m-1} \\ \Delta PFInvIndirect_{m-1} \end{bmatrix} + \begin{bmatrix} ---- & ---- & 0.000 & 0.000 \\ ---- & ---- & (2.564) & (5.137) \\ ---- & ---- & 0.000 & 0.000 \\ ---- & ---- & (3.077) & (2.160) \\ ---- & -9352.315 & -3.584 & -0.076 \\ ---- & (-3.514) & (-26.661) & (-3.244) \\ ---- & 20688.189 & ---- & -3.367 \\ ---- & (3.127) & ---- & (-27.994) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-2} \\ \Delta DYNEF_{m-2} \\ \Delta PFInvDirect_{m-2} \\ \Delta PFInvIndirect_{m-2} \end{bmatrix} +$$

$$+ \begin{bmatrix} ---- & ---- & -0.000 & -0.000 \\ ---- & ---- & (-2.679) & (-4.487) \\ ---- & ---- & -0.000 & ---- \\ ---- & ---- & (-2.978) & ---- \\ ---- & ---- & 1.987 & ---- \\ ---- & ---- & (15.034) & ---- \\ 1380.372 & ---- & -0.580 & 1.694 \\ (2.550) & ---- & (-4.131) & (13.543) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-3} \\ \Delta DYNEF_{m-3} \\ \Delta PFInvDirect_{m-3} \\ \Delta PFInvDirect_{m-3} \end{bmatrix} + \begin{bmatrix} ---- & ---- & ---- & ---- \\ ---- & ---- & ---- & ---- \\ ---- & -0.148 & ---- & ---- \\ ---- & (-1.957) & ---- & ---- \\ ---- & ---- & -0.433 & 0.037 \\ ---- & ---- & (-8.515) & (3.903) \\ 1196.373 & ---- & 0.320 & -0.369 \\ (2.372) & ---- & (3.405) & (-6.689) \end{bmatrix} \begin{bmatrix} \Delta B\_Croatia_{m-4} \\ \Delta DYNEF_{m-4} \\ \Delta PFInvDirect_{m-4} \\ \Delta PFInvIndirect_{m-4} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.629 \\ (4.956) \\ ---- \\ ---- \\ ---- \\ ---- \\ ---- \\ ---- \end{bmatrix} \Delta B\_US_{m-1} + \begin{bmatrix} ---- & ---- & ---- & 16.712 & ---- \\ ---- & ---- & ---- & (2.204) & ---- \\ ---- & ---- & ---- & 4.198 & ---- \\ ---- & ---- & ---- & (3.443) & ---- \\ -67411.957 & ---- & ---- & ---- & ---- \\ (-2.499) & ---- & ---- & ---- & ---- \\ ---- & ---- & 798349.796 & ---- & ---- \\ ---- & ---- & (13.650) & ---- & ---- \end{bmatrix} \begin{bmatrix} D.dummy1_{m-1} \\ D.dummy2_{m-1} \\ D.dummy3_{m-1} \\ D.dummy4_{m-1} \\ D.dummy\_DYNEF_{m-1} \end{bmatrix}$$

$$Corr = \begin{bmatrix} 1 & -0.12803 & 0.01956 & 0.03868 \\ & 1 & 0.19738 & -0.24266 \\ & & 1 & -0.86694 \\ & & & 1 \end{bmatrix}, \det(\text{cov}) = 3.133590e+20$$

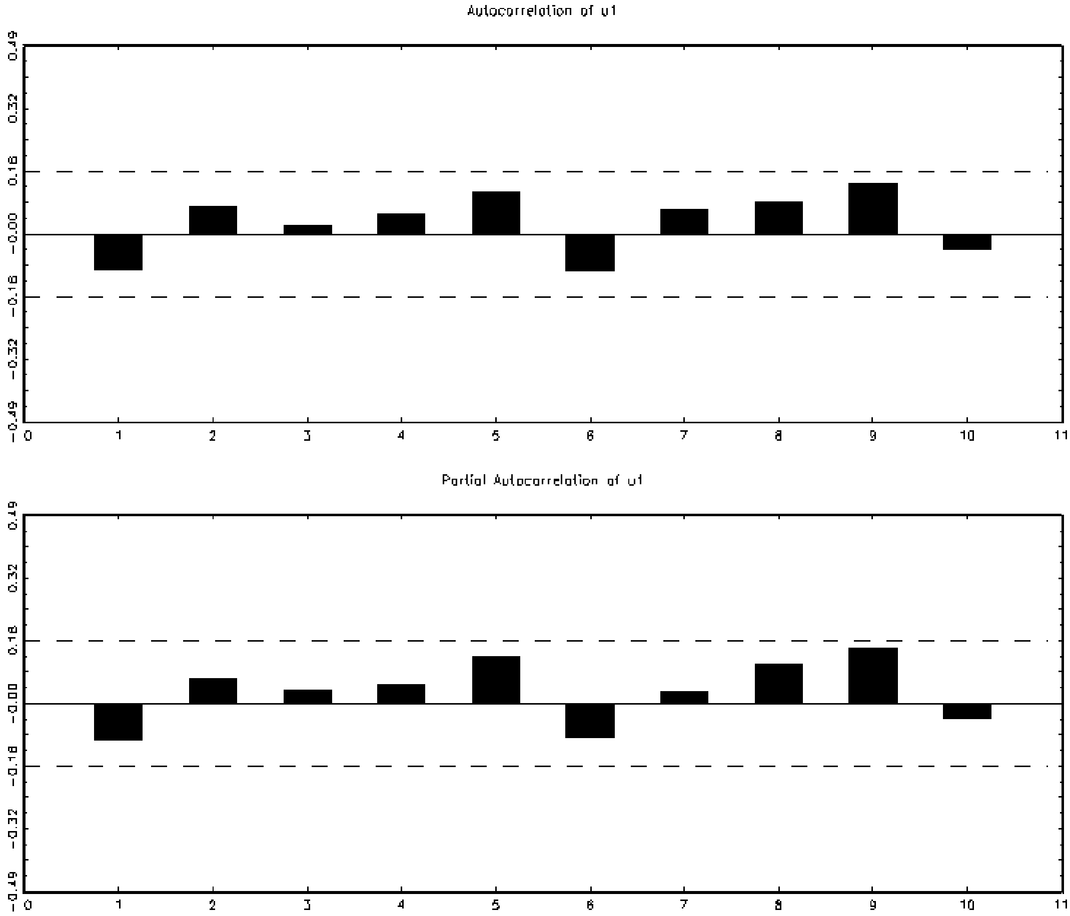
LR test (H1: unrestricted model) p-value = 0.9627 with 73 degrees of freedom

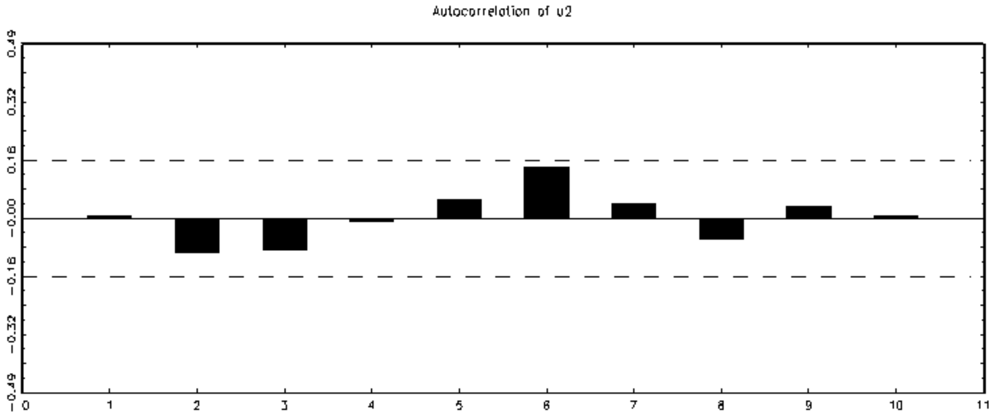
Source: Author, using JMulti 4.24

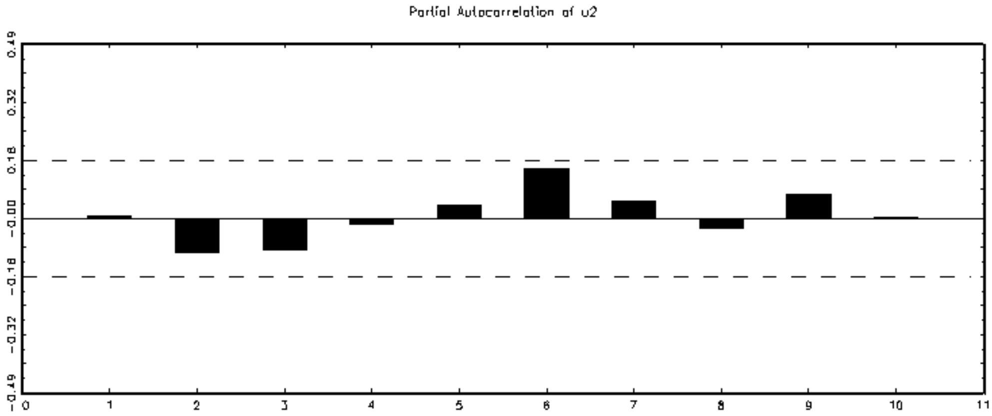
4.7 VEC Model diagnostic tests results

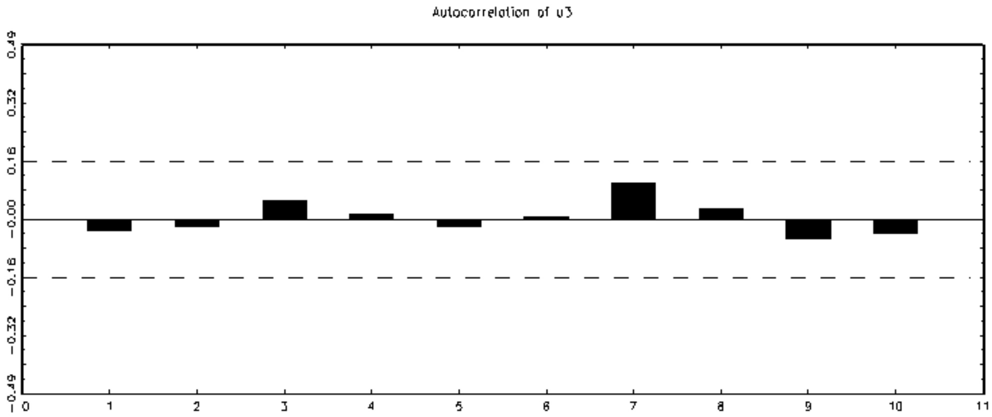
Having estimated our restricted subset model, we need to make sure that its diagnostic tests confirm that we could rely on it being unbiased. We already undertook the formal autocorrelation tests based on the Lagrange Multiplier test, which showed that the residuals are not serially correlated (Table 4.6.3). We confirm this by checking the auto-correlation plots of the residuals from each of the individual equations of the subset VECM, with their lagged values up to the tenth previous month. We present the outcomes in Figure 4.7.1

Figure 4.7.1 Subset VEC Model residual autocorrelations up to the 10th residual lag (with a 99% critical value bands): autocorrelation and partial correlation coefficients (u1 – u4, the residuals from equations 1-4 in the VECM)




Autocorrelation of u2
0.49
0.32
0.16
-0.00
-0.16
-0.32
-0.49
0
1
2
3
4
5
6
7
8
9
10
11


Partial Autocorrelation of u2
0.49
0.32
0.16
-0.00
-0.16
-0.32
-0.49
0
1
2
3
4
5
6
7
8
9
10
11


Autocorrelation of u3
0.49
0.32
0.16
-0.00
-0.16
-0.32
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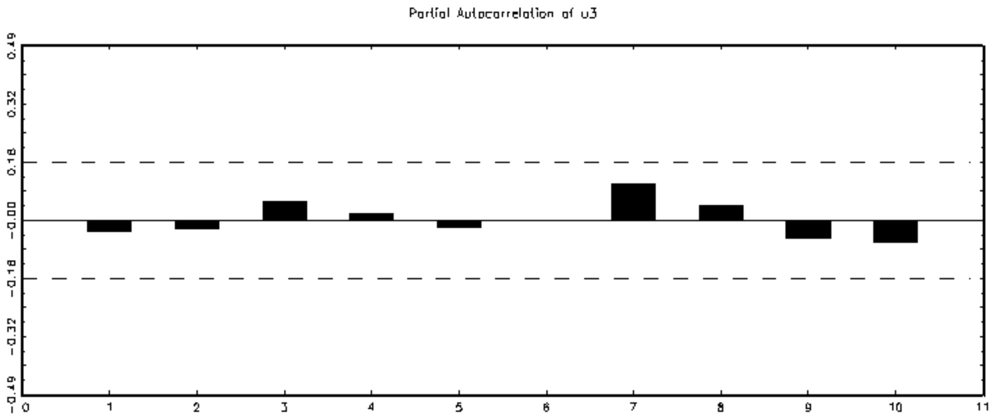
Partial Autocorrelation of u3
0.49
0.32
0.16
-0.00
-0.16
-0.32
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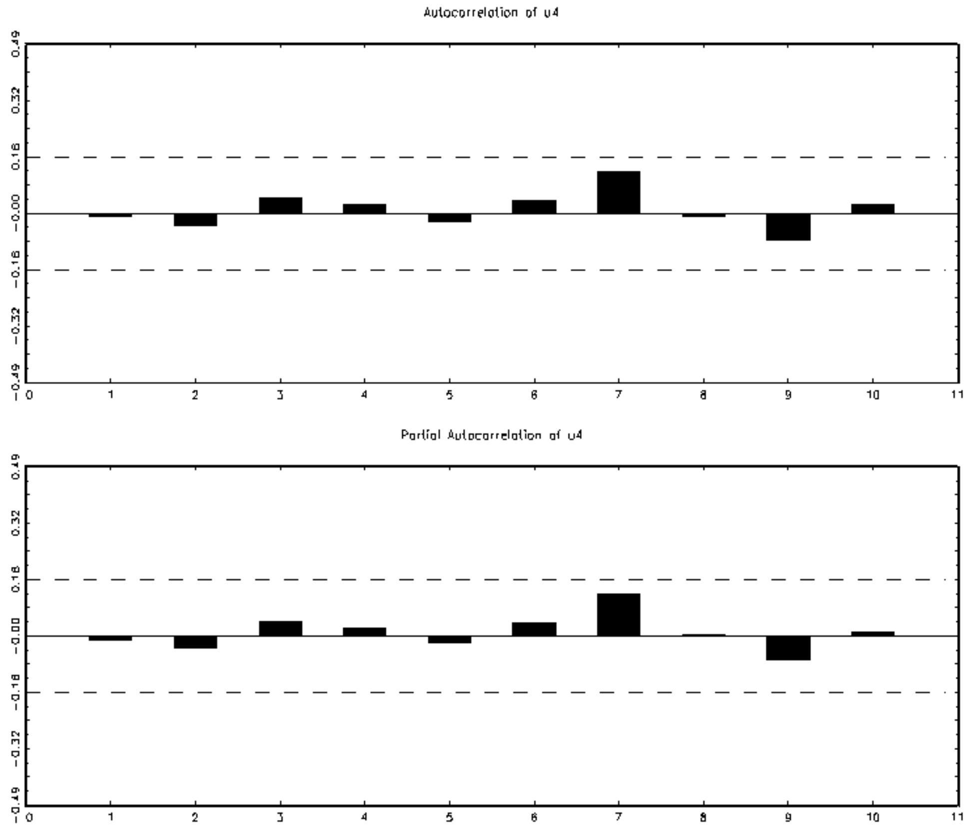


Source: Author, using JMulti 4.24

We find that the graphed residuals autocorrelations of all four VECM equations confirm the outcomes of the LM tests of autocorrelation as presented in Table 4.6.3. Looking at the graphs, there is no statistically significant autocorrelation, breaking the critical +/- 2/sqrt(T) bounds, at any lag of the residuals in any of the four equation in the model. In Table 4.7.1, we report the results of the residuals autocorrelation test and normality test, and in line with the formal LM test, we conclude that there is no substantial autocorrelation of the regression residuals.

Table 4.7.1: Residuals autocorrelation and non-normality tests of our restricted VEC model

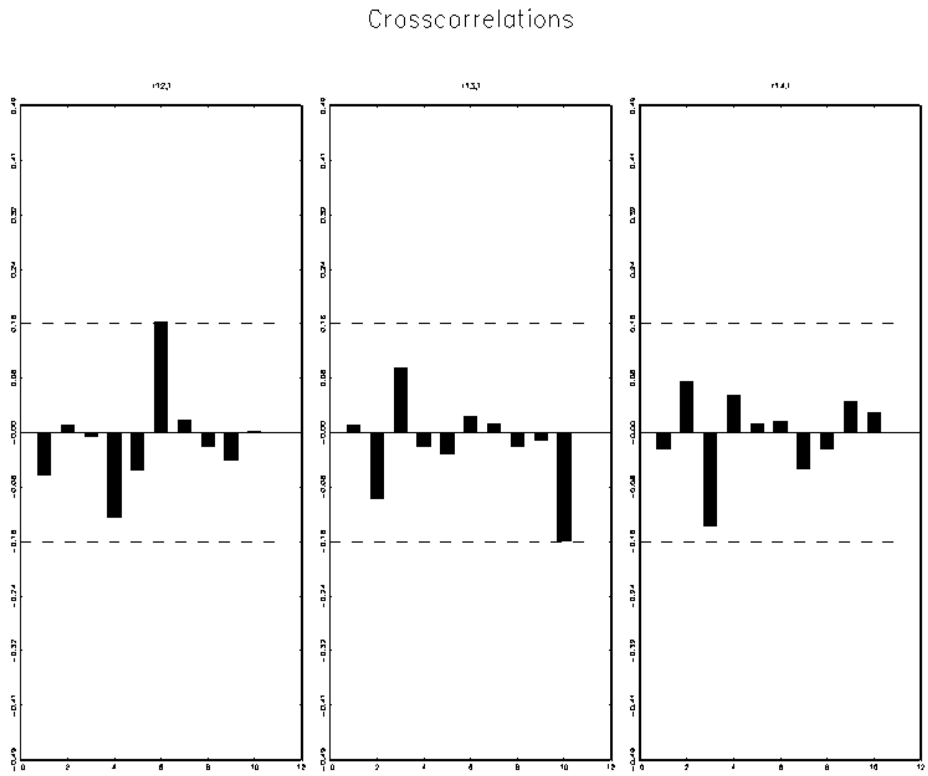
```
LM-TYPE TEST FOR AUTOCORRELATION with 4 lags

LM statistic:             42.4349
p-value:                 0.9828
df:                      64.0000

TESTS FOR NONNORMALITY

Reference: Doornik & Hansen (1994)
joint test statistic:     11168.4728
p-value:                 0.0000
degrees of freedom:       8.0000
skewness only:            721.2511
p-value:                 0.0000
kurtosis only:            10447.2216
p-value:                 0.0000

Reference: Lütkepohl (1993), Introduction to Multiple Time Series Analysis, 2ed, p. 153
joint test statistic:     6645.8887
p-value:                 0.0000
degrees of freedom:       8.0000
skewness only:            296.5880
p-value:                 0.0000
kurtosis only:            6349.3007
p-value:                 0.0000

*** Tue, 28 Jul 2015 16:46:11 ***
JARQUE-BERA TEST

variable        teststat   p-Value(Chi^2)  skewness   kurtosis
u1              29.3274    0.0000           0.3476     5.0365
u2              3820.0687  0.0000          -2.7762     26.9236
u3              2507.8390  0.0000          -0.2039     22.8949
u4              4729.6457  0.0000          -2.7046     29.7867
```

Source: Author, using JMulti 4.24

We also check for the presence of cross-residual correlation. This is especially important for the further investigation of the results by impulse response function analysis. For this purpose, we look at the covariance matrix of our restricted model in Table 4.6.4. Our critical value for deciding if some contemporaneous cross-residual correlation is significant, is determined by the 2/sqrt(T) ratio, which equals 0.162. Comparing the critical value with the residuals correlation matrix in Table 4.6.4 we notice that the residuals of "B_Croatia" are not significantly correlated with any of the other equation residuals, while the strongest cross-residual correlation exists between the residuals of the equation of "PFInvDirect" and "PFInvIndirect", amounting at -0.87. This negative cross correlation among the two investment variables occurs because of their generation process. Both variables are interpolated from their yearly frequency to a monthly frequency using a cubic formula, which induces greater similarity in the data generating process. This is why, even applying a structural VAR model estimation, with contemporaneous interactions among the variables, we still find similar negative correlation of the residuals of the two investment equations. Based on the fact of the presence of the residual cross-correlation, we should be careful when analysing the impulse response functions.

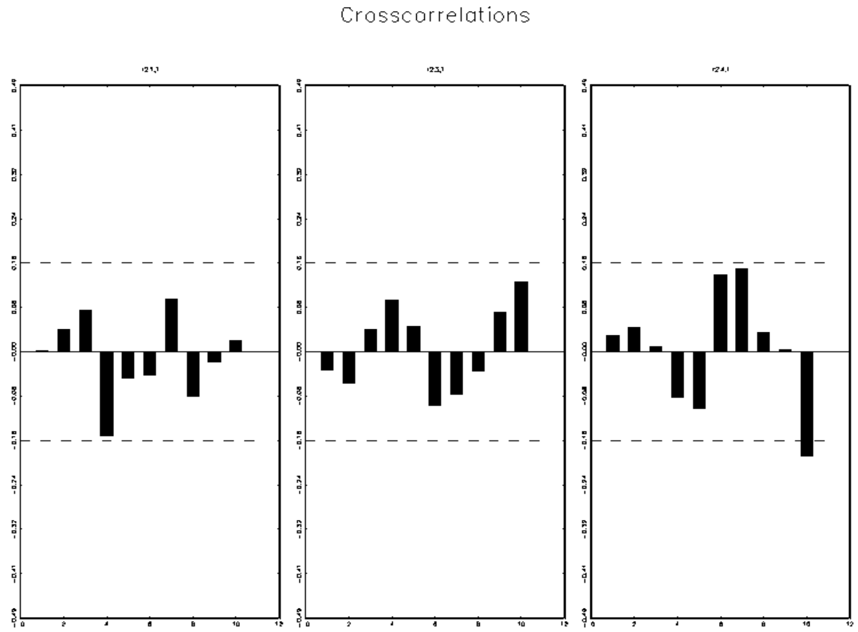
In order to investigate further the existence of lagged cross-correlation of residuals, we also present the plot of cross correlations for all four residuals with each other. We see in Figure 4.7.2a-b that the residuals of "B_Croatia" and "DYNEF" equations are not cross-correlated with the other equation residuals. This is especially important for our main equation of interest explaining "B_Croatia". We also could not spot significant residual cross correlations of lags between the residuals for the equations describing "PFInvDirect" and "PFInvIndirect" presented in Figure 4.7.2c-d.

Figure 4.7.2a Cross-residuals correlation of the "B_Croatia" residuals with the other equation residuals up to 10 lags



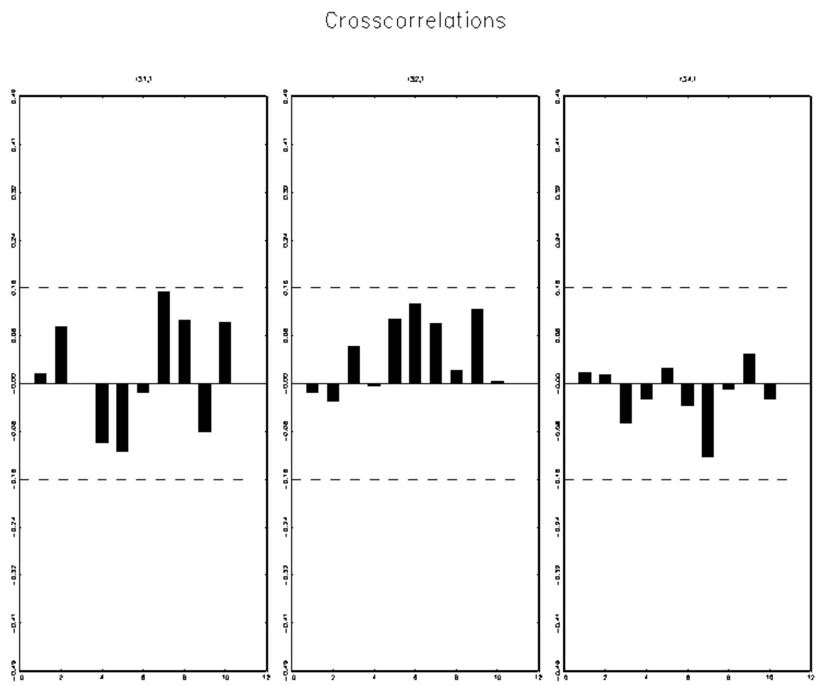
Source: Author, using JMulTi 4.24 cross-correlation graphs

Figure 4.7.2b Cross-residuals correlation of the "DYNEF" residuals with the other equation residuals up to 10 lags



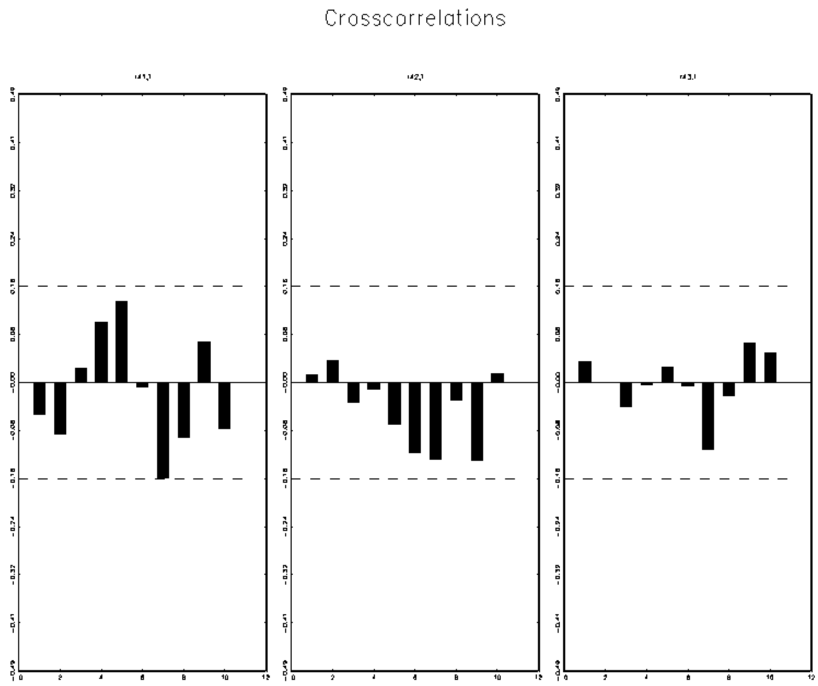
Source: Author, using JMulTi 4.24 cross-correlation graphs

Figure 4.7.2c Cross-residuals correlation of the "PFInvDirect" residuals with the other equation residuals up to 10 lags



Source: Author, using JMulTi 4.24 cross-correlation graphs

Figure 4.7.2d Cross-residuals correlation of the "PFInvIndirect" residuals with the other equation residuals up to 10 lags

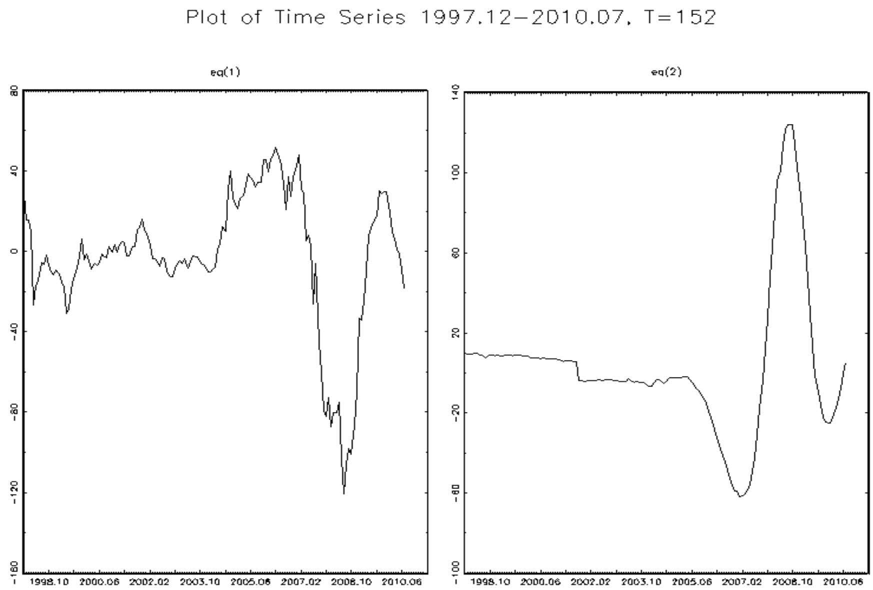


Source: Author, using JMulTi 4.24 cross-correlation graphs

Referring to our test results in Table 4.7.1, we confirm that all residual distributions are statistically different from the normal distribution. The visual inspection of the residuals from all equations in the system is presented in Figure 4.7.4.

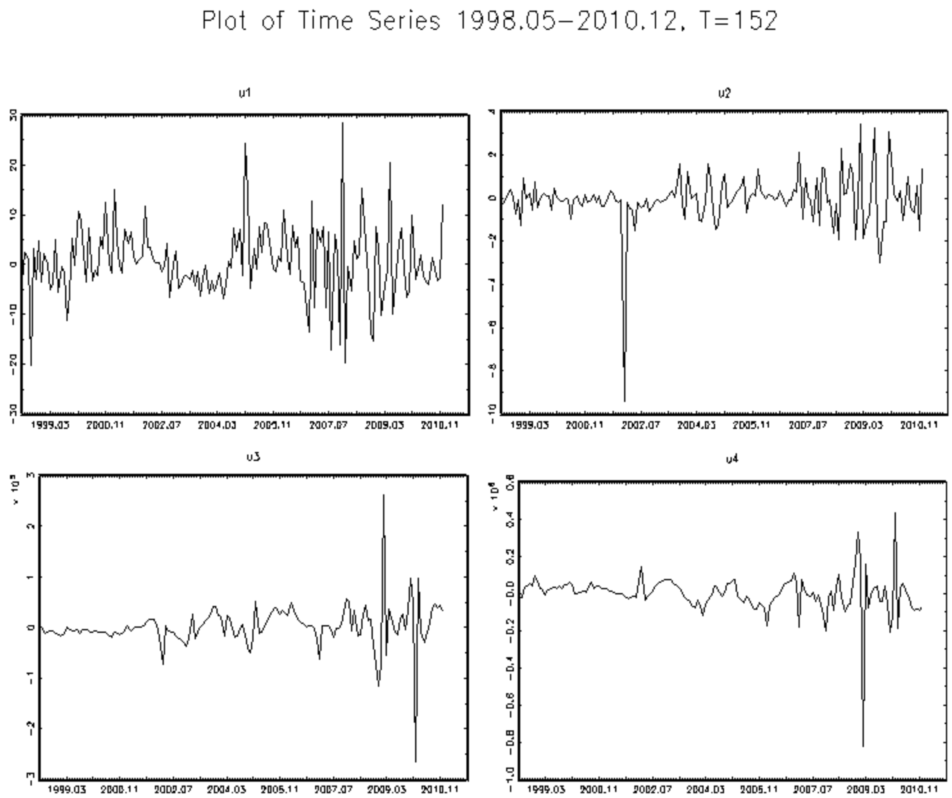
We also plot the Error Correction mechanisms from our two co-integration vectors including a constant and a structural break dummy variable. We note the apparently mean reverting, hence non-trended nature of each co-integrating vector (as required), which is consistent with stationarity (Figure 4.7.3).

Figure 4.7.3 Dynamics of the residual from the Error-Correction equation 1 and 2



Source: Author, using JMulti 4.24

Figure 4.7.4: Dynamics of the residuals in each equation in the VECM (4)



Source: Author, using JMulTi 4.24

4.8. Testing VECM structural stability

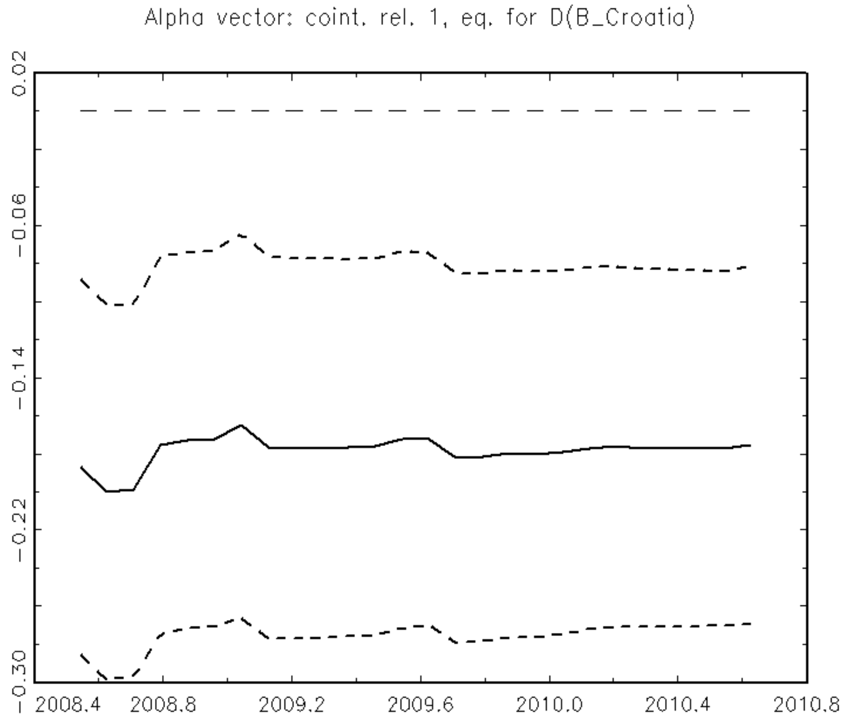
The next important step in our empirical testing is the stability analysis of the model, i.e. of the estimated coefficients. Here we check whether the estimated model coefficients are stable as we expand our data series or across different subsets of our full data series. Stability of the coefficients is a requirement to confirm that the model is time-consistent. There are several visual checks and formal tests for the stability of the coefficients (Lütkepohl & Krätzig, 2004). We first visually analyse the stability of the most important “loading” coefficients in our model using recursive coefficients estimation and then we evaluate the formal Chow test for overall coefficient stability in the model.

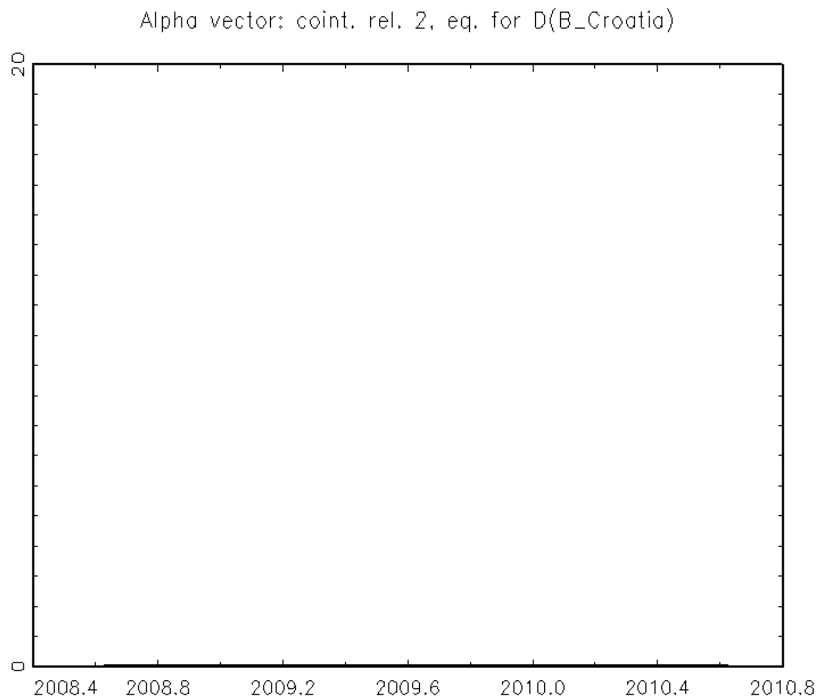
The recursive coefficients estimation technique, as described by Lütkepohl and Krätzig, estimates the regression coefficients starting with the minimum required data set beginning with the first observation (because of the degrees of freedom, estimation requires a minimum length of the data series). Then, the technique gradually expands the data set, finally reaching the complete data set of observations (Lütkepohl & Krätzig, 2004). JMulTi re-estimates the coefficients at each stage at which it expands the dataset, and presents the dynamics of the consecutively estimated coefficients with their confidence interval.

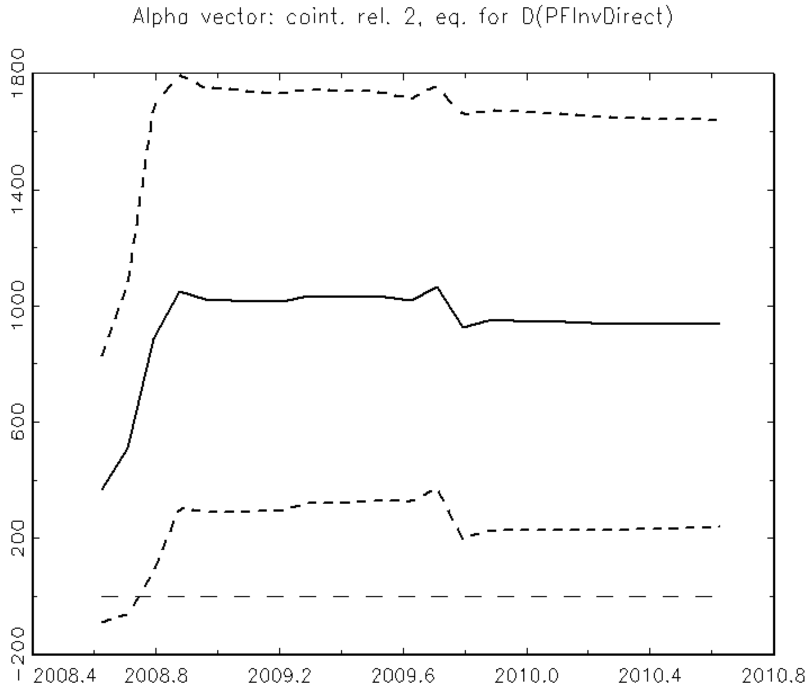
It is expected for the coefficients to demonstrate non-stability at their initial estimations, because of the small subsets being used for estimation, but they should soon stabilize as we expand the data-set and approach the complete data set. Any abnormal shift that occurs after the initial stabilisation of the estimated coefficients signifies that there might be some structural shift in the data generation process that should be accounted for in the empirical analysis. Since we have 60 coefficients in our subset model presented in Table 4.6.4 and just a little over 150 time periods we note that it could be normal to expect non-stable coefficients on our visual inspection for the first half of the dataset. We use visual investigation only on the statistically significant loading coefficients from the “α” matrix, which determines the effects of disequilibrium in either or both of the co-integration vectors. There are 5 such coefficients in our subset model. JMulti determines the co-integration vector coefficients from the full data set. After showing recursive loading coefficients, we continue by applying the Chow testing procedures, which test the null hypothesis of stability of the overall model coefficients against the alternative of non-stable coefficients.

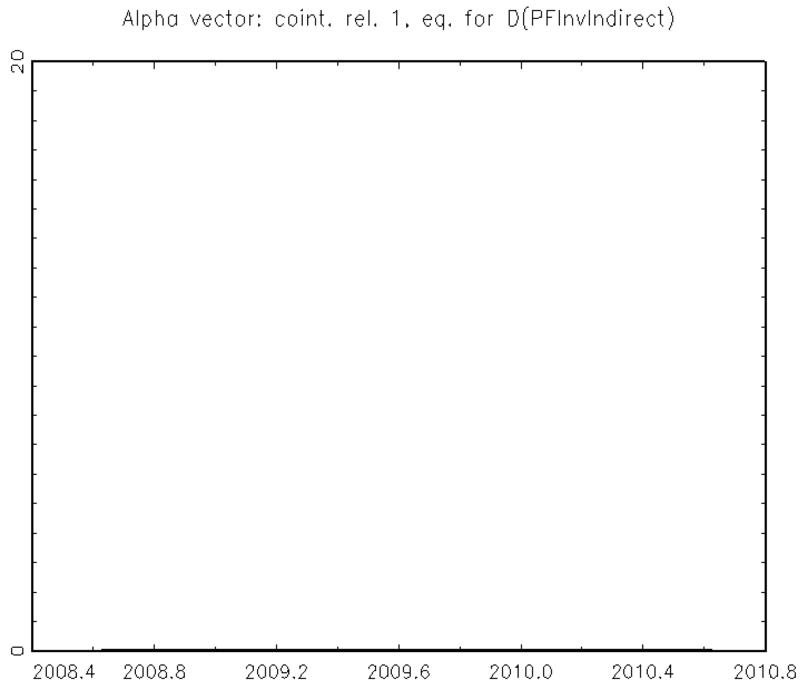
Looking at the recursive coefficient graphs of the alpha matrix loading coefficients, based on the short-run error correction mechanism of the two established co-integration relationships between the relative value of the Croatian equity bubble, the dynamic efficiency of the Croatian financial market and the two DC pension fund direct and indirect equity investment variables, we find that the error-correction loading coefficients, which affect the relative value of the Croatian equity bubble are relatively stable through time. The first graph in Figure 4.8.1 shows that after the initial period when the coefficient stabilized, in the later years the loading coefficient of the first error correction mechanism in "B_Croatia" was relatively stable taking values close to the average value of -0.175. This is very encouraging, especially because our main interest is the consistency of the long-term equilibrium equation with respect to the value of the Croatian equity bubble. We could not obtain a graphical presentation for the loading coefficient of the error correction mechanism of our second co-integration vector into "B_Croatia" which had a value of -0.361 estimated using the full data set. The third "loading" coefficient is for the second co-integration equation affecting the contemporaneous short-term dynamics of the direct investments of DC pension funds into Croatian equity "PFInvDirect". This coefficient, after its initial instability, showed stable dynamics around its average of 789.687. Finally, looking at the last two graphs in Figure 4.8.1, we couldn't obtain a graphical presentation of the impact of the first error correction mechanism on "PFInvIndirect", but the impact of the second error correction mechanism showed the highest instability.

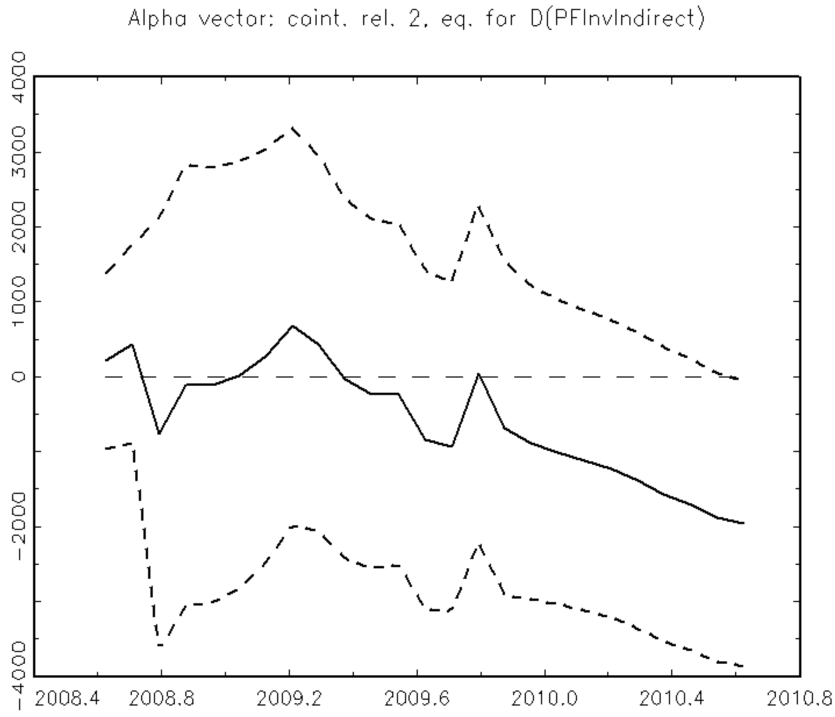
Figure 4.8.1 Recursive alpha vector estimation and coefficient stability






Alpha vector: coint. rel. 2, eq. for D(PFInvDirect)
1800
1400
1000
600
200
-200
2008.4
2008.8
2009.2
2009.6
2010.0
2010.4
2010.8


Alpha vector: coint. rel. 1, eq. for D(PFInvIndirect)
20
0
2008.4
2008.8
2009.2
2009.6
2010.0
2010.4
2010.8



Source: Author, using JMulTi 4.24

The instability of the loading coefficient for the second error correction mechanism on "PFInvIndirect" could be explained by the fact that it was not until 2006/2007 that the DC pension funds actually started investing in the SEE and Croatian Equity focused Open End Investment Funds. So it is normal to expect that the estimated loading coefficient would drift because the initial variation in this variable occurred in the period of 2007 and after. In other words, the DC pension fund indirect investment variable had values other than zero only after 2006/2007, which produces a shift in the value of its loading coefficient. This means that the stabilisation of the coefficient is expected to occur later for this investment variable.

In order to check for the stability of the coefficients of the whole subset VEC model, we also apply two variants of the Chow test. We present in Figure 4.8.2 the sample-split (SS) and the breaking-point (BP) Chow tests p-values for the null hypothesis of having time-consistent coefficient estimates in the VECM. The bootstrapped p-values of this null hypothesis are estimated for the period 2003-2008 with 1000 replications in JMulTi. The logic behind the Chow time consistency sample-split test is to compare the log of the determinants of the residual covariance matrix of the model with the same explicit structure estimated with the whole data set to the one estimated with the two consecutive period subsets from our complete data set (Lütkepohl & Krätzig, 2004). The formal definition of the test statistic is based on first finding the determinants of the covariance matrix based on the complete and partial data sets as presented in Equation 4.12 below, and then using them to calculate the two model time-consistency statistics, the Chow sample-split (SS) and the Chow breaking-point (BP) as presented in Equation 4.13

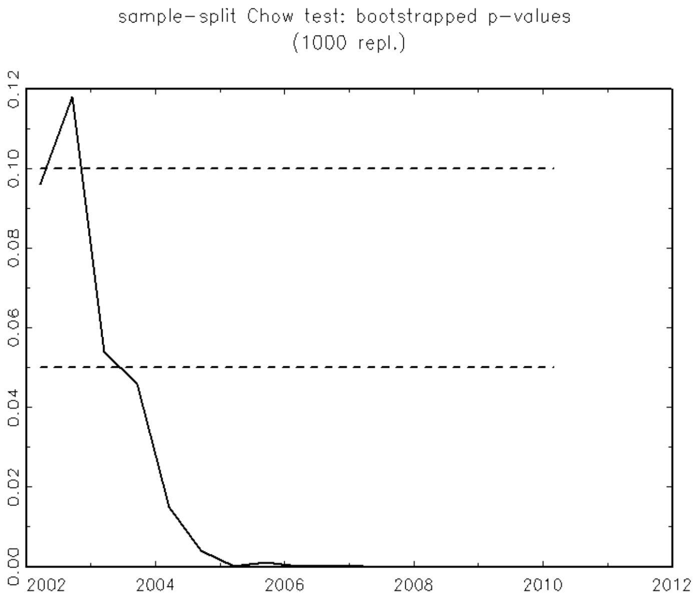
$$
\begin{aligned}
\tilde{\Sigma}_u &= T^{-1}\sum_{t=1}^{T}\hat{u}_t\hat{u}_t' \\
\tilde{\Sigma}_{1,2} &= (T_1+T_2)^{-1}\left(\sum_{t=1}^{T_1}\hat{u}_t\hat{u}_t' + \sum_{t=T-T_2+1}^{T}\hat{u}_t\hat{u}_t'\right) \\
\tilde{\Sigma}_{(1,2)} &= T_1^{-1}\sum_{t=1}^{T_1}\hat{u}_t\hat{u}_t' + T_2^{-1}\sum_{t=T-T_2+1}^{T}\hat{u}_t\hat{u}_t' \\
\tilde{\Sigma}_{(1)} &= T_1^{-1}\sum_{t=1}^{T_1}\hat{u}_t^{(1)}\hat{u}_t'^{(1)} \\
\tilde{\Sigma}_{(2)} &= T_2^{-1}\sum_{t=T-T_2+1}^{T}\hat{u}_t^{(2)}\hat{u}_t'^{(2)}
\end{aligned}
\tag{4.12}
$$

In Equation 4.12, we present the calculation of the different residual covariance matrices based on the same explicit model structure, but obtained from estimation applied on different data subsets. Here $\sum_u$,$\sum_{(1)}$ and $\sum_{(2)}$,$\sum_{1,2}$,$\sum_{(1,2)}$represent covariance matrices of the regression residuals obtained from the full data set, from the data set from T=0 to T_1 , from the data set from the point T_2+1 to the last observation T, and for the two combined data subsets sets together respectively.

$$
\begin{aligned}
\lambda_{SS} &= (T_1+T_2)\left[\log\det\tilde{\Sigma}_{1,2} - \log\det\left\{(T_1+T_2)^{-1}(T_1\tilde{\Sigma}_{(1)} + T_2\tilde{\Sigma}_{(2)})\right\}\right] \\
\lambda_{BP} &= (T_1+T_2)\log\det\tilde{\Sigma}_{(1,2)} - T_1\log\det\tilde{\Sigma}_{(1)} - T_2\log\det\tilde{\Sigma}_{(2)}
\end{aligned}
\tag{4.13}
$$

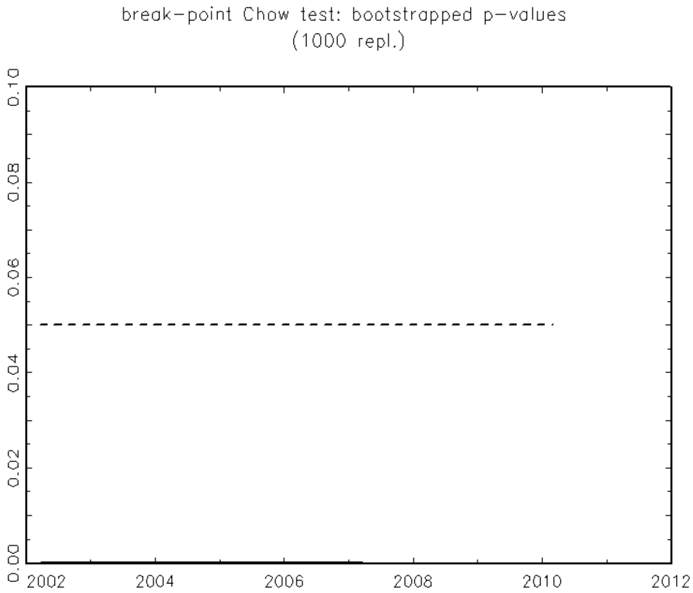
Using the determinants of the covariance matrices of the model estimated on different subsets of the complete data set, the two Chow test coefficients for the sample-split (λ_{SS}) and for the break-point (λ_{BP}) are calculated. Both test statistics presented in Equation 4.13 are asymptotically χ^2 distributed. We used JMulti, to obtain the values of the two test statistics for our subset model. The JMulti software expands the initial data set T_1 by an increment of one month at a time and displays p-values for both test statistics of the stability null hypothesis. We see the graphed p-values of the bootstrapped estimation of the sample-split and break-point Chow test statistic in Figure 4.8.2a-b. The graph gives the p-values of the tests on the null hypothesis of stability of the coefficients as the software expands the data set from the highest viable T_1 value to the lowest viable T_2 value, determined by the number of parameters in the model. For the periods when the p-value exceeds the critical level of the test, we cannot reject the null that the overall model coefficients are stable across the two data subsets. We notice that, based on the Chow SS test statistics, except for the period before 2002 when the pension funds were introduced, we could reject the coefficients stability hypothesis and accept the alternative that the coefficients in the model are not-stable. The same is confirmed with the Chow BP test statistic where the p value equals zero.

Figure 4.8.2a Sample-split Chow test statistic for the coefficient in our VECM (4)



Source: Author, using JMulti 4.24

Figure 4.8.2b Break-point Chow test statistic for the coefficient in our VECM (4)



Source: Author, using JMulTi 4.24

Before we explain the intuition behind our estimated model results, we note that in the next empirical Chapter 5 we will present another model where, instead of using interpolated lower to higher frequency data of DC Pension fund investments (“PFInvDirect” and “PFInvIndirect”) we use an alternative DC pension fund investment explanatory variable, the variable measuring the investment of the Croatian DC pension funds into the potentially bubbly instrument the Croatian open-end mutual funds focused on Croatian and SEE equity investments (“OMF_AUMIndex”). Finally we will compare the two models and try to gain more insights based on their results.

We continue by adding some intuitive explanation of the regression results. We explain the economic meaning of the long-term co-integration relationships as well as some of the short-term impact coefficients on the relative value of the Croatian equity bubble.

4.9. Economic meaning of the regression results

4.9.1. Long term equilibrium relationships

As we previously noted, we use the results from our theoretical model in Chapter 2 to determine the order of the endogenous variables for the estimation of our two co-integrating vectors. We set the two co-integrating long-term relationships normalized to the relative value of the Croatian equity bubble "B_Croatia" in the first co-integration vector, and to the dynamic efficiency of the Croatian financial market "DYNEF" in the second co-integration vector. This is because, based on the theoretical model, there are two logical long-term relationships governing the dynamics of the value of the bubbly asset: first, between the relative value of the Croatian equity bubble "B_Croatia" and the two investment variables of the DC pension funds ("PFInvDirect" and "PFInvIndirect"); and the second between the investment variables and the dynamic efficiency of the Croatian equity market "DYNEF". For the investments in the bubbly asset to become rational, the necessary condition is determined by the dynamic inefficiency of the financial market, and this relationship is captured in the second co-integrating vector. The sufficient condition for the bubble to rise in its value is determined by the actual investments of the DC pension funds in the bubbly asset, which is captured by the first co-integrating relationship. We analyse those two estimated long-term relationships separately in the following lines.

A) Error Correction Mechanism normalized on the relative value of the Croatian equity bubble "B_Croatia"

Looking at the first co-integration vector in our estimated subset model in Table 4.6.4, we find the most important equilibrium relationship between the variables of interest, the relative value of the Croatian equity bubble "B_Croatia" and the two DC pension funds investment variables, "PFInvDirect" representing the direct investments in Croatian equity and "PFInvIndirect" representing the indirect investments through the SEE and Croatian equity focused open-end mutual funds. Our first Long-term equilibrium relation based on the estimated VECM in Table 4.6.4 is presented in equation 4.14a-b (with t-statistics in parentheses).

$$Error1_m = B_{m-1}(Croatia) - \underset{(-3.603)}{0.0000008234} PFInvDirect_{m-1} - \underset{(-0.413)}{0.0000002228} PFInvIndirect_{m-1} + \underset{(0.642)}{20.774} l\_shift\_m9_{m-1} + \underset{(6.151)}{24.882}$$

$$B_{m-1}(Croatia) = \underset{(3.603)}{0.0000008234} PFInvDirect_{m-1} + \underset{(0.413)}{0.0000002228} PFInvIndirect_{m-1} - \underset{(-0.642)}{20.774} l\_shift\_m9_{m-1} - \underset{(-6.151)}{24.882} + Error1_m$$

(4.14a-b)

We notice that both direct investment in Croatian equity "PFInvDirect" and indirect investment of the DC pension funds through the Croatian and SEE equity focused open-end mutual funds "PFInvIndirect" have a positive long-term relationship with the relative value of the Croatian equity bubble (Equation 4.14b). We also find that this relationship is stronger and statistically more significant for the direct equity investments. However, we must note that the t-statistics in this context cannot be compared with the t-student distribution to determine significance, because in the presence of stochastic trends within the error correction vectors the critical values are nonstandard and unknown (Stock, 1987).

Based on the co-integration coefficients, a one million EUR additional yearly investment by the DC pension funds in Croatian equity corresponds to a 0.8 percentage point higher relative value of the Croatian equity bubble. This same amount additionally invested in the Croatian and SEE equity focused open-end mutual funds, corresponds to a 0.2 percentage point increase in the relative value of the Croatian equity bubble. This shows that the introduction of the DC pension funds with positive investments on the Croatian equity market positively affects the possible occurrence of the bubbly asset and its relative value. With an intercept of -24.8 percentage points, for the relative value of the Croatian equity bubble to be non-negative, DC pension funds would be required – ceteris paribus - to invest over 50 million euro's yearly (25*0.8+25*0.2), when the amount is equally split between the direct and indirect equity investments. For the relative value of the bubble to exceed 100 percentage points as it was the case during the Croatian bubbly episode, this investment amount should be five times higher or over 250 million euro's a year.

Finally, looking at the first long-term equilibrium relationship, we notice that the structural shift caused by the Global Financial Crisis (GFC) had a strong effect on the long-term balance between the relative value of the Croatian equity bubble and the Croatian DC pension fund investments. The occurrence of the GFC, creating a positive shift in the error correction value by 20.8 percentage points, required either a strong correction in the value of the Croatian equity bubble, or a strong increase in the DC pension fund direct and indirect investments into the Croatian equity required to sustain the relative value of the asset bubble to its value just before the GFC. The Global Financial Crisis, ceteris paribus moved the equilibrating value of the Croatian equity bubble by 20.8 percentage points lower compared to its previous value.

To see how the system adjusts to the new equilibrium when the first error correction mechanism is disturbed as demonstrated by the occurrence of the GFC, we look at the statistically significant loading coefficients, which explain the first order adjustment mechanisms. Of course, the second, third and fourth order adjustments are present because of the higher order lagged influence of the change in the endogenous variables in the system and their effects will be completely described by the impulse response analysis.

If we impose a positive disturbance in the first error correction mechanism, as was the case with the occurrence of the GFC, then in order for the system to return to its long-term equilibrium, it needs to either negatively affect the relative value of the Croatian equity bubble “B_Croatia” or positively affect the two investment variables through the loading coefficients. We find that the first error correction mechanism negatively feeds into the system through the correction of the relative value of the Croatian equity bubble “B_Croatia” and, at the same time, positively feeds into the system through the investments of the DC pension funds in the Croatian and SEE equity focused open-end mutual funds “PFInvIndirect”. So, although with different signs, the two loading coefficients are both stabilising the disturbance of the first error correction mechanism. One interesting insight at this point is that the direct equity investments are not used to bring the bubbly asset to its long-term equilibrating relationship with the investment variables, but only the indirect investments through the Croatian and SEE open-end mutual funds. This is our first sign that the Croatian and SEE equity focused open-end mutual funds “PFInvIndirect” are our candidate bubbly asset. Obviously, when equity becomes expensively valuated in speculative valuation territory, this is identified by the DC pension funds only through the direct equity investments.

To find the intensity by which the system corrects itself, we analyse the absolute and relative impact of the two loading coefficients. The first loading coefficient, impacting the relative value of the Croatian equity bubble "B_Croatia", has the value of -0.175 (t statistic of -3.73) and is the most intensive one. This means that ceteris paribus, a unit error disturbance from the first co-integration relationship, measured in percentage points of the relative value of the Croatian equity bubble, is corrected through its effect on the relative value of the Croatian equity bubble by more than 85% during a period of 10 months $(1-((1-0.175)^{\wedge 10})=0.854)$ or over 90% in a period of one year. On the other side, the second statistically significant loading coefficient is the one affecting the indirect DC pension fund investments through the Croatian and SEE equity focused open end mutual funds "PFInvIndirect". Although with a positive value of 947.48 (t statistic 2.71), or less than one thousand euro's per unit of the first error correction term, it also has stabilizing role on the dynamics of the Croatian equity bubble. This is because "PFInvIndirect" negatively affects the error generated within the first co-integrating relationship (equation 4.14a). In other words, although its loading coefficient is positive, multiplied by its coefficient within the first error correction mechanism, which is negative, changes in "PFInvIndirect" create a decreasing next period effect on the value of the error correction term. Finally, comparing the two loading coefficients, we note that the adjustment through the relative value of the bubble itself dominates the adjustment mechanism.

We conclude that, to be sustained, the value of the bubble must be maintained by the investment of the DC pension funds either directly in Croatian equity and/or indirectly through SEE and Croatian equity focused open-end mutual funds. But if there is a positive shock on the first co-integrating relationship, we either need some irrationally high increase in DC pension fund investment or the system will tend to stabilize to the new equilibrium by decreasing the relative value of the bubble through the collapse of its price which, based on the loading coefficients, is the more likely outcome.

B) Error Correction Mechanism normalized on the dynamic efficiency of the Croatian equity market "DYNEF"

The second co-integration vector is the one that explains the platform, or the fertile environment for the occurrence of the bubbly investments. It actually explains how the DC pension fund investment variables in the productive and the bubbly assets co-exist with the dynamic efficiency of the Croatian financial market.

We previously explained that although both direct and indirect investments in Croatian equity represent the same ultimate allocation of financial assets in the Croatian equity, perceptions about the two investment vehicles are different, reflecting the extended agency problem and information asymmetries present in the case of indirect investments through the Open End Mutual Funds. Due to this, indirect investments through the Open End Mutual Funds might be more closely associated with the bubbly part of the value of the Croatian equity compared to the direct equity investments. Moreover, as both investment vehicles are available to the DC pension funds, their use of the two investment vehicles in a different manner at a same point of time could signal that, although allocating investments to the same underlying asset, those two vehicles are treated and viewed differently by the institutional investors: respectively, as direct investment in the intrinsic value of productive equity investment; and indirectly – via the Mutual Funds – as a way to invest in the bubbly component of the price of equity.

We also stress that although the DC pension funds, as previously explained in Figure 1.2.2 in Chapter 1, are present on the demand side for investment assets, absorbing part of the supply of the bubbly asset, they also can be on the supply side. In cases when they are required to sell a number of units of the bubbly asset in excess of the number of units they have to purchase, they actually play the role of a net supplier of the bubbly asset to the financial market from their own accumulated stock of units of the bubbly assets.

As was hypothesized by our theoretical model, the financial market environment could be stimulating or restricting the occurrence of the asset bubble. The long-term relationship between the DC pension fund investments and the dynamic efficiency of the Croatian financial market is presented by the second co-integrating equation (4.15a-b).

$$Error2_m = DYNEF_{m-1} + \underset{(+1.1926)}{0.00000023095} PFInvDirect_{m-1} - \underset{(-0.8804)}{0.00000040280} PFInvIndirect_{m-1} - \underset{(-0.264)}{7.234} l\_shift\_m9_{m-1} + \underset{(+0.062)}{0.213}$$

$$DYNEF_{m-1} = -\underset{(-1.1926)}{0.00000023095} PFInvDirect_{m-1} + \underset{(0.8804)}{0.00000040280} PFInvIndirect_{m-1} + \underset{(0.264)}{7.234} l\_shift\_m9_{m-1} - \underset{(-0.062)}{0.213} + Error2_m$$

(4.15a-b)

Looking at the second co-integrating long-term relationship, we see that the low and negative dynamic efficiency co-exists in the long-term equilibrium with a high level of DC pension funds direct equity investments. This suggests that direct equity investments represent the productive investments from our theoretical model in Chapter 2. As the productive investments increase, their diminishing rate of return negatively impacts the dynamic efficiency of the financial market, which is consistent with the theoretical model. In contrast, the indirect investments presented by "PFInvIndirect" provide an alternative investment vehicle crowding out part of the productive investments, thereby having an indirectly positive impact on their investment returns and solving the problem of dynamic inefficiency of the financial market.

This identification of the direct investments "PFInvDirect" with the productive asset in our theoretical model, compared with the indirect investments "PFInvIndirect" as the bubbly asset in our theoretical model, is consistent with the different agency problems and informational characteristics of the two asset classes. The agency problem and the imperfect information issues increase in the case of indirect investments using open-end mutual funds, which helps to explain their potentially more speculative character making them better candidates for being accepted as a bubbly asset. Conversely, DC pension funds are more able to understand the fundamental value of the Croatian companies when directly investing in their equity, more likely characterizing them as a productive investment vehicle.

The overinvestment into productive capital (direct equity investments), as shown in our theoretical model, is associated with the financial market becoming dynamically inefficient and this could be seen in our second co-integrating relationship. As the amount of direct equity investments increase, the financial market returns decrease, which indicates that the financial market becomes less and less dynamically efficient. Namely, from the equation 4.19b, we see that 1 million euro's of additional yearly direct equity investments coexist with reduced financial market efficiency by 0.2 measurement units. Such interaction of the productive investments and financial market dynamic efficiency is described in the theoretical model in Chapter 2. When the financial market becomes dynamically inefficient due to over-accumulation of productive investments, this creates a fertile platform for the bubbly asset to occur, thereby crowding out part of the productive investments. The rational consequence of a dynamically inefficient financial market, as shown in our theoretical model, is the occurrence of the rational need for the introduction of bubbly assets, which will bring the financial market to its dynamically optimal “golden steady state”. Such a bubbly asset plays its role of solving the problem of dynamic inefficiency of the financial market, by crowding out part of the overly accumulated productive investments (Chapter2 - Figure 2.4). Looking back at the second co-integrating vector, it is clear that the bubbly asset improving the dynamic efficiency of the Croatian financial market is the investment into the Croatian and SEE equity oriented open-end mutual funds “PFInvIndirect”.

Again it is important to see how the disturbance in the long-term relationship between the financial market dynamic efficiency and the two investment variables corrects itself. Looking at the loading coefficients in the second column of the “α” loading coefficient matrix in Table 4.6.4, we find that the disturbance does not correct itself through the next period impact on the dynamic efficiency “DYNEF”. The correction is made through the change in the two investment variables, and indirectly through the relative value of the equity asset bubble. This is the expected outcome, as dynamic inefficiency is not a consequence, but the cause of the occurrence of and investments in the bubbly asset. We notice that there are only indirect short-run effects - from the lagged changes in the investment variables - on the Dynamic efficiency of the Croatian equity market. We notice also that the loading coefficients on the investment variables in the case of the disturbances of the second co-integrating relationship have a destabilizing effect. In the case of a positive error in the second co-integrating relationship (i.e. increased dynamic efficiency), adjustment requires either decreasing direct investments “PFInvDirect” or increasing indirect investments “PFInvIndirect”. But the loading coefficients are moving the dynamics of the system in the opposite direction, increasing direct investments “PFInvDirect” and decreasing indirect investments “PFInvIndirect”, which has a destabilizing direct effect on the second error-correction term. When we look at the absolute values of adjustments initiated by a unit disturbance of the second error correction mechanism on the investment variables, we notice that the total adjustment of around 3 thousand euro’s (equal to the sum of the two loading coefficients of the second error correction mechanism on the two investment variables) per measurement unit of the dynamic efficiency is negligible[12]. Most significant adjustment is made through the relative value of the Croatian equity bubble (the effect on the prices of the bubbly asset). Namely, a positive unit error in the second error correction mechanism

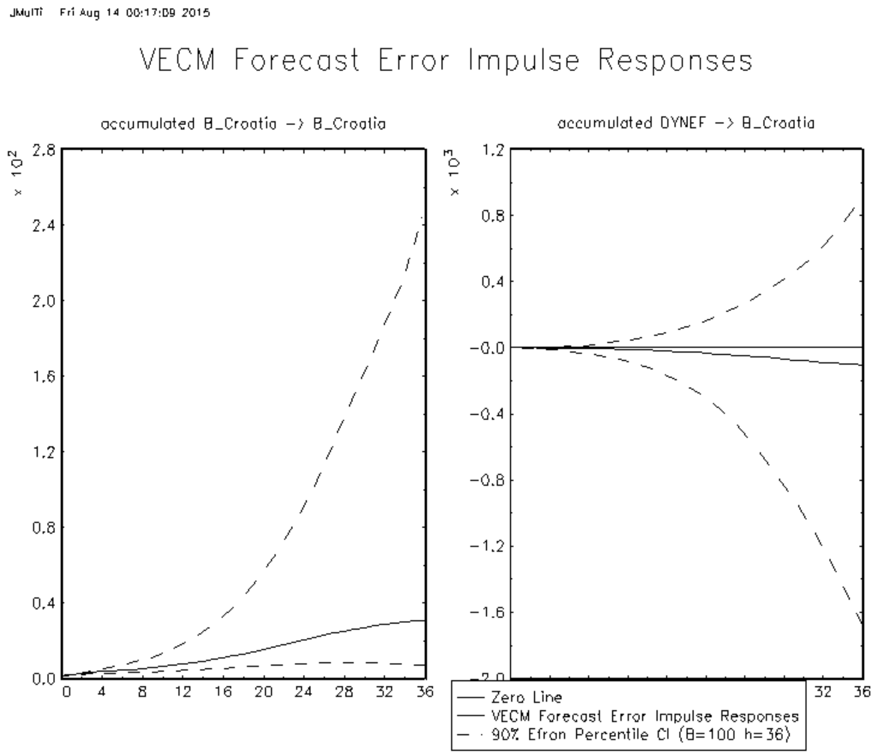
[12] Given that both coefficients are destabilizing, we add them together to obtain the combined effect.

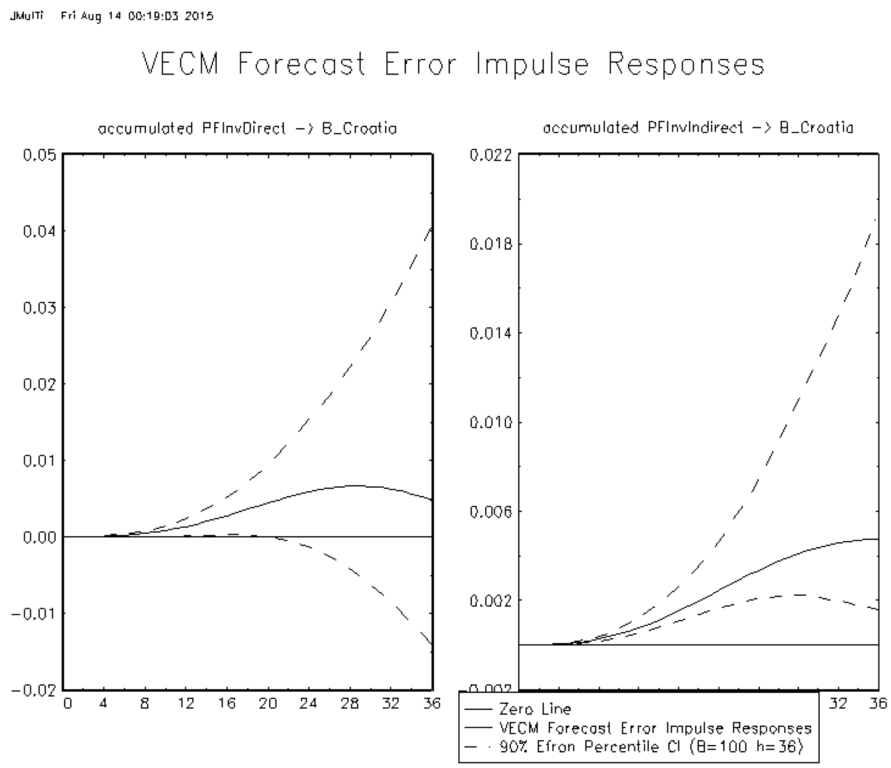
produces around a quarter percentage point decrease of the relative value of the Croatian equity bubble in the following period. Such a negative effect of the increase in the dynamic efficiency of the Croatian equity market on the relative value of the Croatian equity bubble is an expected outcome from our theoretical model. Because "B_Croatia" doesn't appear in the second co-integrating vector, the stabilisation is then effected though the dynamics of the whole system, which will be analysed though IR analysis.

We also have a structural shift in the second co-integrating vector occurring with the incidence of the Global Financial Crisis in September 2008 (*I_shift_m9* = 1). It produces a negative error correction term (-7.234) requiring a strong increase of the dynamic efficiency of the Croatian financial market in order to bring the second long-term relationship into equilibrium. To get a complete image on the interaction of our endogenous variables in the model, with the confidence intervals describing their significance, we will look at the Impulse Response Analysis.

To analyze the holistic picture about the interactions of our endogenous variables in the VECM (4) system, we complete our analysis by the use of Impulse Response Function analysis. For this purpose, we will introduce a unit impulse on each endogenous variable in the model, and analyse its accumulated effect on the value of itself, and on the rest of the endogenous variables. The effect is "accumulated", because it accumulates through time in the system based on the holistic interaction of all endogenous variables. We present the outcomes in Figure 4.9.1a-d, where four separate graphs for each endogenous variable in the model are shown. Each graph represents the accumulated response of the observed variable (y- axis) during the period of 36 months (x- axis), caused by an independent unit shock introduced to one of the four endogenous variables at the month 0. 90% confidence interval bands are constructed around the expected impact using the Efron bootstrapped CI method (Diciccio & Efron, 1996).

Figure 4.9.1a: Accumulated IR on "B_Croatia" induced by a unit shock introduced to (a) "B_Croatia", (b) "DYNEF", (c) "PFInvDirect" and (d) "PFInvIndirect" (T=36, conf. interval 90%)

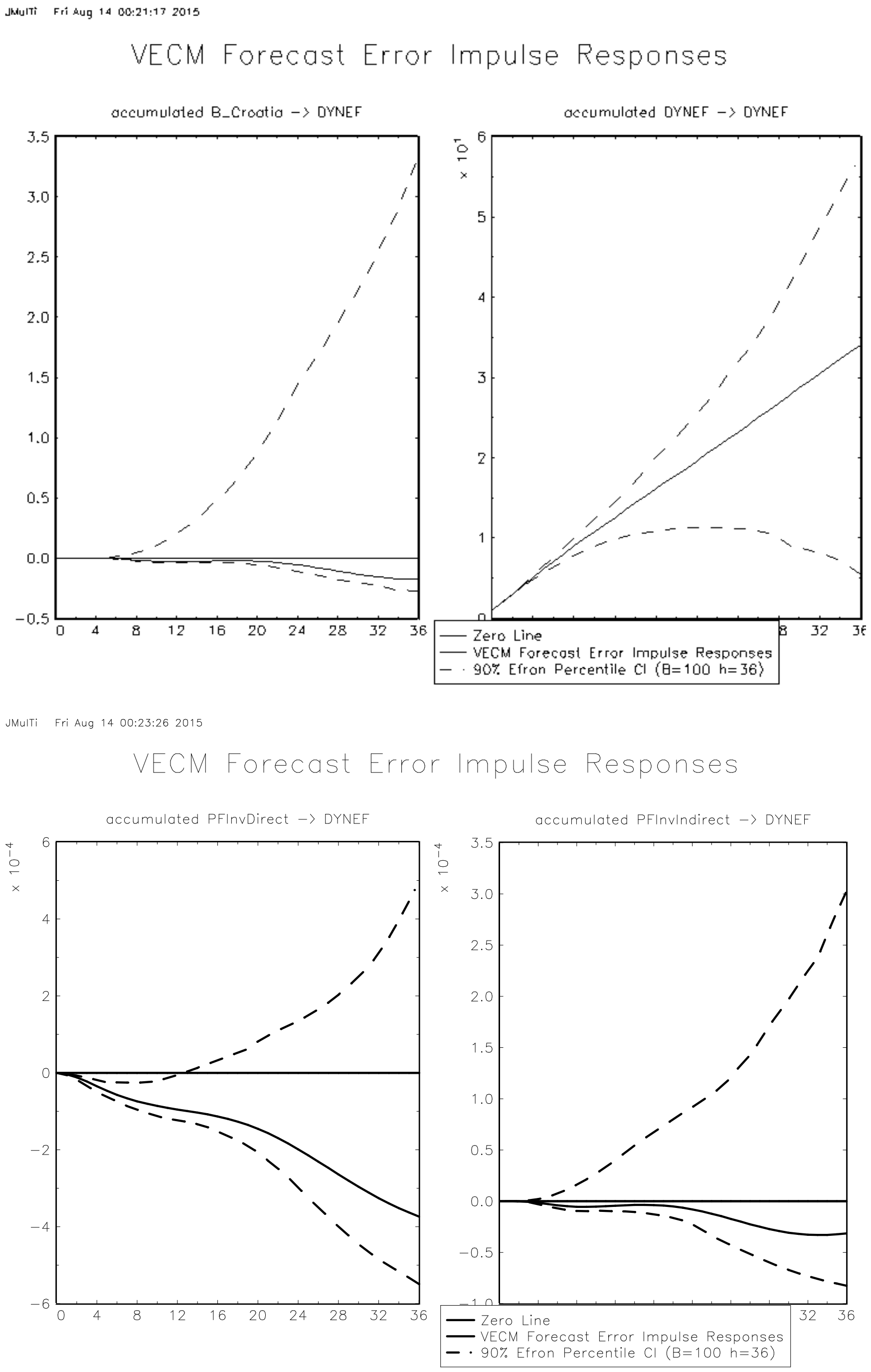




Source: Author, using JMulti 4.24

Looking at the IR analysis in Figure 4.9.1a, from the upper left corner panel [a], we find that the relative value of the Croatian equity bubble tends to persist in its value. A unit shock introduced on the relative value of the Croatian equity bubble “B_Croatia”, tends to create a higher cumulative effect on itself in the following 36 months. This captures a trending behaviour of the relative value of the Croatian equity bubble where a positive shock induces additional growth of the bubble itself. The mechanism also works in the opposite direction, when the crash of the bubble begins it intensifies itself in the following 36 months. In the upper right corner panel [b] of Figure 4.9.1a, we see that although a unit shock introduced on the value of the dynamic efficiency of the Croatian financial market “DYNEF” has a negative average effect on the relative value of the Croatian equity bubble, this effect is statistically insignificant. Finally, we analyse the impact of the introduction of the DC pension fund investment on the relative value of the Croatian equity bubble presented in the lower left and right corner panel [c] and panel [d] of Figure 4.9.1a. We find that both variables have a positive and significant impact on the relative value of the Croatian equity bubble. Especially in the case of a unit positive shock introduced to the indirect DC pension fund investments through the Croatian and SEE equity focused open-end mutual funds “PFInvIndirect”, the effect on the relative value of the Croatian equity bubble “B_Croatia” is positive and highly significant. The same positive unit shock introduced on the direct Croatian equity investments “PFInvDirect” becomes statistically insignificant 2 years after the shock is introduced (panel [c]). This once again contributes to the argument that the indirect DC pension fund investments through the open-end mutual funds investing in the Croatian and SEE equity are representing the bubbly asset. As we warned in the previous lines, IR analysis is affected by the negative correlation of the residuals of the two investment variables, this is why we are not going to comment on the absolute effects of a unit shock introduced on each investment variable, because such a unit shock might, at the same time, coexist with an opposite shock on the other investment variable. This means that a correct absolute effect analysis of a unit currency invested in “PFInvIndirect” should account for the over 0.8 units of currency disinvested from other “PFInvDirect” investment variable at the same time. What is important at this point, especially related to the hypothesis on the impact of the DC pension funds from the theoretical model, is that the total increase of the DC pension fund direct and indirect investments in Croatian equity market (“PFInvDirect” and “PFInvIndirect”), corresponding to the introduction of the DC pension funds on the Croatian equity market, have a positive and significant effect on the relative value of the Croatia equity bubble “B_Croatia”. This is shown in panels [c] and [d] in Figure 4.9.1a, and it confirms our main hypothesis defined in Chapter 2.

Figure 4.9.1b: Accumulated IR on “DYNEF” induced by a unit shock introduced to (a) “B_Croatia”, (b) “DYNEF”, (c) “PFInvDirect” and (d) “PFInvIndirect” (T=36, conf. interval 90%)



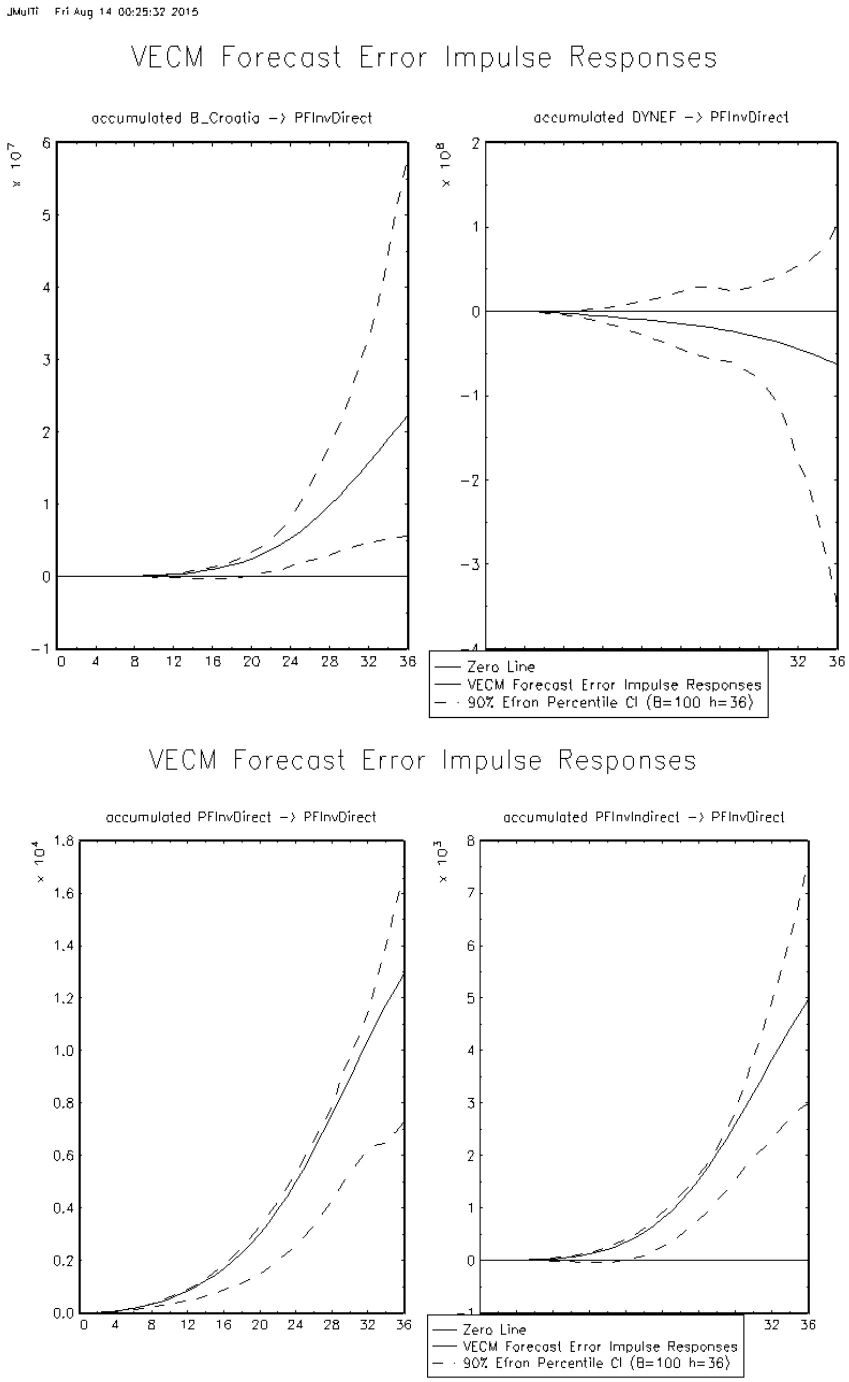
Source: Author, using JMulti 4.24

Figure 4.9.1b shows the accumulated impact to the value of the Croatian financial market dynamic efficiency “DYNEF” caused by a unit shock introduced to each endogenous variable in the system. In the upper left panel [a] of Figure 4.9.1b, we find that a unit shock introduced to the relative value of the Croatian equity bubble “B_Croatia” has no statistically significant effect on the dynamic efficiency of the Croatian equity market “DYNEF”.

On the other side, the dynamic efficiency “DYNEF” shows a high persistence in its value, which can be seen on the upper right panel [b] in Figure 4.9.1b. A unit shock introduced to the value of the Croatian financial market dynamic efficiency “DYNEF” persists and even accumulates over its initial unit shock in the following 36 months. This also works in the opposite direction when a negative shock is introduced to the value of the Croatian financial market dynamic efficiency.

We are also interested to analyze the impact of a positive unit shocks introduced to the DC pension fund investment variables on the dynamic efficiency of the Croatian financial market. Our theoretical model assumes that the over-accumulation of the productive investments lowers the dynamic efficiency of the financial market. This hypothesis is confirmed by the accumulated impact on the dynamic efficiency “DYNEF” caused by a unit shock introduced on the direct investments of the DC pension fund “PFInvDirect”. As shown in the panel [c], a positive unit shock introduced on the DC pension fund direct equity investment variable, has a negative and significant impact on the dynamic efficiency of the Croatian financial market. This implies that the DC pension funds direct equity investments “PFInvDirect” are less speculative and more corresponding to the productive investments from our theoretical model. On the other side, a unit positive shock introduced to the indirect DC pension fund equity investments through the Croatian and SEE equity focused open-end mutual funds, “PFInvIndirect”, has no statistically significant effect on the dynamic efficiency of the Croatian financial market “DYNEF” as shown in the lower right panel [d] in Figure 4.9.1b. This result once again confirms that the indirect equity investments are a better bubbly asset candidate. Although the direct equity investment of the DC pension funds “PFInvDirect” decreases Croatian financial market dynamic efficiency, the indirect equity investments “PFInvIndirect” and the relative value of the bubble itself “B_Croatia” have no statistically significant impact on the Croatian financial market dynamic efficiency. Our theoretical model implies that the bubbly asset is a rational consequence of the dynamic inefficiency of the financial market solving the systemic problem of dynamic efficiency but not affecting dynamic efficiency itself. This corresponds to the non-significant impact of a unit shock introduced on the bubbly indirect equity investment and on the relative value of the Croatian equity bubble on the Croatian financial market dynamic efficiency.

Figure 4.9.1c: Accumulated IR on "PFInvDirect" induced by a unit shock introduced to (a)"B_Croatia", (b)"DYNEF", (c) "PFInvDirect" and (d) "PFInvIndirect" (T=36, conf. interval 90%)



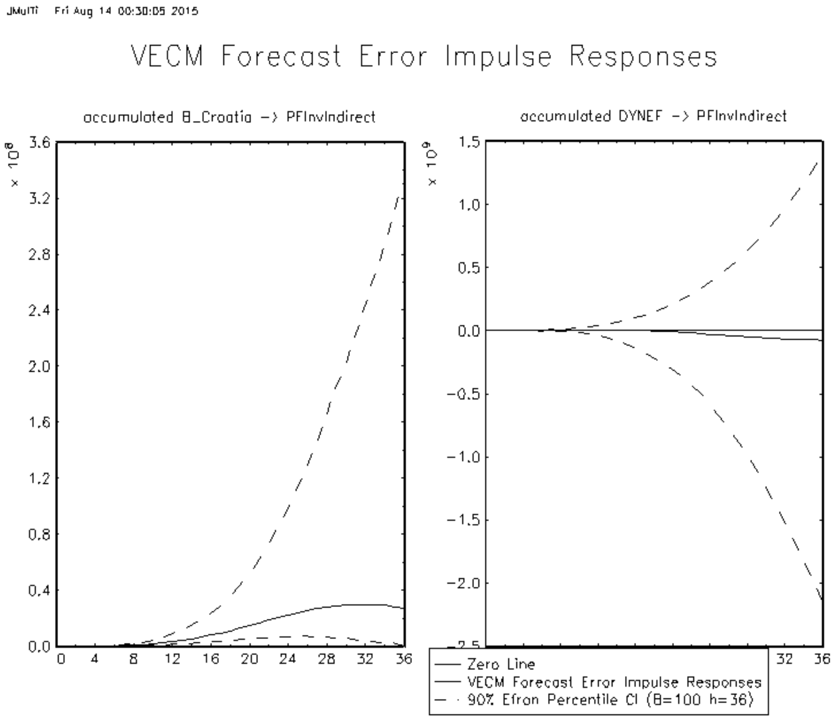
Source: Author, using JMulti 4.24

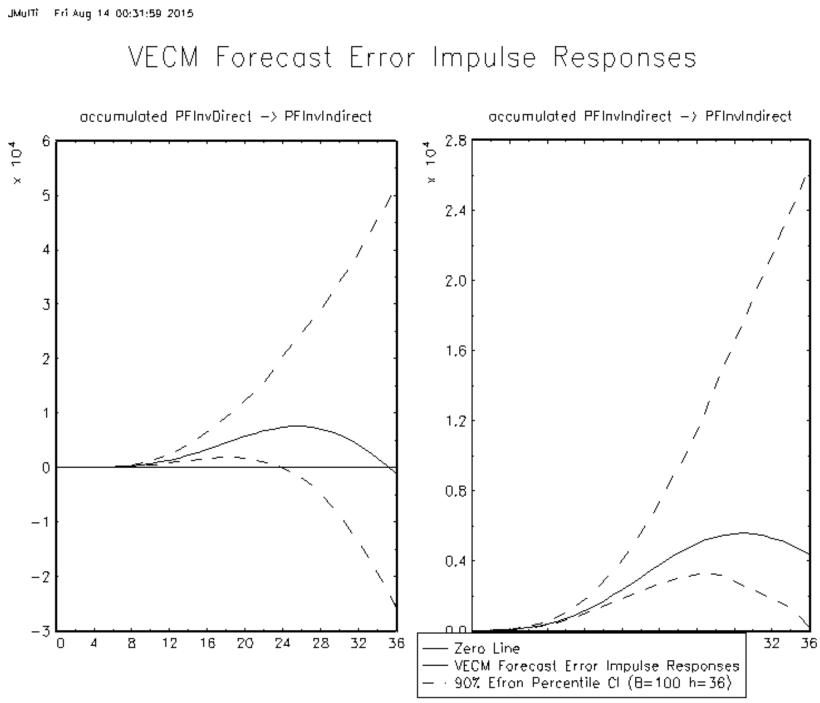
In Figure 4.9.1c above, and in Figure 4.9.1d below, we analyze the accumulated effect of a unit shock introduced to each individual endogenous variable in the model on, respectively, the DC pension fund investment variables “PFInvDirect” and “PFInvIndirect”. Because the accumulated effects are very similar for both direct and indirect DC pension fund investment variables, we will comment on them jointly. In the upper left panel [a] of both Figure 4.9.1c and Figure 4.9.1d, we find that a positive unit shock in the relative value of the Croatian equity bubble “B_Croatia” accumulates to a positive effect on the direct and indirect DC pension fund investments. The effect is felt sooner (in 6 months) in the case of the indirect investment (Figure 4.9.1d panel [a]) compared to the direct investment variable (Figure 4.9.1c [a]).

We also find in the upper right panel [b] in Figure 4.9.1c and Figure 4.9.1d, that although the positive unit shock introduced on the Croatian financial market dynamic efficiency “DYNEF” has a statistically insignificant accumulated effect on both direct and indirect DC pension fund investment variables, the average accumulated impact on both investment variables is negative. Based on the hypothesis drawn from our theoretical model, we should expect the dynamic efficiency to have an especially negative and significant effect on the bubbly asset in our case best represented by the indirect DC pension fund investments through the Croatian and SEE equity focused open-end mutual funds “PFInvIndirect”. This is not confirmed by the confidence intervals in panel [b] in both Figure 4.9.1c and in Figure 4.9.1d. The issue of strong negative cross-correlation of the residuals among our two estimated equations for the direct and indirect investment variables (Table 4.6.4), maybe affecting our IR analysis results. This problem will be addressed by our second empirical model in Chapter 5 where instead of using two, we will be using a single variable for the DC pension fund speculative investments.

Finally, looking at the lower panels [c] and [d] in Figure 4.9.1c and in Figure 4.9.1d, we find that a positive unit shocks introduced on each DC pension fund direct or indirect investment variable, tends to self-induce both higher direct and indirect DC pension fund investments in the next periods. This shows that DC pension fund investments are self-inducing in both positive and negative directions; i.e. not only during the rise of the stock market but also in a negative direction during the crash of the financial markets and during the crash of the speculative bubble.

Figure 4.9.1d: Accumulated IR on "PFInvIndirect" induced by a unit shock introduced to (a)"B_Croatia", (b)"DYNEF", (c) "PFInvDirect" and (d) "PFInvIndirect" (T=36, conf. interval 90%)





Source: Author, using JMulti 4.24

Finally, analyzing the impact of the exogenous and dummy variables in our estimated model in Table 4.6.4, we find that the US equity market, as the representative of the global financial markets, has a one month lagged positive short-term effect on the change in the relative value of the Croatian equity bubble. One percentage point increase in the relative value of the US equity market bubble causes a 0.629 percentage point rise in the relative value of the Croatian equity bubble after one month. This captures the interconnection of the Croatian with the global financial markets and suggests that the significant correction of the US equity markets during the GFC caused an additional negative impact on the Croatian equity bubble, decreasing the valuations on the Croatian equity market.

The introduction of regulation increasing the transparency of the DC pension fund investments, denoted by "Dummy4=1", had a shocking impact on the relative value of the bubble (+16.7 pp) and on the dynamic efficiency of the Croatian equity market (+4.2). This event causes disturbing dynamics on the Croatian equity market, increasing the value of the bubble, but also increasing the dynamic efficiency of the financial market, which decreases the value of the bubble.

Both dummy variables representing the initiation of the DC pension funds "Dummy1=1" and the legislative acts widening the investment horizon of the DC pension funds on the Croatian equity market "Dummy3=1" have one-time negative and a one-time positive effects on the direct and on the indirect DC pension fund investments respectively.

4.10. Concluding remarks

Estimating our theoretical model using data that describes the actual direct and indirect DC pension funds equity investments, and the relative value of the equity bubble in Croatian equity market, we find supportive results confirming the factors driving the bubbly equity market dynamics hypothesised in our theoretical model in Chapter 2.

We found that the VECM (4) with the two investment variables describing the introduction of the DC pension fund on the Croatian financial market should be derived from a VAR (5) levels model, and that a structural shift in the long-term equilibrium caused by the Global Financial Crisis has a meaningful role in the VECM with two co-integrating relationships. We set the suggested two long-term relationships (co-integrating vectors) based on theoretical reasoning and their estimation showed us the following two long-term equilibrating relationships:

- the first, signifying the positive long-run relationship among the relative value of the equity bubble and both DC pension fund direct equity investments and indirect equity investments through the Croatian and SEE equity focused open-end mutual funds; and
- the second, signifying the long term relationship between the dynamic efficiency of the Croatian equity market and the direct and indirect equity investments of the Croatian DC pension funds.

From the second relationship we found that direct equity investments have a negative relation to the Croatian financial market dynamic efficiency, which characterizes them as a productive investment vehicle. The negative impact of the DC pension fund direct equity investments on the dynamic efficiency of the Croatian financial market was also confirmed using IR analysis. The indirect investments have an opposite – positive (corrective) impact on dynamic efficiency – characterizing them as a bubbly asset. The loading coefficients confirm that the shocks in the first and in the second long-term relationship mostly adjust through the relative value of the Croatian equity bubble. This means that a positive shock in the first error correction mechanism produced by lower DC pension fund direct and indirect investments, or a positive shock in the second error correction term caused by higher dynamic efficiency, will adjust mainly through the decrease in the relative value of the Croatian equity bubble. From our results we also find that the introduction of the DC pension funds, measured by their direct and indirect investments, produced a deterioration of the dynamic efficiency of the Croatian financial market and created a fertile ground for the occurrence of the bubbly assets. The indirect investments, through the Croatian and SEE equity focused open-end mutual funds, played the role of the bubbly asset increasing the relative value of the Croatian equity bubble and solving the problem of dynamic inefficiency of the Croatian financial market.

The Global Financial Crisis played a critical role for the collapse of the Croatian equity bubble. It produced an immediate negative effect on the relative value of the Croatian equity bubble by making its value overpriced compared to the amount of DC pension fund investments and by making the dynamic efficiency of the Croatian equity market lower than its long-term equilibrating value. This interaction together with the direct external market impact produced a swift collapse of the equity asset bubble. Having found that the indirect DC pension fund investments to the Croatian equity, through the Croatian and SEE equity focused open-end mutual funds, are the best candidate representative of the bubbly asset used by the DC pension funds, in the following chapter we will develop a simpler model that substitutes our lower to higher frequency interpolated direct and indirect investment variables with a single variable representing an index of assets under management of the open-end mutual funds used by the DC pension funds with a genuine higher frequency. The use of a single investment variable will produce a simpler model and will solve the issue of residual cross-correlation existent among the two interpolated investment variables used in this chapter.

5. Model 2: Explaining the Croatian equity bubble with the DC pension fund investments represented by a single investment variable "OMF_AUMIndex"

Introduction

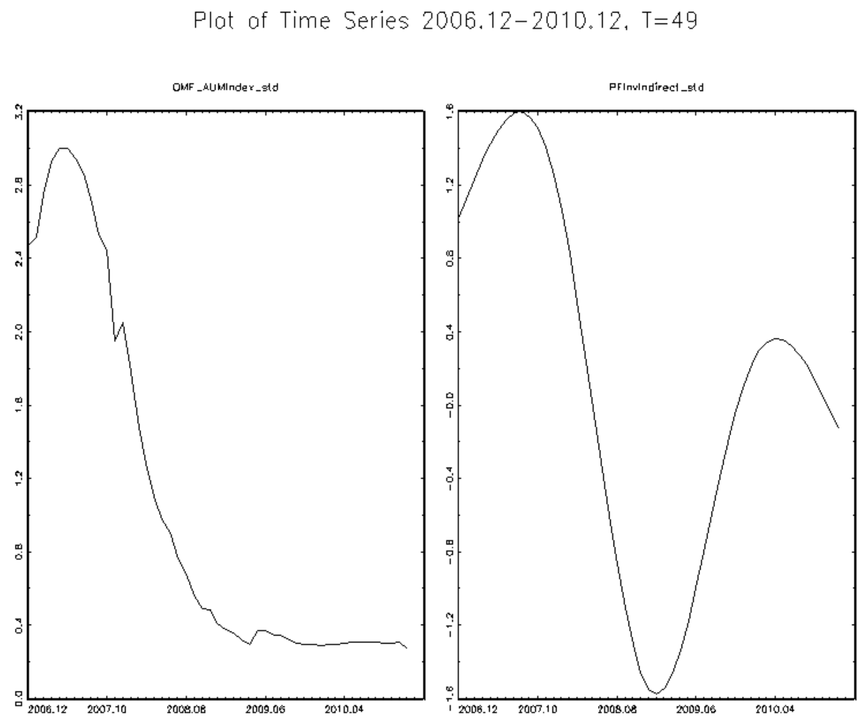
In this fifth Chapter we present an empirical model explaining the dynamics of the Croatian equity bubble using an endogenous explanatory variable, the index of the assets under management of the Croatian and SEE equity focused open-end mutual funds "OMF_AUMIndex", instead of the two lower to higher frequency interpolated investment variables "PFInvDirect" and "PFInvIndirect". We showed the detailed derivation of this I(1) variable in Chapter 3, and here we recall its main characteristics.

The variable "OMF_AUMIndex" is a genuine monthly index variable (Dec. 2004 = 100), which explains the relative dynamics of the value of Assets under Management (AUM) of the Croatian and SEE equity focused open-end mutual funds invested by DC pension funds. Its main idea is to present a good estimate of the genuine monthly investments of the Croatian DC pension funds, being the dominant investors in the identified bubbly asset, the Croatian and SEE equity focused open-end mutual funds. In order to present only net investment amounts, net of the equity price changes, the index is discounted by the monthly dynamics of the "Crobex", the Croatian general equity market price index. In order to represent the DC pension fund investments, the index is calculated as a weighted average of the monthly dynamics of the AUM of all open-end mutual funds investing in Croatian and SEE equity, weighted by their yearly weights in the DC pension fund portfolios. This way, this index closely represents the monthly index of investments of the DC pension funds into the bubbly indirect investment vehicle, the Croatian and SEE equity focused open-end mutual funds. By using this index variable instead of the two investment variables "PFInvDirect" and "PFInvIndirect", we gain an explanatory variable with a genuine higher frequency whose monthly variance is much richer with information. Because we are not using interpolation, we also exclude the residuals cross-correlation issue, which we had in our previous model for the residuals of the two interpolated investment variables.

But we also lose, because this variable also contains noise from the other investors who are also investing money in those open-end mutual funds together with the DC pension funds. In addition, we also have a much shorter data set starting at the end of 2006, almost at the peak of the asset bubble when the monthly assets under management of the open-end mutual funds started being reported by the regulatory agency on a monthly basis (previously only yearly data was reported).

Comparing the “OMF_AUMIndex” variable with the dynamics of the DC pension fund indirect investments in the SEE and Croatian equity focused open-end mutual funds represented by “PFInvIndirect”, we notice very similar dynamics. The only significant difference following after the collapse of the bubble in the mid of 2008, when the “PFInvIndirect” exhibits higher recovery than “OMF_AUMIndex”, could partly be due to the noise contained in the value of “OMF_AUMIndex” coming from the actions of other investors –“herd followers” – who were also investing/disinvesting in the Croatian and SEE equity focused open-end mutual funds. We present the dynamics of the two variables in Figure 5.0.1.

Figure 5.0.1 Plot of “OMF_AUMIndex”(LH panel) and “PFInvIndirect” (RH panel) standardized by their standard deviation



Source: Author, using JMulTi 4.24

5.1. Lag order of the underlying VAR model

In order to estimate the model using the "OMF_AUMIndex" variable instead of the "PFInvDirect" and "PFInvIndirect" variables, we proceed by first investigating the optimal order of the underlying VAR model. We do this by checking the suggested lag using information criteria, and then the diagnostics at different lags of the VAR model, where only "B_Croatia", "DYNEF" and "OMF_AUMIndex" are endogenous variables. The results from the investigation of the optimal order of the levels VAR model based on the information criteria are presented in Table 5.1.1.

Table 5.1.1 Suggested VAR lag order for a model without a constant, with a constant, and with both trend and a constant using information criterion

```
endogenous variables:     B_Croatia DYNEF OMF_AUMIndex
exogenous variables:      B_US_d1
exogenous lags (fixed):   2
deterministic variables:  dummy4_L1_d1 dummy3_L1_d1
sample range:             [2007 M6, 2010 M12], T = 43

optimal number of lags (searched up to 6 lags of levels):
Akaike Info Criterion:    1
Final Prediction Error:   1
Hannan-Quinn Criterion:   1
Schwarz Criterion:        1


endogenous variables:     B_Croatia DYNEF OMF_AUMIndex
exogenous variables:      B_US_d1
exogenous lags (fixed):   2
deterministic variables:  dummy4_L1_d1 dummy3_L1_d1 CONST
sample range:             [2007 M6, 2010 M12], T = 43

optimal number of lags (searched up to 6 lags of levels):
Akaike Info Criterion:    6
Final Prediction Error:   1
Hannan-Quinn Criterion:   1
Schwarz Criterion:        1


endogenous variables:     B_Croatia DYNEF OMF_AUMIndex
exogenous variables:      B_US_d1
exogenous lags (fixed):   2
deterministic variables:  dummy4_L1_d1 dummy3_L1_d1 CONST TREND
sample range:             [2007 M6, 2010 M12], T = 43

optimal number of lags (searched up to 6 lags of levels):
Akaike Info Criterion:    6
Final Prediction Error:   1
Hannan-Quinn Criterion:   1
Schwarz Criterion:        1

*info-criteria estimation made up to the 6th lag because of the short dataset T=43
```

Source: Author, using JMulti 4.24

Although we find that the lag order of 1 in the underlying levels VAR model is dominantly suggested by the information criterion, we continue with the lag order selection procedure by comparing the diagnostic tests and checks of the underlying levels VAR model at different lag orders. We first present Table 5.1.2 where the LMF residuals autocorrelation test p-values are presented for different VAR lag orders of the models without a constant, with only a constant, and with a constant and a trend. The LMF test p-values represent the probability of rejecting the null hypothesis of no residual autocorrelation and making a Type 1 error (rejecting the null when actually the null hypothesis is valid). We conclude that the VAR model with the lag order of one has the most favourable residual non-autocorrelation test statistics.

Table 5.1.2 LMF tests of residual auto-correlation for different lag order of the VAR model

	no constant or trend		constant, no trend		constant and trend	
	LMF(2)	LMF(3)	LMF(2)	LMF(3)	LMF(2)	LMF(3)
VAR(1)	**0.1723**	**0.2068**	**0.4738**	**0.5097**	**0.5154**	**0.4681**
VAR(2)	0.0801	0.0143	0.0609	0.0344	0.0383	0.0212
VAR(3)	0.0145	0.0038	0.0113	0.0059	0.0027	0.0005
VAR(4)	0.0643	0.0694	**0.1739**	0.0737	**0.1713**	0.0605
VAR(5)	**0.2285**	**0.1906**	0.0029	0.0057	0.0267	0.0307
VAR(6)	0.0015	0.0265	0.037	0.1423	**0.1579**	**0.2677**

Source: Author, using JMulTi 4.24

We continue with the residuals normality tests. In Table 5.1.3 we present the p-values of the normality tests (H0: Existence of residuals normality) of the model and of each equations' residuals. We find that the VAR models with 5 and 6 lags have more favourable residual normality results compared to the VAR model with only one lag.

Table 5.1.3: Test of the normality of residuals of the VAR model at different lag order

		no const. or trend	constant, no trend	constant and trend
VAR(1)	Jarque-Bera			
	U1	0.1513	0.035	0.0038
	U2	0.0116	0.0071	0.0274
	U3	0	0	0
	Joint test LJB	0	0	0
VAR(4)	Jarque-Bera			
	U1	0.315	0.6644	0.5128
	U2	0.0066	0.003	0.0029
	U3	0	0	0
	Joint test LJB	0	0	0
VAR(5)	Jarque-Bera			
	U1	0.2548	0.8946	0.9994
	U2	0.4823	0.4391	0.2097
	U3	0	0	0
	Joint test	0	0	0.0286
VAR(6)	Jarque-Bera			
	U1	0.2715	0.8618	0.6536
	U2	0.4459	0.2279	0.1551
	U3	0	0	0.005
	Joint test	0	0.0008	0.6106

Source: Author, using JMulti 4.24

Although the VAR (1) models show inferior residuals normality test results, because of them being preferred by the Information Criterion and, in particular, because of their superior LMF test results of no residual autocorrelation, and especially because the small data set favours the use of a smaller number of lags, we analyse co-integration based on the assumed VAR (1) model.

5.2. Estimating the rank of the co-integration matrix

We present in Table 5.2.1 the Johansen test results for the rank of the co-integration matrix using the VAR (1) levels model as our underlying model. We also inspect models with and without a structural break occurring with the Global Financial Crisis on September 2008, which we suspect might cause a significant structural break in the model.

Table 5.2.1 Johansen Trace Test for the rank of co-integration of our underlying VAR (1) model

a) Not allowing for a structural break: intercept included; trend and intercept included

```
Johansen Trace Test for:  B_Croatia DYNEF OMF_AUMIndex
unrestricted dummies:     dummy3_L1_d1 dummy4_L1_d1 Dummy_DYNEF_L1_d1
restricted dummies:
sample range:             [2007 M1, 2010 M12], T = 48
included lags (levels):   1
dimension of the process: 3
intercept included
response surface computed:
-----------------------------------------------
r0  LR       pval     90%      95%      99%
-----------------------------------------------
 0   53.24    0.0001   32.25    35.07    40.78
1   15.90    0.1823   17.98    20.16    24.69
 2   5.44     0.2468   7.60     9.14     12.53

Johansen Trace Test for:  B_Croatia DYNEF OMF_AUMIndex
unrestricted dummies:     dummy3_L1_d1 dummy4_L1_d1 Dummy_DYNEF_L1_d1
restricted dummies:
sample range:             [2007 M1, 2010 M12], T = 48
included lags (levels):   1
dimension of the process: 3
trend and intercept included
response surface computed:
-----------------------------------------------
r0  LR       pval     90%      95%      99%
-----------------------------------------------
 0   49.79    0.0077   39.73    42.77    48.87
 1   15.11    0.5726   23.32    25.73    30.67
 2   5.13     0.5860   10.68    12.45    16.22
```

b) Testing in the presence of a structural break: intercept included; trend and intercept included

```
Johansen Trace Test for:  B_Croatia DYNEF OMF_AUMIndex
unrestricted dummies:     dummy3_L1_d1 dummy4_L1_d1 Dummy_DYNEF_L1_d1 D[2008 M9]
restricted dummies:       S[2008 M9]
sample range:             [2007 M1, 2010 M12], T = 48
included lags (levels):   1
dimension of the process: 3
intercept included
response surface computed:
-----------------------------------------------
r0  LR       pval     90%      95%      99%
-----------------------------------------------
 0   57.52    0.0000   37.61    39.81    44.17
 1   20.75    0.1658   22.29    24.18    28.00
 2   6.35     0.4568   11.02    12.82    16.66
```

Johansen Trace Test for: B_Croatia DYNEF OMF_AUMIndex
unrestricted dummies: dummy3_L1_d1 dummy4_L1_d1 Dummy_DYNEF_L1_d1 D[2008 M9]
restricted dummies: S[2008 M9]
sample range: [2007 M1, 2010 M12], T = 48
included lags (levels): 1
dimension of the process: 3
trend and intercept included
response surface computed:

r0	LR	pval	90%	95%	99%
0	66.13	0.0002	43.19	46.40	52.84
1	30.49	0.0262	25.72	28.29	33.54
2	5.28	0.6802	12.02	13.89	17.88

Source: Author, using JMulti 4.24

Looking at the test results from Table 5.2.1, we find that the co-integration matrix of the underlying VAR (1) model has a rank of one in a model without a structural break. This means that without the structural break, there is only one independent co-integration vector of constants transforming our three endogenous variables into a non-stationary process.

We also note that by adding a trend in the co-integration matrix to the model with a structural break, we find evidence that the rank of the co-integration matrix equals two. Although the value of the bubble should not demonstrate a trending dynamics in the long term, when we are analysing a shorter data series covering only the periods of the rise or the collapse of the bubble, such as in our case, and not the full bubbly cycle, then we could expect a trend to play a descriptive role in the relationship among our endogenous variables. Otherwise, as in our previous model, there is no justification for the inclusion of a long-term trend as in our previous empirical model presented in Chapter 4. The dynamics of the system governed by the long-term relations among our endogenous variables brings the system to its long-term equilibrium and it stays at that point until there are disruptive changes such as the GFC that cause a new dynamic path, collapse the bubble and bring the market to its new dynamic equilibrium. Only in the case where we include a shorter data series, covering only a part of the bubbly cycle, do we find a theoretical justification for including a trend in the co-integrating relationships among our endogenous variables.

5.3. VECM (0) model estimation

We chose to proceed with the VECM model with a restricted constant and a trend in the co-integration matrix, a structural shift occurring with the Global Financial Crisis (GFC) and with two co-integration vectors. Our VAR(1) level model reduces to a VECM(0) model, with two error correction mechanisms including a constant and a trend in the co-integration matrix. As previously explained, we believe that the trend gained significance in the model, compared to the previous model with the interpolated investment variables, because of the shorter time series, which includes mostly the period of the crash of the bubble. In this period, the value of the Croatian equity bubble and investments into the bubbly asset exhibit a joint negative trend. Because of the shorter time series not including proportionally the full bubbly cycle, we support the inclusion of the trend in the co-integration matrix. We will also add one lag of the exogenous variable, the value of the US equity market bubble, because we found in our previous modelling that the external markets represented by the US market have their most significant impact on the Croatian financial market with a lag of one month. In Table 5.3.1, we present the unrestricted VECM (0) estimation results.

Table 5.3.1 Unrestricted VECM (0) with the "OMF_AUMIndex" variable representing the DC pension fund SEE and Croatian equity investments in the bubbly asset (t-statistics in brackets)

$$\begin{bmatrix} \Delta B\_Croatia_m \\ \Delta DYNEF_m \\ \Delta OMF\_AUMIndex_m \end{bmatrix} = \left[\begin{bmatrix} -0.435 & 0.225 \\ (-5.195) & (0.387) \\ 0.001 & -0.272 \\ (0.052) & (-2.984) \\ -1.748 & -11.848 \\ (-3.794) & (-3.707) \end{bmatrix} \left[\begin{bmatrix} 1 & / & -0.08549 \\ & & (-8.45421) \\ / & 1 & 0.00308 \\ & & (1.8556) \end{bmatrix} \begin{bmatrix} B\_Croatia_{m-1} \\ DYNEF_{m-1} \\ OMF\_AUMIndex_{m-1} \end{bmatrix} + \begin{bmatrix} 45.707 & 16.649 & -0.949 \\ (4.257) & (1.087) & (-2.257) \\ -8.877 & -2.095 & 0.286 \\ (-5.029) & (-0.832) & (4.138) \end{bmatrix} \begin{bmatrix} l\_shift\_m9_{m-1} \\ Const \\ Trend_{m-1} \end{bmatrix} \right] \right]$$

$$+ \begin{bmatrix} -0.567 \\ (-1.805) \\ -0.025 \\ (-0.511) \\ -0.432 \\ (-0.250) \end{bmatrix} \left[\Delta B\_US_m \right] + \begin{bmatrix} 1.199 \\ (3.387) \\ -0.024 \\ (-0.439) \\ 1.097 \\ (0.564) \end{bmatrix} \left[\Delta B\_US_{m-1} \right] + \begin{bmatrix} -0.679 & 9.070 & -5.438 \\ (-0.078) & (1.010) & (-0.881) \\ -0.325 & 2.996 & 1.579 \\ (-0.237) & (2.127) & (1.631) \\ -92.707 & -44.065 & -6.980 \\ (-1.928) & (-0.893) & (-0.206) \end{bmatrix} \begin{bmatrix} \Delta dummy3_{m-1} \\ \Delta dummy4_{m-1} \\ \Delta dummy\_DYNEF_{m-1} \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$Corr = \begin{bmatrix} 1 & -0.01636 & 0.3790 \\ & 1 & -0.0317 \\ & & 1 \end{bmatrix}, \det(Cov) = 2.345089E+05$$

*Estimated VECM (0) model with T=47 [2007 M2, 2010 M12]

Source: Author, using JMulTi

Having estimated the unrestricted model, we perform an analysis for selecting the optimally restricted model. For this purpose, we check the diagnostics of all suggested subset models by the subset selection procedures available in JMulti4.24 based on different information criteria. In Table 5.3.2, we present the diagnostic test p-values for the tests of residual no-autocorrelation and normality, for all suggested subset models, restricted using different selection criteria. Again, similar to the selection process in our first model presented in Chapter 4, we found our optimal subset model using the System SER method (HQ) which, in this case, gave us the same model restrictions as the System SER(AIC), SER/Testing procedure (AIC), SER/Testing procedure (HQ) and the Top down – AIC procedure.

Table 5.3.2 Subset model selection criteria and the model diagnostic characteristics

Model	Number of variables	Testing residuals autocorrelation		Testing joint residuals joint normality (p-value)	LR-test	Det(Σ(u))
		LM(2) p-value	LM(3) p-value			
Unrestricted	33	0.5021	0.3944	0	1	2.35E+05
System SER (AIC)	21	0.3143	0.35	0	0.9665	2.59E+05
System SER (HQ)	21	0.3143	0.35	0	0.9665	2.59E+05
SER/Testing procedure (AIC)	21	0.3143	0.35	0	0.9665	2.59E+05
SER/Testing procedure (HQ)	21	0.3143	0.35	0	0.9665	2.59E+05
Top down -AIC	21	0.3143	0.35	0	0.9665	2.59E+05
Top down -HQ	20	0.433	0.6558	0	0.8648	2.75E+05
System SER (SC)	19	0.6239	0.7845	0	0.621	3.00E+05
SER/Testing procedure (SC)	19	0.6239	0.7845	0	0.621	3.00E+05
Top down -SC	19	0.6239	0.7845	0	0.621	3.00E+05
System testing proc. (t>2.00)	19	0.6239	0.7845	0	0.621	3.00E+05

Source: Author, using JMulTi 4.24

Having chosen the restrictions determined by the System SER (HQ) selection criterion as our optimal selection criteria, we place the suggested restrictions on the unrestricted model and repeat the estimation. We present the estimated subset model in Table 5.3.3. This model is characterised by 21 regression coefficients (seven within the co-integration matrix) and based on the diagnostic tests has no residual auto correlation (LMF (2) test p-value of 0.4355). There are four statistically significant “loading” coefficients in the model, two for each error correction mechanism.

Table 5.3.3 Estimation results of the optimally restricted VECM with the "OMF_AUMIndex" variable representing DC pension fund bubbly asset investments

$$
\begin{bmatrix} \Delta B\_Croatia_{m} \\ \Delta DYNEF_{m} \\ \Delta OMF\_AUMIndex_{m} \end{bmatrix}
=
\left[
\begin{bmatrix} -0.418 & ---- \\ (-5.300) & ---- \\ ---- & -0.285 \\ ---- & (-3.363) \\ -1.884 & -11.525 \\ (-5.010) & (-4.023) \end{bmatrix}
\left[
\begin{bmatrix} 1 & / & -0.08549 \\ & & (-8.454) \\ / & 1 & 0.00308 \\ & & (1.855) \end{bmatrix}
\begin{bmatrix} B\_Croatia_{m-1} \\ DYNEF_{m-1} \\ OMF\_AUMIndex_{m-1} \end{bmatrix}
+
\begin{bmatrix} 45.707 & 16.649 & -0.949 \\ (4.257) & (1.087) & (-2.257) \\ -8.877 & -2.095 & 0.286 \\ (-5.029) & (-0.832) & (4.138) \end{bmatrix}
\begin{bmatrix} l\_shift\_m9_{m-1} \\ Const \\ Trend_{m-1} \end{bmatrix}
\right]
\right]
$$

$$
+
\begin{bmatrix} -0.647 \\ (-2.256) \\ ---- \\ ---- \\ ---- \\ ---- \end{bmatrix}
\left[\Delta B\_US_{m} \right]
+
\begin{bmatrix} 1.296 \\ (4.174) \\ ---- \\ ---- \\ ---- \\ ---- \end{bmatrix}
\left[\Delta B\_US_{m-1} \right]
+
\begin{bmatrix} ---- & ---- & ---- \\ ---- & ---- & ---- \\ ---- & 3.167 & 1.669 \\ ---- & (2.361) & (1.755) \\ -94.191 & ---- & ---- \\ (-2.082) & ---- & ---- \end{bmatrix}
\begin{bmatrix} \Delta dummy3_{m-1} \\ \Delta dummy4_{m-1} \\ \Delta dummy\_DYNEF_{m-1} \end{bmatrix}
+
\begin{bmatrix} u_{1} \\ u_{2} \\ u_{3} \end{bmatrix}
$$

$$
Corr = \begin{bmatrix} 1 & -0.01766 & 0.3552 \\ & 1 & -0.03353 \\ & & 1 \end{bmatrix}, \det(Cov) = 2.587792E+05
$$

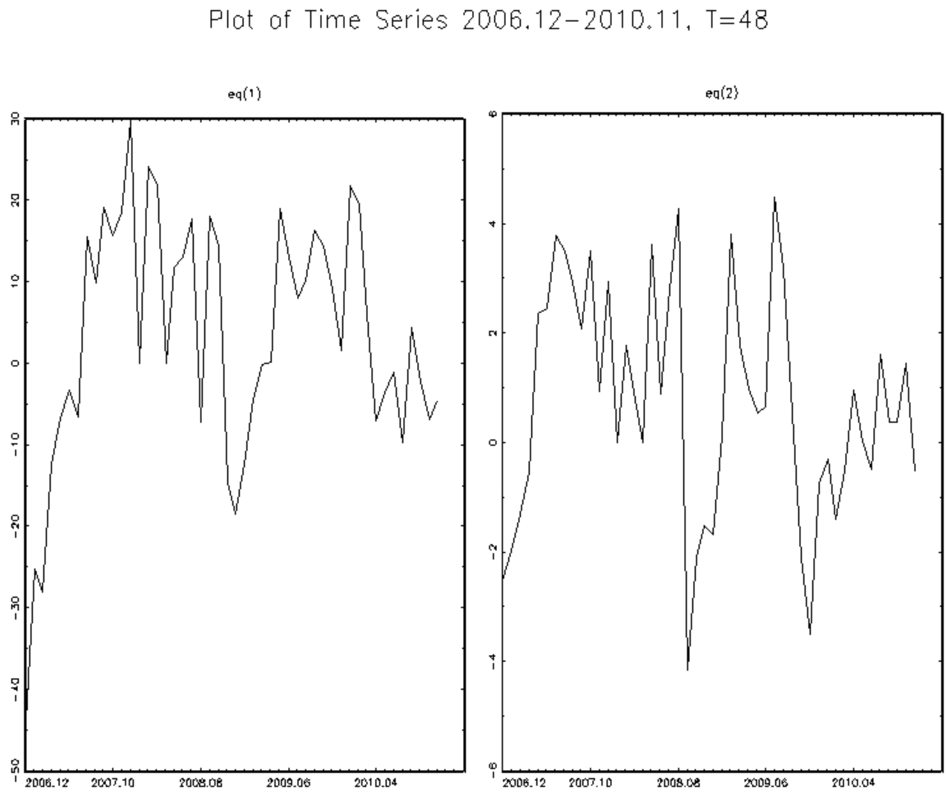
*Estimated VECM (0) model with T=47 [2007 M2, 2010 M12]

Source: Author, using JMulTi 4.24

5.4. VECM (0) model diagnostics

Before we make any statistical or economic inferences from our estimated model, we need to make sure that the model is valid as a statistical generating mechanism. For this purpose, we perform diagnostic tests and checks on the estimated model. We first visually check the dynamics of the two Error Correction (EC) variables, finding that the two variables resemble the dynamics of a stationary process. We present the EC dynamics of the restricted model in Figure 5.4.1

Figure 5.4.1 Error Correction dynamics of the restricted model



Source: Author, using JMulti 4.24

We further present the diagnostic tests for the null hypothesis of no-autocorrelation of the residuals and residual normality in Table 5.4.1. We conclude that the estimated restricted model does not generate auto-correlated residuals, which suggests that the estimated coefficients are non-biased and consistent. Although the model suffers from non-normality of the regression residuals, this is considered as an issue of second-order importance not affecting the consistency of the regression coefficients (Wooldridge, 2000).

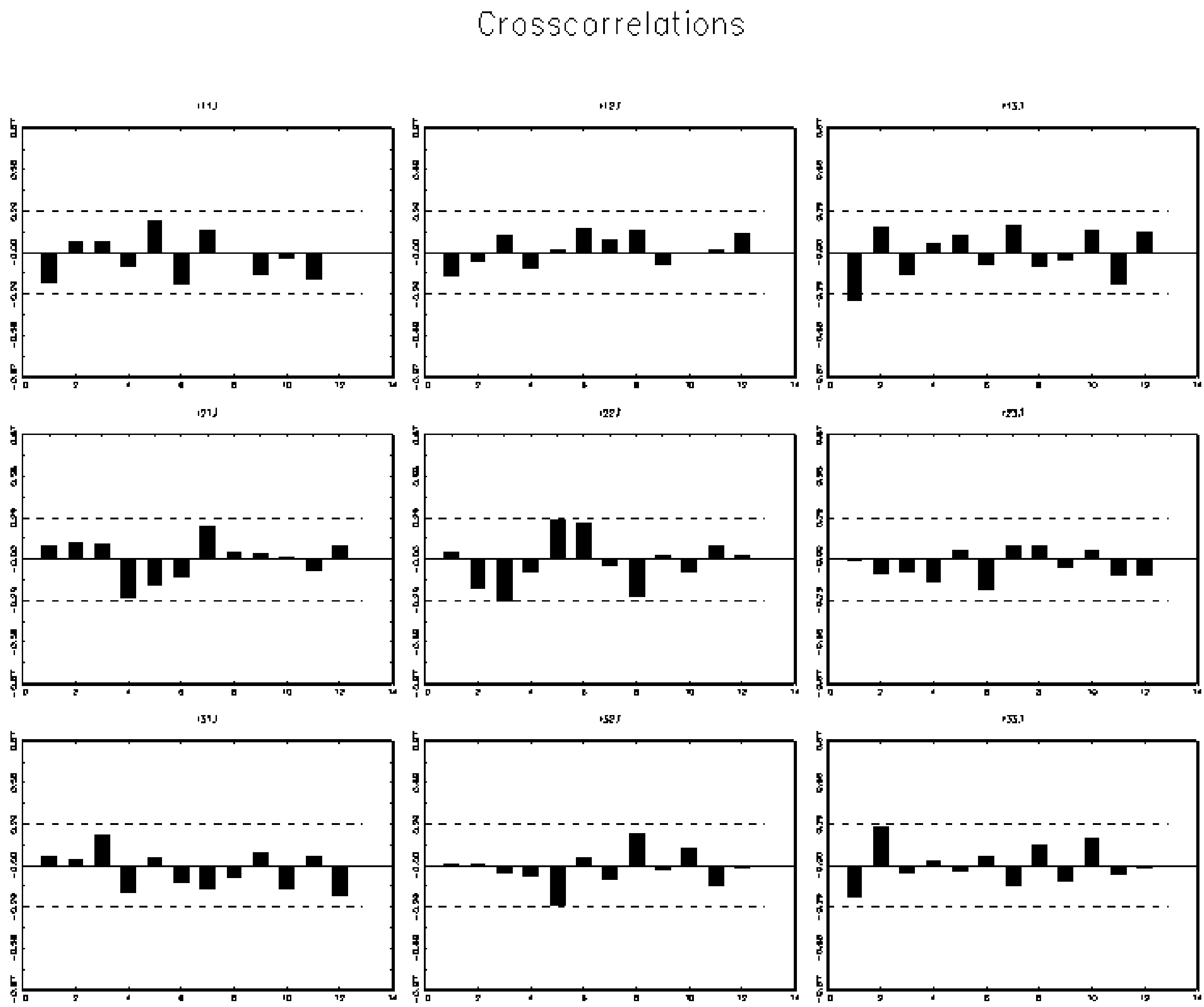
Table 5.4.1 LMF test of residual no-autocorrelation and tests of residual normality for the restricted model

```
LM-TYPE TEST FOR AUTOCORRELATION with 2 lags

LM statistic:             20.3348
p-value:                 0.3143
df:                      18.0000

LM-TYPE TEST FOR AUTOCORRELATION with 3 lags

LM statistic:             29.2257
p-value:                 0.3500
df:                      27.0000

TESTS FOR NONNORMALITY

Reference: Doornik & Hansen (1994)
joint test statistic:     59.2961
p-value:                 0.0000
degrees of freedom:       6.0000
skewness only:            20.1134
p-value:                 0.0002
kurtosis only:            39.1827
p-value:                 0.0000

Reference: Lütkepohl (1993), Introduction to Multiple Time Series Analysis, 2ed, p. 153
joint test statistic:     71.2622
p-value:                 0.0000
degrees of freedom:       6.0000
skewness only:            32.0941
p-value:                 0.0000
kurtosis only:            39.1682
p-value:                 0.0000

JARQUE-BERA TEST

variable        teststat   p-Value(Chi^2)  skewness   kurtosis
u1              21.2502    0.0000           1.1289     5.3511
u2              7.7045     0.0212           0.8126     4.1005
u3              45.3110    0.0000          -1.0654     7.2562
```

Source: Author, using JMulti 4.24

We also performed visual inspection on the residual cross-correlations and auto-correlation presented in Figure 5.4.2. Looking at the diagonal, where the residual auto-correlation is presented, we conclude that none of the three equations in the system has a significant positive or negative auto-correlation of its residuals at the first 12 lags. We also find no significant cross-correlation of the residuals among different equations, with the exception of the negative first lag correlation between the residual from the first and the third equation in the system. All other residuals show no cross-correlation.

Figure 5.4.2 Auto-correlation and cross-correlation of residuals of the restricted model with "B_Croatia", "DYNEF" and "OMF_AUMIndex" as endogenous variables



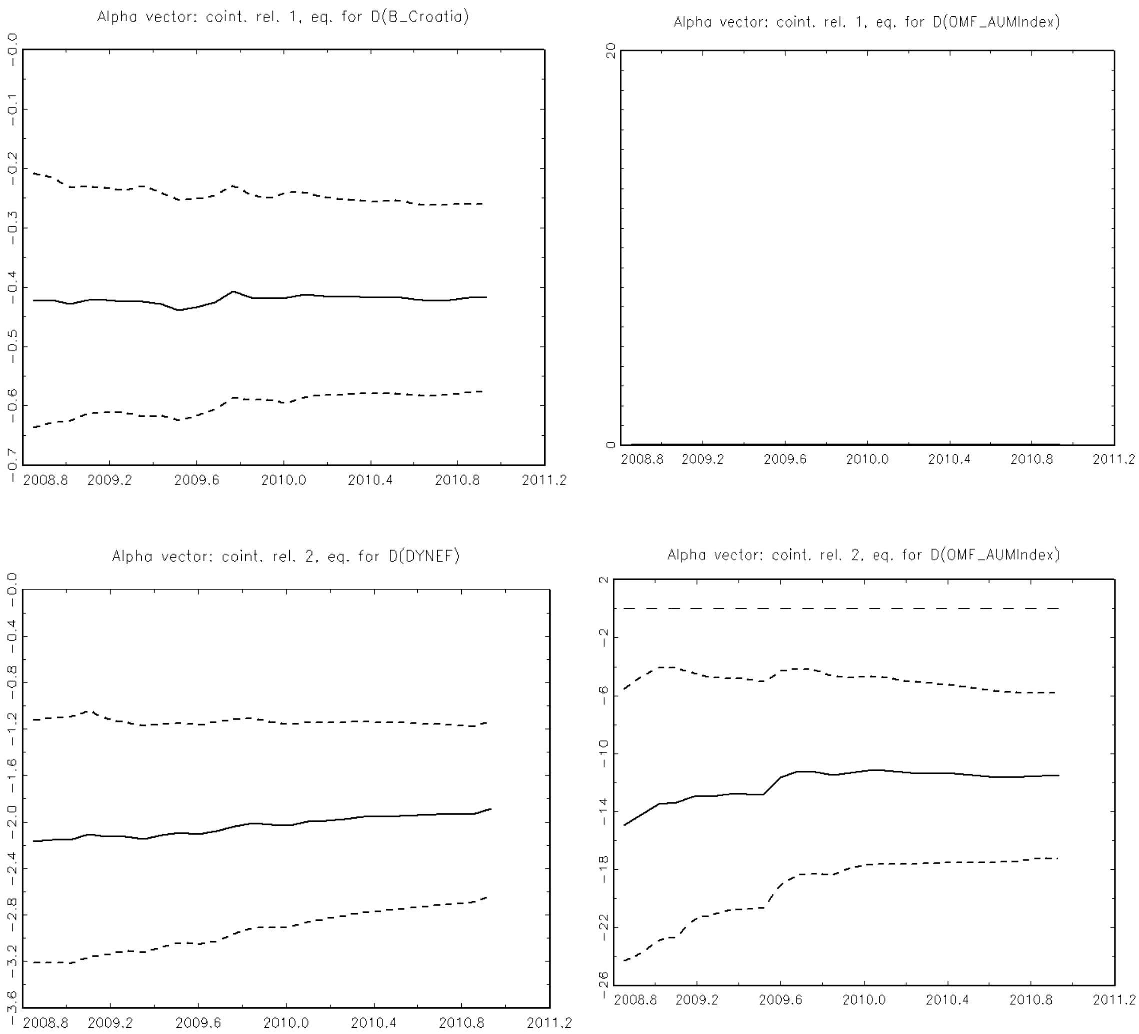
*each graph shows the cross-correlation of residuals of each one of the three endogenous variable equations, including the 95% critical value band. Cross-correlation of the residuals from one equation, with the residuals of another equation is estimated for different lags up to 12 months. Graphs on the diagonal represent the auto correlations of residuals in the three equations of the system.

Source: Author, using JMulti 4.24

5.5. VECM (0) coefficient stability

Examining the stability of the regression coefficients, we find that the regression coefficients of our restricted model "loading matrix" are stable as we recursively expand the data set on which the coefficients are re-estimated. These recursive estimations and the graphical presentations of the coefficients as performed in JMulti 4.24 are presented in Figure 5.5.1.

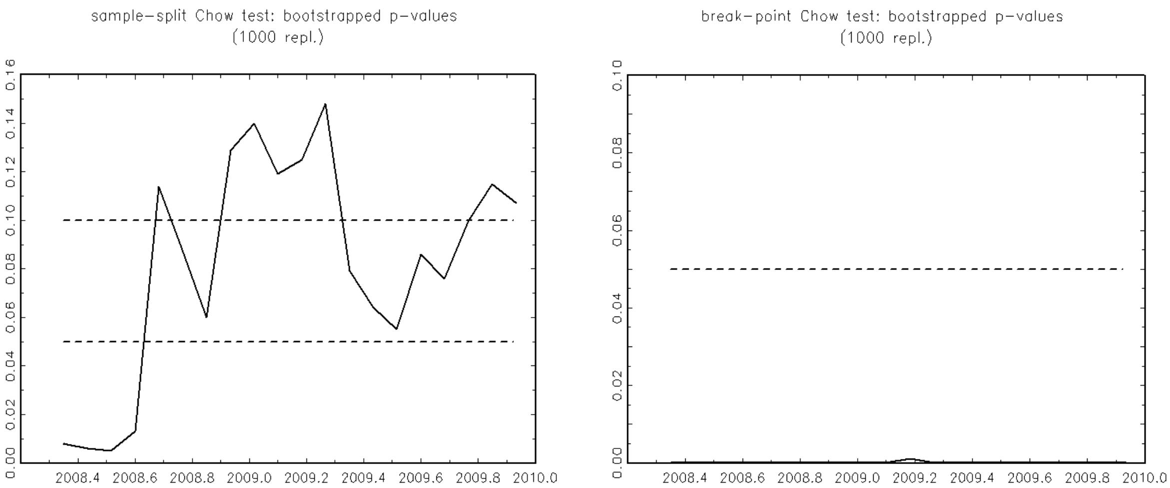
Figure 5.5.1 Recursive estimation of the "loading matrix" coefficients of our VEC Model with "B_Croatia", "DYNEF" and "OMF_AUMIndex" as endogenous variables



Source: Author, using JMulti 4.24

We also performed the Chow sample split (SS) and Chow break point (BP) tests for coefficient stability in the model. The test p-values of the Chow SS statistic with 95% level of confidence show that, in contrast to the initial periods (as expected), in the later periods we could not reject the null hypothesis that the coefficients of the model are stable. This result confirms that the overall model stability is better compared to our first model reported in Chapter 4. Conversely, the Chow BP test results reject the stability null, because of the instability of the coefficients other than the loading coefficient matrix. The Chow test p-values are presented in Figure 5.5.2.

Figure 5.5.2Chow SS and Chow BP test results for all viable months between June 2008 and January 2010 performed in JMulti 4.24



Source: Author, using JMulti 4.24

We conclude that this model shows much more robust results in terms of the stability of its coefficients compared to the model presented in Chapter 4.

5.6. Economic meaning of the estimation results

Finally, we interpret the regression results of our estimated restricted VECM (0) model. We first comment on the long-term equilibrium relationships represented by the two co-integration vectors. Then we analyse the interactions of the endogenous variables in the model by the use of Impulse Response (IR) analysis and, finally, comment on the role of the exogenous and deterministic variables.

Before discussing the long-term co-integrating relationships, we must note that, similar to our modelling in Chapter 4, we determined the structural order of variables in the two co-integration vectors based on theoretical reasoning. Our theoretical model in Chapter 2, suggested that the first long-term relationship should describe the relation between the relative value of the Croatian equity bubble "B_Croatia" and the DC pension fund speculative investments "OMF_AUMIndex", while the second long-term relationship should relate the DC pension fund speculative investments "OMF_AUMIndex" and the Dynamic efficiency of the Croatian financial market "DYNEF". This imposed restrictions on the order of the variables in the vector of endogenous variables used in the VECM (0) estimation process. In order to properly capture those two long-term relationships, we followed the same principle as in our estimation of the first empirical model in Chapter 4. We normalized the first co-integrating relationship on the unit relative value of the Croatian equity bubble "B_Croatia" and we placed a zero restriction on the dynamic efficiency variable "DYNEF". This way, the first co-integrating vector related "B_Croatia" only to the DC pension fund investment variable "OMF_AUMIndex". Then we normalize our second co-integrating relationship on the unit value of the dynamic efficiency of the Croatian financial market "DYNEF", and we set a zero restriction on the relative value of the Croatian equity bubble "B_Croatia". This way, in our second co-integrating relationship we focus on the long-term relationship between the Dynamic efficiency of the Croatian financial market "DYNEF" and the investments of the DC pension funds in the Croatian and SEE equity focused open-end mutual funds "OMF_AUMIndex". These two long-term relationships reflected in our co-integrating vectors are supported by our theoretical model. On the one hand, the introduction of the DC pension funds, measured by the investment variables of the DC pension funds "OMF_AUMIndex", should relate to the relative value the Croatian equity market bubble "B_Croatia" as a sufficient condition for the occurrence of the bubble; while, on the other, the dynamic efficiency of the Croatian equity market "DYNEF" should relate to the DC pension fund investments "OMF_AUMIndex" as the necessary condition for rational investments in the bubbly asset.

Based on our estimated VECM (0) using the suggested structure of the co-integrating vectors, the OLS estimated long-term co-integrating relationships are presented in equation 5.1a-b and 5.2a-b (t-statistics in parenthesis).

$$Error1_{m-1} = B\_Croatia_{m-1} + \underset{(1.087)}{16.649} - \underset{(8.454)}{0.08549} OMF\_AUMIndex_{m-1} + \underset{(4.257)}{45.707} l\_Shift\_m9_{m-1} - \underset{(2.257)}{0.949} Trend_{m-1}$$

$$B\_Croatia_{m-1} = -\underset{(1.087)}{16.634} + \underset{(8.454)}{0.08549} OMF\_AUMIndex_{m-1} - \underset{(4.257)}{45.707} l\_Shift\_m9_{m-1} + \underset{(2.257)}{0.949} Trend_{m-1} + Error1_{m-1}$$

(5.1a-b)

$$Error2_{m-1} = DYNEF_{m-1} - \underset{(0.832)}{2.095} + \underset{(1.855)}{0.00308} OMF\_AUMIndex_{m-1} - \underset{(5.029)}{8.877} l\_Shift\_m9_{m-1} + \underset{(4.138)}{0.286} Trend_{m-1}$$

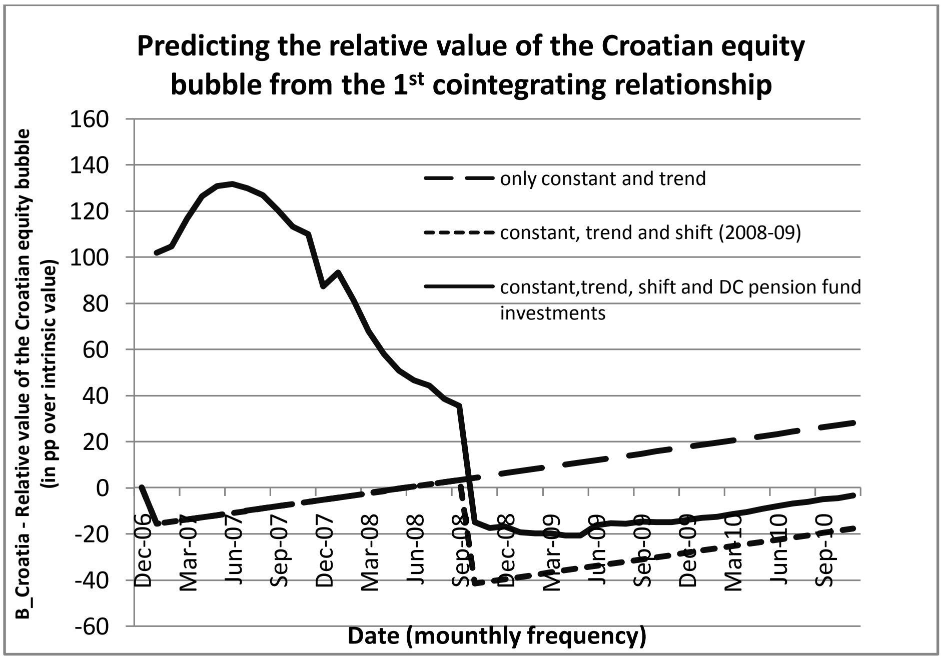
$$DYNEF_{m-1} = \underset{(0.832)}{2.095} - \underset{(1.855)}{0.00308} OMF\_AUMIndex_{m-1} + \underset{(5.029)}{8.877} l\_Shift\_m9_{m-1} - \underset{(4.138)}{0.286} Trend_{m-1} + Error2_{m-1}$$

(5.2a-b)

Looking at the first long-term relationship (equation 5.1a-b), we notice that the index explaining the DC pension fund investments in the Croatian and SEE open-end mutual funds "OMF_AUMIndex" is positively related to the relative value of the Croatian equity bubble "B_Croatia". This means that as the demand for the bubbly asset investment by the DC pension funds increases, this is followed by an increase in the value of the bubbly assets. This demand is realized through the DC pension fund investment in the indirect investment vehicle, the Open End Mutual Funds focused on Croatian equity. The impact of changes in "OMF_AUMIndex" can be highlighted by considering the first co-integrating vector set out with "B_Croatia" as the left-hand side variable and setting the other terms to zero. If we set the value of the "OMF_AUMIndex" to100, which corresponds to zero investments of the DC pension fund in the bubbly open-end mutual funds, the corresponding level of bubbly asset investments by the DC pension funds is a marginally positive value of the Croatian equity bubble of only 8.5 percentage points (0.08549*100=8.549). At the other extreme, when the

"OMF_AUMIndex" reaches high speculative investment values of over 1500 index points, then the corresponding relative value of the Croatian equity bubble, ceteris paribus, reaches nearly 130 percentage points (1500*0.08549=128.235). Given that "*B_Croatia*" ranges between -55.7 and 138.1, we see the prime importance of indirect DC pension fund equity investment "OMF_AUMIndex" in the development of the equity bubble. If we now consider just the deterministic components, the constant (-16.63) and time trend (0.949), excluding "OMF_AUMIndex" and the structural shift term "$l\_shift\_m9_{m-1}$" representing the impact of the GFC, we see that the large negative constant enables an upward trend that captures otherwise un-modelled growth in the relative value of the Croatian equity bubble. Finally, we find that the occurrence of the Global Financial Crisis (GFC), ceteris paribus, caused a strong negative shock to the value of variable "*B_Croatia*" (by -45.7 percentage points). The following Figure 5.6.1 shows the time path of "*B_Croatia*" predicted by Co-integrating Vector 1 taking into account: (1) only the constant and time trend; (2) all of the deterministic components (constant, time trend and shift dummy); and (3) all variables (the deterministic components together with the impact of "OMF_AUMIndex").

Figure 5.6.1 Predicted time paths of "*B_Croatia*" from the Co-integrating Vector 1



Source: Author, using co-integration equation 5.1b

We notice from Figure 5.6.1, that the level of the relative value of the Croatian equity bubble "B_Croatia" reaches positive values only after we include the DC pension fund investments in the equation (solid line). Otherwise, including only the deterministic variables, the trend and the constant, the relative value of the Croatian equity bubble shows much lower long-term values, which could have been reached had there been no structural break caused by the GFC. When we include the structural shift caused by the GFC along with the other deterministic variables (constant and trend), without the introduction of the DC pension funds speculative investments, the first co-integration relationship predicts that the Croatian equity bubble would have not occurred. This result supports the hypothesis drawn from the theoretical model in Chapter 2, which stated that the introduction of the DC pension funds increase the intensity and the level of the asset bubble in the financial market.

Having interpreted the long-run equilibrium relationship from the first co-integrating vector, we now explore its corresponding adjustment processes. In order to understand the way such disequilibrium in the first long-term relationship is removed by adjustments within the system, we need to look at the loading coefficients in the first column of the "α" loading matrix (Table 5.3.3). Those coefficients correspond to the first co-integrating vector. Looking at the loading coefficients, we find that the strongest error correction mechanism works through the correction of the relative value of the bubble itself "B_Croatia". Namely, a unit error term from the first co-integrating vector, expressed in units of relative value of the Croatian equity bubble, corrects by 41.8% during the following month by changing the relative value of the Croatian equity bubble "B_Croatia".[13] This means that over 95% of the error is corrected by the change of the relative value of the Croatian equity bubble in only 6 months [(1-0.418)^6=0.96]. On the other side, the other loading coefficient affecting the next period value of the DC pension fund investments in the bubbly asset "OMF_AUMIndex", although with a negative value of -1.884, has a destabilizing effect on the equilibrium in the first co-integrating vector.[14]A unit error term from the first co-integrating vector, expressed in

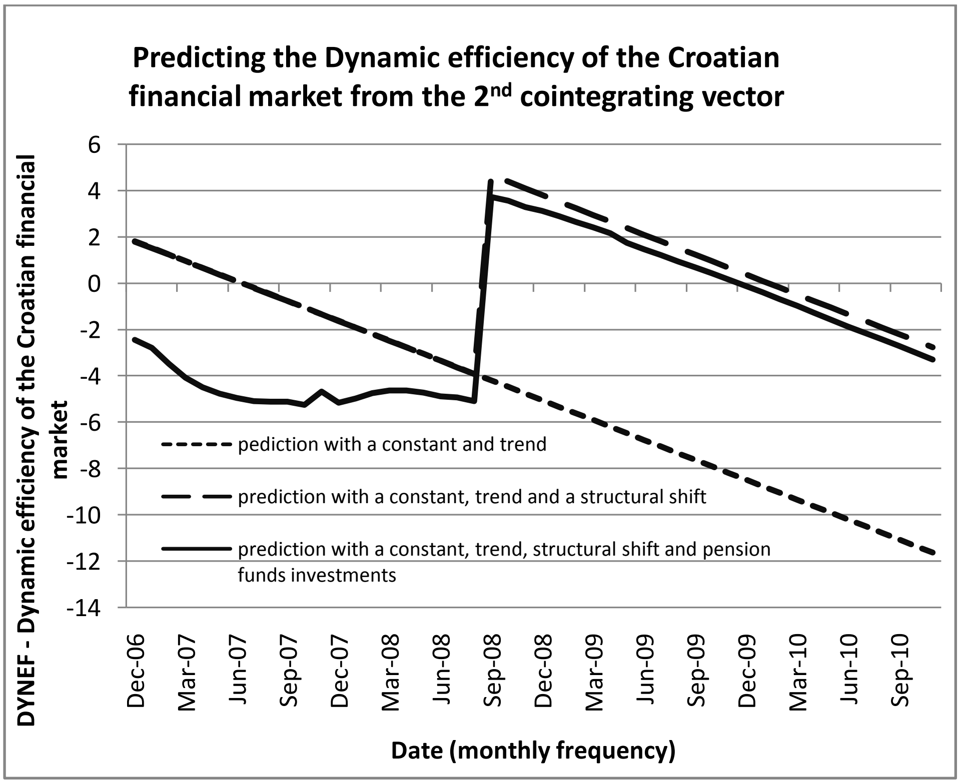
[13]The dependent variable is the absolute change per month of "B_Croatia". A unit error term from the first co-integrating vector, expressed in units of relative value of the Croatian equity bubble, corrects by 0.418 units per month. Hence, this is a stabilising rate of change of 41.8% per month.

[14] The adjustment (loading) coefficients on the differenced variable on the left-hand side of the (vector) error-correction model and the long-run coefficients on the corresponding level within the co-integrating vector need to be of opposite signs for the process to be stable (Juselius, 2006, p. 122).

units of relative value of the Croatian equity bubble, takes the first co-integrating vector away from equilibrium at a monthly rate of 1.884 index points of the “OMF_AUMIndex”. However, because “OMF_AUMIndex” varies between around 100 and 1700, this is a rather small effect (proportionally small when at the lower end and very small at the higher end of its range).Accordingly, although the stabilising coefficient on “B_Croatia” is smaller in absolute value than the destabilising coefficient on “OMF_AUMIndex”, once it is taken into account that they are acting on different variables within the system, and that the scale of measurement of these two variables is different by an order of magnitude, then it becomes clear that the former signifies a large effect and the latter a small effect. Hence, the overall process is stabilising. The speculative DC pension fund investments tend to destabilize the adjustment process, but their effect is much lower compared to the dominating adjustment through the value of the Croatian equity bubble “B_Croatia”.

Next, we analyse the second co-integrating relationship (equation 5.2a-b). This explains the long-term equilibrating relationship between the dynamic efficiency of the Croatian financial market “DYNEF” and the DC pension fund investments in the bubbly asset “OMF_AUMIndex”. The second co-integrating vector shows that there exists a negative long-term relationship between the DC pension fund investments in the bubbly Croatian and SEE equity focused open-end mutual funds “OMF_AUMIndex” and the Dynamic efficiency of the Croatian financial market “DYNEF”. Ceteris paribus, the value of 100 index points of the DC pension fund investment variable “OMF_AUMIndex” corresponds to a 0.3 unit of the dynamic efficiency of the Croatian financial market (0.003*100). This means that at the beginning of the sample period, at m=1, and without the introduction of the DC pension funds speculative investments, represented by “OMF_AUMIndex=100”, the dynamic efficiency of the Croatian equity market “DYNEF” would be positive (2.095-0.3=1.795), although this positive intercept (constant) is the point of departure for a negative time trend (-0.286*Trend, which given the range of “DYNEF” – between about -7.3 and +9.6 – represents a substantial monthly rate of decline). Such a relatively fast negative time trend in the dynamic efficiency of the Croatian financial market could be a logical consequence of a significant accumulation of productive investments, which makes the financial market dynamically inefficient as productive investment opportunities become scarce. As the dynamic efficiency of the Croatian equity market decreases through the passage of the time, the system requires higher speculative investment of the DC pension funds through the Croatian and SEE equity focused open-end mutual funds "OMF_AUMIndex". Namely, to compensate a unit fall of the dynamic efficiency (a large fall) then, ceteris paribus, the DC pension funds need to increase their speculative "OMF_AUMIndex" investments by over 324 percentage points (-0.003081*324=-1), bringing the dynamic efficiency error correction mechanism into equilibrium. This relationship corresponds to the role of the dynamic efficiency of the financial market suggested by the theoretical model in Chapter 2, where lower dynamic efficiency creates a fertile environment for rational investment into the bubbly asset. In addition, part of the fall of the dynamic efficiency would be compensated by the trend variable (0.286 units per month), which might signify the rise in other investment opportunities on the Croatian financial market, which widen the investment horizon and improve the dynamic efficiency of the financial market. The following Figure 5.6.2 shows the time path of "*DYNEF"* predicted by Co-integrating Vector two taking into account: (1) only the constant and time trend; (2) all of the deterministic components (constant, time trend and shift dummy) and (3) all variables (the deterministic components together with "OMF_AUMIndex").

Figure 5.6.2 Predicted time paths of "*DYNEF*" from the Co-integrating Vector 2



Source: Author, using equation 5.2a-b

Figure 5.6.2 show that the Croatian financial market has a decreasing trend in its dynamic efficiency, which is an expected consequence of a maturing financial market. The dynamic efficiency is worsened by the introduction of the DC pension funds, because they put additional pressure on the demand for saving/investment vehicles in the market. We also see that the GFC created a significant shift in dynamic efficiency, mainly by increasing the investment risk premiums. The occurrence of the GFC turned the market into a state of being dynamically efficient, a state in which investments in the bubbly asset become irrational. Such a change in the dynamic efficiency of the Croatian financial market, based on the theoretical model in Chapter 2, is the major factor causing the crash of the Croatian equity bubble (Figure 2.5)

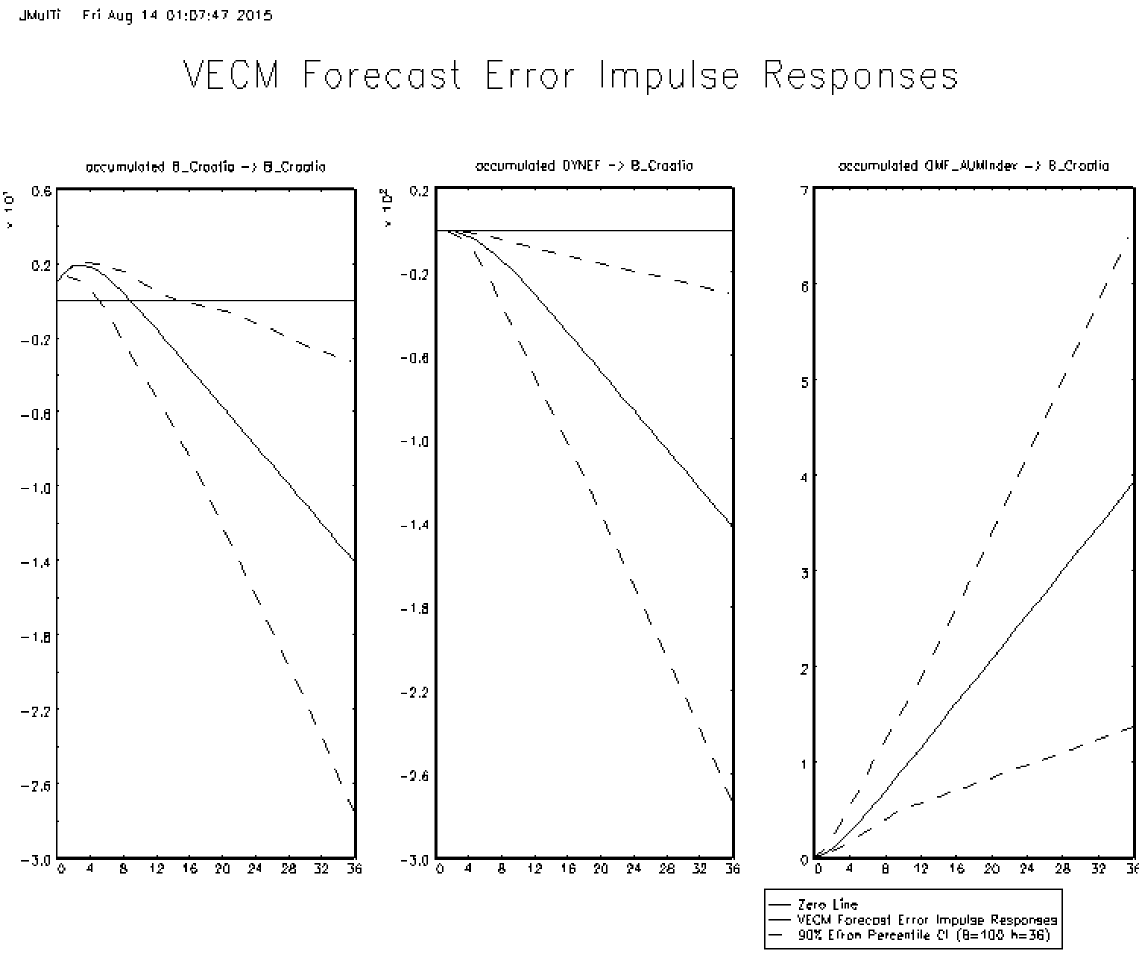
We analyse the second column of the “α” loading coefficient matrix in order to learn how a unit shock in the second co-integrating vector feeds back into the system, bringing the second long-term relationship back to equilibrium (Table 5.3.3). We see that a positive unit shock in the second co-integrating relationship corrects the next period value of the dynamic efficiency by -0.285, decreasing the unit error by 28.5% in the next period. This means that a unit disturbance in the second error correction term adjusts itself by over 85% in only six months [(1-0.285)^6=0.866], mostly through the change in the value of the dynamic efficiency of the Croatian financial market “DYNEF”. A unit of positive disequilibrium in the second long-term relationship also corrects through the change in the value of the investment variable “OMF_AUMIndex”. This adjustment is also stabilizing (the long-run coefficient is 0.003 and the adjustment coefficient is -11.525, hence of the opposite signs required for negative – i.e. stabilising – feedback). Namely, a positive unit shock in the second long-term relationship causes a 11.5 index points decrease in the DC pension fund speculative investments “OMF_AUMIndex” in the next period. In effect, this decrease in the DC pension fund investments, ceteris paribus, corrects the unit disequilibrium in the second long-term relationship by -0.035497 (-11.525*0.00308), or -3,5%, which presents a much slower adjustment compared to the direct adjustment through the dynamic efficiency variable “DYNEF”. Although with a slower adjustment, this shows that the DC pension fund speculative investments also react to the disequilibrium in the second long-term relationship normalized on the Croatian financial market dynamic efficiency.

Finally, looking at the effect of the GFC in the second co-integrating vector (equation 5.2a-b), we see that it caused – ceteris paribus – a strong positive shock of 8.877 units (the estimated effect of "*l_shift_m9"*), on the dynamic efficiency of the financial market "DYNEF". Given that the range of values of "DYNEF" is between -8 and +10, this represents an enormous rise in dynamic efficiency associated with the GFC (see Figure 5.6.2). This makes the investment environment suddenly much less favourable for rational bubbles and so triggers the collapse of the bubble.

After we described the two co-integration relationships and their loading mechanisms, we analyse the effect of the occurrence of the GFC in the context of the two co-integrating vectors together. If we now focus on the equilibrium error formulation of the two co-integrating vectors (the first equations in, respectively, 5.1,a-b and 5.2, a-b), we find that the GFC caused a positive structural shock of +45.7 percentage points in the first long-term equilibrium error normalized on the relative value of the Croatian equity bubble (*Error1*), and a negative structural shock of -8.877 units in the second long-term relationship normalized on the dynamic efficiency of the Croatian equity market (*Error2*). Both effects jointly caused a sharp correction in the relative value of the Croatian equity bubble "B_Croatia" and a disappearance of a speculative financial market environment on the Croatian financial market.

To get a better image of the complete causal relationships between the endogenous variables in our estimated VECM (0), respecting all interactions defined by the estimated model, we continue with Impulse Response Analysis. IR analysis is presented in the Figure 5.6.3a-c, where the accumulated effects on each endogenous variable, caused by unit shocks on other variables, are investigated for the period of 36 consecutive months. Graphs contain the average projected effect, together with the 90% Confidence Intervals determined following Diccicio and Efron CI estimation(Diciccio & Efron, 1996).

Figure 5.6.3a Impulse Responses of "B_Croatia" to an imposed unit shock on all three endogenous variables individually: (a) "B_Croatia"; (b) "DYNEF"; and (c) "OMF_AUMIndex"

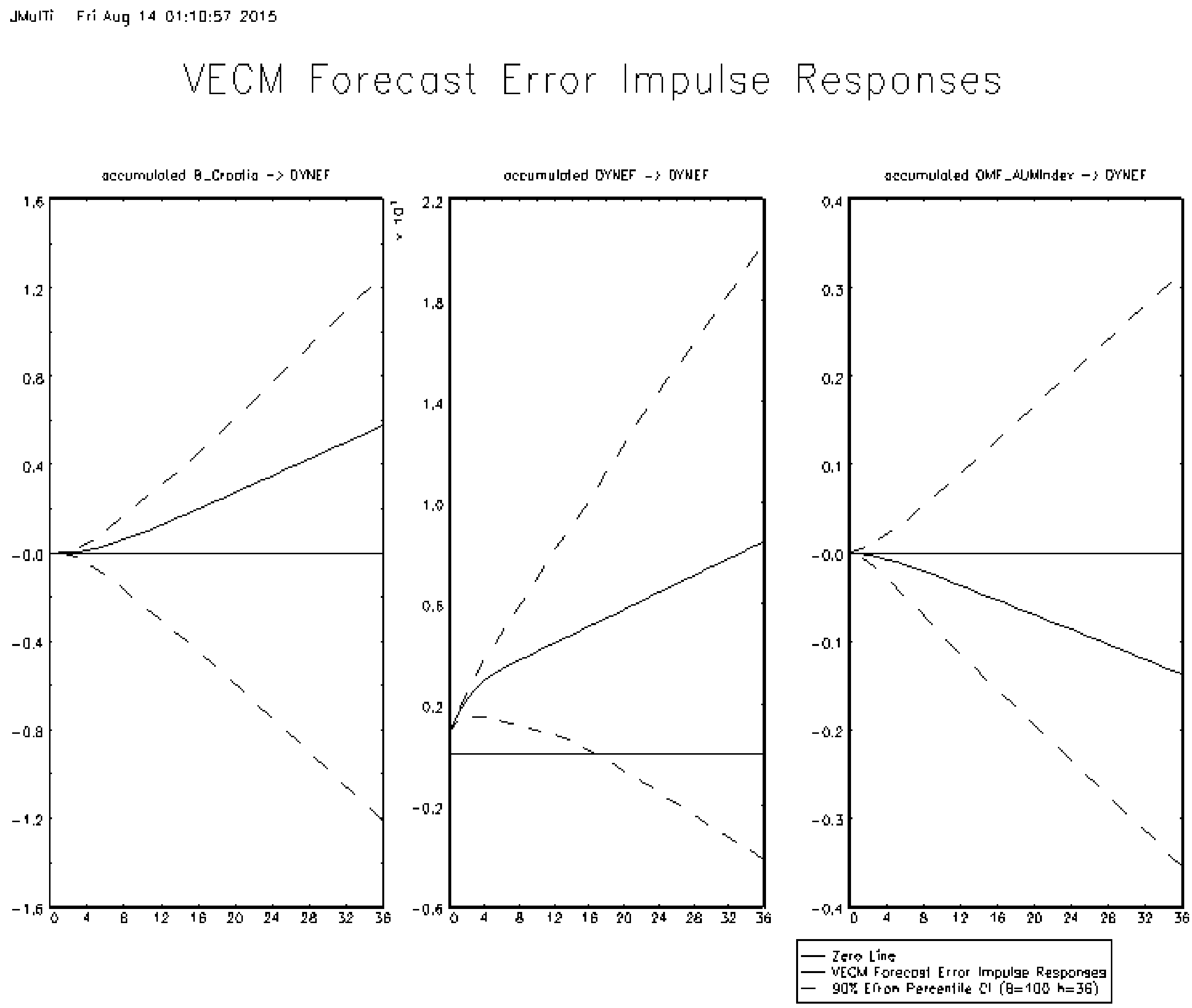


Source: Author, using JMulti 4.24

From the first IR analysis (panel [a] in Figure 5.6.3a), focused on the impacts on the relative value of the Croatian equity bubble "B_Croatia", we find that similar to the rapid adjustment process identified above, a unit positive shock on the relative value of the bubble itself is not self sustaining, meaning that it rapidly declines towards zero. We also find from panel [b], that a unit positive shock introduced to the dynamic efficiency "DYNEF" causes a negative and significant impact on the relative value of the Croatian equity bubble (namely, a unit shock introduced on "DYNEF" causes an average $-0.2 *10^2 = 20$ percentage point accumulated decrease of "B_Croatia" in 12 months). Consistent with the theoretical model, a positive shock to the dynamic efficiency of the Croatian financial market "DYNEF" creates a less favourable environment for the occurrence of the speculative equity bubble "B_Croatia".

And finally, looking at panel [c], we find that a unit positive shock imposed on the DC pension fund investments in the speculative open-end mutual funds, “OMF_AUMIndex”, positively affects the relative value of the Croatian equity bubble “B_Croatia” (causing a unit accumulated increase in the relative value of the bubble in the following 12 months).

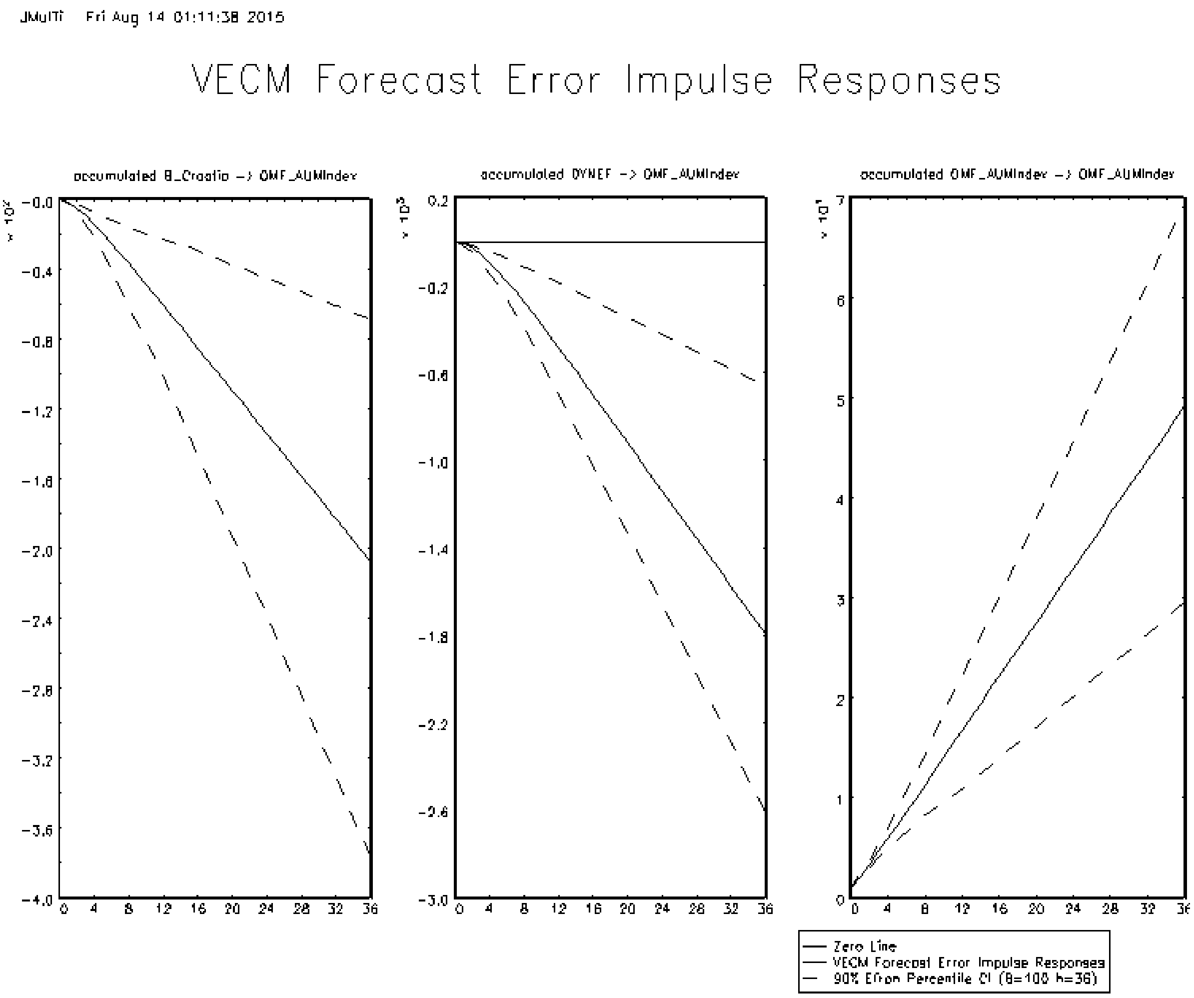
Figure 5.6.3b: Impulse Responses of “DYNEF” to an imposed unit shock on all three endogenous variables individually: (a) “B_Croatia”; (b) “DYNEF”; and (c) “OMF_AUMIndex”



Source: Author, using JMulti 4.24

From the second IR analysis presented in Figure 5.6.3b, in panel [a] on the far left and in panel [c] on the far right of the figure respectively, we see that a unit positive shock introduced to the relative value of the Croatian equity bubble “B_Croatia” and a unit positive shock introduced on the speculative DC pension fund investments in the open-end mutual funds “OMF_AUMIndex” does not significantly affect the dynamic efficiency of the Croatian financial market “DYNEF”. This result is consistent with the theory, where the dynamic efficiency of the financial market is expected to be affected by the productive investments and not by the speculative investments, which are the only type of pension fund investments in this model. This distinction was made in the estimated model in Chapter 4, where we included both direct and indirect DC pension fund investment variables. We also find from panel [b] in the middle, that there exists some persistence in the value of dynamic efficiency “DYNEF”.

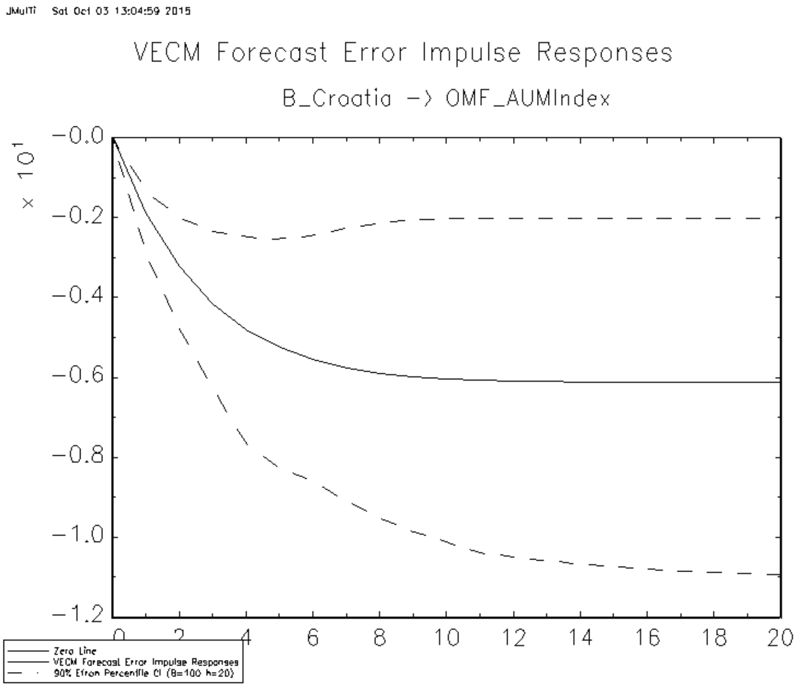
Figure 5.6.3c.Impulse Responses of “OMF_AUMIndex” to an imposed unit shock on all three endogenous variables individually: (a) “B_Croatia”; (b) “DYNEF”; and (c) “OMF_AUMIndex”



Source: Author, using JMulti 4.24

Finally, looking at the third IR analysis presented in Figure 5.73c, from panel [a], we see that the unit shock introduced on the relative value of the Croatian equity bubble "B_Croatia" has a negative and significant effect on the speculative DC pension fund investments in the Croatian and SEE open-end mutual funds "OMF_AUMIndex", our bubbly asset. The theoretical model shows that total speculative investments should approach the bubbly steady state at a rate diminishing in absolute value (Figure 2.4 in Chapter 2 suggests diminishing absolute value of speculative investments as the system approaches its bubbly equilibrium). In other words, at the onset of the increase in the relative value of the bubbly asset, when its return is the highest, the self enforcing effect of the speculative investments should dominate. But this self-enforcing effect on speculative investments would be diminished by the opposite effect of the increasing relative value of the bubble itself. If we compare the two effects caused by a unit shock of the relative value of the Croatian equity bubble "B_Croatia" in panel [a] and a unit shock on the DC pension fund speculative investments "OMF_AUMIndex" in panel [c], on the DC pension fund bubbly investments "OMF_AUMIndex", we notice that the self-enforcing effect of the rise in speculative investments (panel [c]) dominates over the negative effect caused by the increase in the relative value of the equity bubble panel [a]). The diminishing effect on the additional speculative investments caused by a unit increase of the relative value of the bubble could also be seen by looking at the non-accumulated impact presented in Figure 5.6.4.

Figure 5.6.4Non-accumulated impact of a unit shock introduced on "B_Croatia" to the value of "OMF_AUMIndex" in the period of 36 months



Source: Author, using JMulti 4.24

Looking at the second panel [b] in 5.6.3c, we find a negative and significant effect of a unit shock introduced to the value of the dynamic efficiency of the Croatian equity market "DYNEF" on the speculative DC pension fund investments "OMF_AUMIndex". Namely, a unit increase in the value of the dynamic efficiency of the Croatian financial market accumulates to more than a 500 index points decline in the DC pension fund investment in the bubbly Croatian and SEE equity focused open-end mutual funds "OMF_AUMIndex". This shows that an increase of the dynamic efficiency of the Croatian market observed by the GFC could fully discourage and even produce high disinvestment in the bubbly asset "OMF_AUMIndex", making it a major cause of the collapse of the Croatian equity bubble. Our theoretical model predicts that a positive change in the dynamic efficiency of the financial market will affect the rationality of the bubbly investments and will discourage further speculative investments. This is exactly consistent with the result in panel [b]. Finally, panel [c] shows that the bubbly DC pension fund investments are self-inducing, which means that a unit positive shock in the speculative investments "OMF_AUMIndex" creates additional speculative investments in the next period by the DC pension fund; and the opposite happens in the correction process. Looking at the theory, we mentioned that for the bubbly asset to occur, it needs initially to be accepted as an investment vehicle; this is supported by the self-inducing effect of the speculative investments shown in panel [c].

Finally, looking at the estimated model (Table 5.3.3), we also find that the dummy variables "Dummy4" representing the introduction of higher transparency standards in the DC pension fund industry in Croatia, and the dynamic efficiency dummy variable "Dummy_DYNEF" (coefficient with a border line significance), representing the moment when the dynamic efficiency of the financial market rises over the value of zero, both positively affected the dynamic efficiency of the Croatian financial market. This is a logical consequence of the improved transparency of DC pension fund investments and it could show that when the dynamic efficiency crossed the zero thresholds ("dummy_DYNEF=1") it improved even more rapidly in the following period. We also find that the introduction of wider investment opportunities to the DC pension funds, by the increase of their investment horizon on the domestic equity market, indicated by "Dummy3=1",decreased their investment in the speculative Croatian and SEE equity focused open-end mutual funds.

We also find that the dynamics of the relative value of the bubble on global financial markets, such as the US market "B_US", affects the relative value of the Croatian equity bubble both contemporaneously and with lag. A unit positive change in the relative value of the US equity market bubble translates into a 0.65 units positive effect on the relative value of the Croatian equity bubble with a lag of two month periods (-0.647+1.296=0.65). Such a positive effect of the US market could even increase the negative impact of the GFC on the collapse of the Croatian equity bubble.

5.7. Concluding remarks

Following the same estimation procedure as we reported in Chapter 4, we estimate a model explaining the dynamics of the relative value of the Croatian equity bubble using the "OMF_AUMIndex" variable representing the DC pension fund investments in the bubbly asset. We found that the VECM (0) of our system of three endogenous variables is the most representative model. Using the Johansen test, we determined that there exist two co-integrating relationships in the model, which we structured based on the theoretical reasoning from our model developed in Chapter 2. We estimated the model, and applied subset selection techniques to restrict the model to its optimal subset. The subset model was much simpler compared to the estimated model in Chapter 4, and suffered no contemporaneous cross-correlation, which was an issue in the previous model between the two interpolated investment variables. The model also showed much better coefficient stability in the loading coefficients as well as overall. Finally, the dynamics suggested by the empirical model confirmed the suggestions defined by the theory. First, the sufficient condition for the higher levels of the relative value of the Croatian equity bubble was the introduction of the DC pension funds and their acceptance of the bubbly Croatian and SEE equity focused open-end mutual funds in their portfolio. But the necessary condition for the DC pension fund investments in the bubbly asset to be rational and sustainable was the occurrence of dynamic inefficiency of the Croatian financial market. DC pension fund investments in the Croatian and SEE equity focused open-end mutual funds were self-inducing and sustainable if and only if the market was dynamically inefficient. Such a state of the financial market becoming dynamically inefficient created the logical development and growth of the speculative equity bubble. When the market suddenly became dynamically efficient, as in the aftermath of the Global Financial Crisis, the Croatian equity bubble collapsed and speculative investment became irrational. Although initially DC pension funds continued moderately investing in the speculative asset, soon the crash intensified, opening a strong self-inducing sell-off of the DC pension fund investments in the Croatian and SEE equity focused mutual funds.

This empirical second simpler empirical model gives support to several of the hypothesis of our OLG rational asset bubbles model with DC pension funds in Chapter 2. We have observed from our first co-integrating relationship and the Figure 5.6.1, as well as from the panel [c] in Figure 5.6.3 describing the Impulse response of the relative value of the

Croatian equity bubble on the impulse introduced to the DC pension fund investment variable, that the DC pension fund presence on the Croatian financial market, increases the speed of the development of the equity bubble as well its level. This supports the first as well as the second hypothesis suggested by our theoretical model developed in Chapter 2. The ability of the regulator to control DC pension fund investments on the local market and the pension tax imposed on consumer agents, could translate into control of the speed and the level of the speculative market dynamics caused by the DC pension fund. We have seen in Figure 5.6.2 describing the second co-integrating relationship that the DC pension fund investments coexist with a strictly lower dynamic efficiency of the Croatian financial market compared to the environment without the DC pension fund investments in place. On the other side we did not found strong support that the DC pension fund investments had a significant impact on the dynamic efficiency of the financial market. This might be a consequence of the use of only one investment variable, the DC pension funds to the Croatian and the SEE equity oriented open-end investment funds labelled as speculative investments. In our previous model, where we could have distinguished among the two types of direct and indirect investments, direct being identified as productive investments, we found more support that such investments by the DC pension funds decrease the dynamic efficiency of the Croatian financial market. Dynamic inefficiency imposing support on the DC pension fund investment in the bubbly Croatian and SEE equity focused open-end investment funds as observed in panel [b] in Figure 5.6.3c, and the self inducing effect of DC pension fund speculative investments as observed on panel [c] in Figure 5.6.3c, suggest that the impact of the DC pension fund investment in the bubbly asset, would accelerate with the shift of the financial market from the state of being dynamically efficient to a state of being dynamically inefficient. In such environment, as could be seen from Figure 5.6.1, and the panel [c] of Figure 5.6.3a, the presence of the DC pension fund on the financial market with a positive investment in the bubbly asset, would increase the level of the relative value of the equity bubble. This supports the fourth hypothesis suggested by the theoretical model in Chapter 2. Finally, the effects of the structural break identified by the dummy variable representing the occurrence of the Global Financial Crisis, clearly supports our fifth theoretical hypothesis suggesting that shocking market events such as the global financial crisis could rapidly change the financial market environment to a dynamic efficient state, and could simultaneously produce a crash of the relative value of the bubbly asset, which no longer represents a rational investment vehicle.

6. Conclusion

6.1. Summary and contribution to knowledge

Some prominent financial economists, including J.M. Keynes (1936) and the Nobel laureate Robert Shiller (2000), appealed to the “animal spirits” of individual market participants, and their interactions, as being the main source of financial market failure and speculative financial market outcomes. However, our theoretical model and the empirical findings developed in this dissertation suggest that the “irrational exuberance” of consumer agents is not the only potential source of financial market speculation and the development of asset bubbles. Our results suggest that institutional reform, introducing influential market participants, could be another such source.

When we introduce influential institutions such as Defined Contribution pension funds to an underdeveloped and illiquid financial market, as in the case of pension reforms in developing and transition economies, the institution takes command of a significant portion of consumer savings and investment discretion. Such capturing of the saving decision making discretion promotes a higher level of financial market investments, which greatly affects the balance of supply and demand of investable assets, especially visible in less developed financial markets. In our dissertation, we developed an original extension to the Overlapping Generations Model economy with rational asset bubbles, by introducing DC pension funds in order to simulate theoretically market outcomes in such a financial market environment.

The importance of such potential impacts caused by the introduction of DC pension funds onto underdeveloped financial markets, was largely ignored and misunderstood by global reform leaders, such as the World Bank (1994). In contrast to their mainly positive expectations of the outcomes of such institutional reform, suggested on the grounds of the aging population and inadequacy of the current pension systems, we show in this dissertation that the introduction of highly influential financial institutions can induce speculative financial market outcomes, especially when defined contribution mandatory pension funds are introduced into underdeveloped financial markets with shallow (liquidity constrained) equity markets. In a financial market context, where most consumers have no investment experience or interest prior to the introduction of the DC pension fund, the introduction of the

DC pension funds, vastly changing the overall demand for investment vehicles, stimulates the development of bubbly financial market instruments and propels their dynamics to higher speculative valuation levels.

We estimated two Vector Error Correction models based on Croatian market data in order to determine the long-term and short-term interactions between DC pension fund investments, the dynamic efficiency of the Croatian financial market and the relative value of the Croatian equity bubble. Most importantly, we undertook this empirical estimation to test our theoretical hypotheses about the impact of influential institutional investor on dynamic efficiency and speculative market valuations. Our empirical results support the theoretical hypothesis that the introduction of DC pension funds, especially in financial market environments with underdeveloped equity market such as the Croatian financial market, could lead to asset bubbles. Accordingly, financial market reforms such as introducing influential financial institutions should be carefully designed, considering the impact of those new influential institutional investors on the dynamic efficiency of the financial market and on the overall market-saving attitude in order to avoid causing the investment environment to stimulate the development of speculative asset bubbles, which are ultimately fragile. This is suggested in order to prevent costly abandoning of financial market reforms, such as the pension reform in the transition and developing economies, after the occurrence and the collapse of speculative market episodes.

6.2. Findings

In order to develop our argument, we reviewed the foundations of the currently dominating Efficient Capital Markets Hypothesis (ECMH) and its main critics, stressing the arguments for the failure of the efficient market pricing mechanism and the development of speculative asset pricing outcomes. We found that the Overlapping Generations Modelling framework (OLG), introduced by Paul Samuelson, presents one of the most complete modelling frameworks for explaining financial market dynamics by connecting specific micro-characteristics of the financial market with different macro-dynamic outcomes. This model presented the basic framework for the development of the canonical OLG Rational Asset Bubbles model introduced by Jean Tirole in 1985, which is used as the basis of our extension; namely, adding influential institutional investors, such as the DC pension funds, in order to theoretically examine their impact on financial market speculative valuation outcomes.

We divide the findings with respect of the market environment stimulating the development of asset bubbles into two categories. The first category contains the summarized micro characteristics identified by the current literature on asset bubbles as the main drivers of market speculation. These characteristics mainly represent certain anomalies within the investment decision making of market participants that create market miss-valuations. The second category of findings encompasses our original contribution, which is focused on the role of influential institutional investors such as the defined contribution pension funds. In order to reach those findings, we developed a specific theoretical model based on the OLG Rational Bubbles modelling framework. Our theoretical model simulations led us to our theoretical hypotheses, which we tested using Croatian financial market data. This second category of findings contains the originality in the dissertation, both in its theoretical and its empirical approaches, while the first category represents a summary of findings from the most important already established work in the field. Nonetheless, this first category is needed in order to reach a complete understanding of the main factors that promote speculative or bubbly asset market valuation and that might complement our conclusions connected to the impact of influential institutional investors. Hence, before we summarize our original findings with respect to the impact of influential institutional investors such as the DC pension funds, we begin with a summary of the first category of established ideas on asset bubbles and micro market characteristics that stimulate the occurrence of asset bubbles:

a) The asymmetric information problem and its impact on the valuation efficiency in the financial market. The presence of high asymmetry of information about the fundamental value of investment instruments among market participants places institutional investors, especially when introduced to underdeveloped market environments, into a leading – signaling – role, thereby multiplying their market influence by “guiding” the actions of the “herd”. As the market participation of the DC pension funds increases on such a financial market in relative terms, it is critical to introduce strict standards in investment decision making, based on high and sophisticated knowledge in asset valuation, investment ethics and transparency, in order to avoid the potentially speculative market leading role of the DC pension funds. It is also important to impose high ethical standards when it comes to the public presentation of valuation opinions and investment recommendations by the representatives of the DC pension funds.

b) The agency problem within the institutional investor points to the need for setting strict rules in order to overcome issues of "risk shifting" –not only within the investment management decision makers of the DC pension fund management companies but also in cases when institutional investors stimulate market innovation in the direction of the extension of the principal-agent chain in the investment process outside the DC pension fund. Such extension of the principal-agent chain is achieved by further delegating investment authority from the DC pension fund to other institutional investors. Our empirical study, from the case of Croatia, suggests that the extension of such an investment decision making chain may create speculative market innovations, such as was the case with the introduction of the open-end mutual funds investing in Croatian and SEE equity. Those investment vehicles in the case of Croatia served as a suitable speculative-bubbly asset for the Croatian DC pension funds when financial market conditions for the development of such a market innovation were satisfied, such as the dynamic inefficiency of the financial market.

c) Irrational investors are the third influential micro characteristic, signifying the issue and the impact of groups of market participants making decisions based on irrational market expectations. Their high presence on the financial market makes departures from the fundamental value of financial assets even more persistent and harder to arbitrage-out. Their presence and survival could be especially dominant in underdeveloped financial market. In this case, policies preventing waves of abrupt market speculation and policies introducing financial education and knowledge based investment decision making should be applied alongside the introduction of the DC pension funds. Moreover the presence of irrational investors may increases the impact and the importance of the institutional investor in both productive and speculative asset investments.

These micro-market characteristics, stressed by the currently dominating asset bubbles theories, should be taken into account by policy makers in addition to the suggestions arising from our theoretical model, which focus predominantly on the role of the introduction of DC pension funds in creating speculative market dynamics. We did not add any additional micro-market complication in our theoretical modelling, in order to preserve the simplicity of the model, and in order to focus only on the speculative market outcomes arising from the interaction of DC pension funds with financial market dynamic-efficiency. The dynamic efficiency of the financial market represents the financial market balance between the growth of the wealth that is required to be invested or saved, and the investment opportunities available on the financial market. The state of a high imbalance between assets to be invested and investment opportunities available on the market, caused by the introduction of the influential institutional investor, characterizes the financial market as being a fertile platform for the development of rational asset bubbles.

Based on our theoretical OLG model of rational asset bubbles in illiquid financial markets with DC pension funds, developed and examined in Chapter 2, we derive five main hypotheses, which we test using data from the Croatian financial market. We introduced an original approach to measuring the relative value of the Croatian equity bubble, mainly influenced by the ideas presented by authors such as Robert Shiller. Similarly, our measurement of the dynamic efficiency of the Croatian financial market is based on suggestions in the work of Abel et al. (1989) emphasising financial market imbalance between the return on productive assets and the growth rate of investable assets approximated by the growth rate of subscribers to the DC pension funds. We also had to make significant primary data collection on the regulatory process and the investments of the DC pension funds in Croatia in order to investigate their impact on the Croatian financial market. Finally, using our Croatian data set we estimated two empirical time series Vector Error Correction Models; we then tested our main theoretical hypotheses against the long-term equilibrium relationships measured by these models and the Impulse Response analysis performed on the platform of these models.

Our first empirical model presented in Chapter 4 had several issues, such as the cross correlation of the equation residuals from direct DC pension fund equity investments labelled as “productive” and the indirect equity investments labelled as “bubbly”. The model also suffered from being too extensive, consuming many degrees of freedom in our short data set, and showing non-stable estimation coefficients. In order to overcome some of those issues, mainly arising from the fact that our DC pension fund investment variables were interpolated from a yearly to a monthly frequency, we also investigated a second empirical model in Chapter 5. This second VEC model contained only one endogenous investment variable, representing the DC pension fund investments in the Croatian and SEE equity focused open-end mutual funds, which we labelled as “bubbly” in the first model. Although the second

VEC model is based on a much shorter data set, all of its variables have a genuine monthly frequency, which makes the model much richer in information. The second VEC Model is estimated with a zero lag of the endogenous variables, which means that the consumed degrees of freedom are much smaller and, altogether, the model is much simpler. In addition, very important is that the cross correlation issue is not present in the second VEC Model, and that the model is much more robust, showing much better results on the coefficient stability tests.

The difference between the two estimated VECM models is made by the use of two different data sets representing the investments of the Croatian DC pension funds on the Croatian equity market. The first VEC Model made an important distinction between *direct* equity investment in the Croatian equity market and *indirect* equity investments through the Croatian and SEE equity focused Open-end investment funds. The estimation results of the first VEC Model presented in Chapter 4, and the simulations made by the Impulse Response Analysis, suggest that the direct equity investments had a less speculative character on the Croatian financial market, and could be labelled as “productive investments”. On the other side, the results from the first empirical VEC model suggest that the indirect equity investments through the Croatian and SEE equity focused mutual funds were the financial market “bubbly asset” innovation. In the light of the theoretical discussion, this identification of the productive and the bubbly asset in the case of the Croatian market is one of the major benefits of the first VEC Model.

The following hypotheses capture the implications of our original theoretical modelling for the impact of introducing influential institutional investors, DC pension funds, onto underdeveloped financial markets. Following each hypothesis, we present a summary of the supporting empirical evidence based on the two VEC Models.

Hypothesis 1: *Faster and more aggressive rise in the value of the bubbly asset could be expected with the introduction of the DC pension fund on the financial market.*

Looking at the empirical results representing the bubbly asset pricing dynamics in our estimated VEC Models in Chapter 4 and 5, we find supporting for our first theoretical hypothesis. Namely, in the first co-integrating relationship of our first empirical model presented in Chapter 4, both direct (“productive”) and indirect (“bubbly”) investments of the DC pension funds have a strictly positive long-term relationship with the relative value of the

Croatian equity bubble. Such a positive impact of the DC pension fund investments on the relative value of Croatian equity bubble was also confirmed by the use of Impulse Response Analysis applied on our estimated restricted VECM in Chapter 4. Namely, positive impulses introduced in both direct and indirect DC pension fund investment variables, translated into an increase in the relative value of the Croatian equity bubble. The same long-term and short-term impact is confirmed also using our second empirical VEC Model presented in Chapter 5, where we use only one investment variable representing the introduction of the DC pension fund. Its long-term relationship with the relative value of the Croatian equity bubble, as seen from the first co-integrating relationship is also positive. The same positive direction of the impact of the introduction of DC pension funds is likewise confirmed by the Impulse Response Analysis applied on the VEC Model in Chapter 5. Results from our both empirical models based on Croatian financial market data support our first hypothesis, suggesting faster and more aggressive increase in the value of the bubbly asset as a consequence of the introduction of DC pension funds.

Hypothesis 2: *Manipulating the pension fund investment rules* –represented in the theoretical model by the variable τ *–which could be in the hands of the law setting body responsible for the introduction of the DC pension fund in the system, affects the intensity of the effect of the DC pension fund on the financial market and on the dynamics of the productive and bubbly investment assets in the economy.*

The evidence supporting our second theoretical hypothesis is connected to the previously mentioned supporting evidence for the first theoretical hypothesis. As the DC pension fund tax on the gross salary of the young generation and DC pension fund investments increase, it exerts strictly positive effect on the relative value of the Croatian equity bubble. Consequently, a regulatory institution that has control on the extent of such DC pension fund salary tax, and the consequent investments on the domestic financial market in both productive and speculative market instruments, can impact the level and the intensity of the rise of the bubbly asset on the domestic market. The concomitant role of the regulator could be especially important in underdeveloped financial markets. This is an important finding that increases the role and the responsibility of the DC pension fund regulatory body with respect of the occurrence of speculative market episodes.

Hypothesis 3: *The introduction of the mandatory DC pension fund to a closed illiquid financial market increases significantly the probability that the economy becomes dynamically inefficient and that it requires a bubbly asset to be introduced to overcome the potential issue of productive capital over-accumulation.*

This hypothesis is treated by the second long-term co-integrating relationship in both estimated VEC models in Chapter 4 and in Chapter 5. By looking at the second co-integrating relationship of the first VEC Model in Chapter 4, we find that the increase of productive investments by the DC pension fund produces decrease in the dynamic efficiency of the financial market. This is the expected theoretical consequence of productive capital over-accumulation, and it confirms our third hypothesis; namely, that the introduction of DC pension funds strictly increases the probability that the financial market becomes dynamically inefficient. Moreover, in the first VEC Model in Chapter 4, we also find that the "bubbly" asset has the opposite impact, crowding out part of the productive investments and thereby solving the problem of productive capital over-accumulation. Although this theory-consistent distinction between the productive and "bubbly" investments of the DC pension funds is not preserved in the second VEC Model in Chapter 5, because we used a single investment variable, we again find that the investment variable has a negative interaction with the dynamic efficiency of the financial market. We conclude that the long-term equilibrium relationships in the two estimated VEC Models, in Chapters 4 and 5, suggest that the introduction of DC pension funds intensify productive capital over-accumulation and increase the probability that the financial market becomes less dynamically efficient or, ultimately, dynamically inefficient. Such dynamic inefficiency of the financial market is simulative for the occurrence of rational asset bubbles.

Hypothesis 4: *The increase of the value of the bubbly assets, consequent upon the introduction of DC pension funds, would be relatively much more dramatic and unsustainable in the case when the starting point is a dynamically efficient economy.*

Here, the idea is that the DC Pension Funds increase saving– changing behaviour by capturing a substantial part of the income of the young generation – and, hence, investment. The consequent over accumulation of productive assets leads – via diminishing returns – to the financial market becoming dynamically inefficient. In turn, this favours investment in the bubbly asset, thereby partially crowding out productive investment. Finally, this raises the rate of return on productive assets and – through the non-arbitrage condition – increases the rate return on the bubbly asset and thus incentivises increasing investment in the bubbly asset and inaugurating an asset price bubble. The corollary is that the greater the intervention of DC Pension Funds, the more dramatic and the less sustainable the bubble. In particular, the bubble will be vulnerable to any movement back towards dynamic efficiency in the financial market.

This hypothesis finds support with our second VEC Model presented in Chapter 5, where we model the impact of the DC pension funds with the Croatian data set using only one investment variable, the DC pension fund investments into the Croatian and SEE equity focused open-end mutual funds. In this second VEC model, using Impulse Response Analysis, we find that the negative shocks introduced on the dynamic efficiency of the Croatian financial market translate into positive relative valuation levels of the Croatian equity bubble. This means that increased dynamic efficiency (decreased dynamic inefficiency) negatively affects the value of the bubbly assets, which also means that if the starting point is a dynamically inefficient financial market economy, and if it changes to becoming dynamically efficient, or at least more dynamically efficient, this could trigger lower values of the bubbly asset. This can clearly be seen in panel [b] of Figure 5.6.3a, which confirms the negative response of the relative value of the Croatian Equity bubble caused by a positive unit shock to the dynamic efficiency of the Croatian financial market. By implication – given that the empirical model is linear – this also works the other way round; i.e., reduced dynamic efficiency (increased dynamic inefficiency) triggers higher values of the bubbly asset.

Moreover, based on Figure 5.6.3c panel [b], a positive shock introduced on the dynamic efficiency of the Croatian financial market also triggers lower DC pension fund investments in the bubbly asset through the Open End Mutual Funds. As explained above, this also works the other way round, such that an increase in dynamic inefficiency stimulates the DC pension funds to invest more in the bubbly asset. As seen from the analysis of the first co-integrating relationship presented in Figure 5.6.1, this may be one of the main drivers to higher valuation levels of the bubbly asset. Such mechanisms initiated by the decreased dynamic efficiency of the Croatian financial market stimulate increase of the relative value of the Croatian equity bubble.

Finally, if we analyse the long-term relationship presented by the second co-integrating relationship of our VEC Model presented in Chapter 5, describing the long-term relationship between the dynamic efficiency of the Croatian financial market and the DC pension fund bubbly asset investments, and depicted in Figure 5.6.2, we find that without the Croatian DC pension funds, the Croatian financial market could have remained dynamically efficient even in the period before the Global Financial Crisis. This evidence of a long-run relationship between the introduction of the DC pension funds and the dynamic efficiency of the Croatian financial market is consistent with the implication of our theoretical model that the introduction of the DC pension funds could have shifted the overall Croatian financial market savings attitude, thereby changing the state of the financial market from being dynamically efficient into being dynamically inefficient (Figure 5.6.2).If the extent of such a change in the state of dynamic efficiency was large enough, as suggested by our empirical analysis, ceteris paribus, such a change might have made the Croatian equity market become a fertile environment for higher speculative equity valuations, additionally stimulated by the higher DC pension fund investments in the bubbly Croatian and SEE equity focused open-end mutual funds.

This leads to the conclusion that introducing massive DC pension schemes and so significantly changing savings behaviour on underdeveloped and illiquid financial markets, which might inherently be dynamically efficient, might create a market environment for aggressive adoption and valuation of potentially bubbly assets.

Hypothesis 5: *The crash of the bubbly asset could result from a sudden change of financial market dynamic efficiency caused either by a termination of the DC pension scheme or a shift in the inter-temporal consumption preferences of individual investors affected by some domestic or foreign risk event such as the Global Financial Crisis.*

Finally, in order to test our fifth hypothesis, in both VEC Models we introduced a shift dummy variable representing a structural break in the co-integrating relationships. The estimated impact of the structural shift at the time of the Global Financial Crisis was the same in both VEC Models. Namely, from the first co-integrating relationship, the occurrence of the Global Financial Crisis caused highly negative pressure on the pricing levels of the bubbly asset in both our models by making the bubbly asset overpriced. This is especially intensive and visible in our simpler VEC Model presented in Chapter 5. From the second co-integrating relationship in both VEC Models, the Global Financial Crisis produced also a sudden and significant positive shift in the dynamic efficiency of the Croatian financial market. This in addition intensified the dynamics of the crash of the bubble, making further investment in the bubbly Croatian and SEE equity focused open-end mutual funds irrational.

6.3. Final thoughts and directions for further research

By augmenting the canonical OLG model of rational asset bubbles of Tirole (1985) with influential DC pension fund investors, and simulating their impact on the underdeveloped financial markets, we have contributed to theoretical debate on the consequences of the pension reform advocated by the WB in the 1990s. Moreover, we report significant empirical evidence consistent with the theoretically simulated outcomes of such reforms, which introduced influential DC pension funds onto small and underdeveloped financial markets. This thesis shows that the financial innovation and market deepening caused by the introduction of the DC pension funds, especially when introduced into illiquid and underdeveloped financial markets, could cause the rise of financial market asset bubbles that, upon their collapse, may have high costs on the real economy and on the pension reform process. We believe that further research has to bring this debate to a higher level with empirical and theoretical work focused on the impact of fully-funded pension schemes on the global financial market. We have seen that the institutional investors and especially pension funds are becoming the leading investment participants on global financial markets (Allen, 2001). Such a position makes them an inevitable feature that must be considered when modelling market valuation outcomes not only on the financial markets of the developing and transition economies, but also on global financial markets.

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APPENDICES

APPENDIX II - Derivation of the OLG model economy with rational asset bubbles and DC pension funds

In this Appendix, we represent the solution for the augmented OLG model with DC pension fund. There are two Equilibrium Law of Motion (ELM's) derived from our OLG model economy, which govern the dynamics of the financial market. First, is the ELM for the productive capital investment asset, and second is the ELM for the speculative or bubbly investment instrument. To begin with, we first need to define those two saving assets:

I) Saving Assets (Productive Capital Investments and Bubbly Asset)

a. **Productive Capital Investment** K_t, and Productive Capital per young agent $k_t = \frac{K_t}{N_t}$.

Resources invested today in productive capital become capital investments in the next period k_{t+1}, bringing a financial interest as a return. Assuming a Constant Returns on Scale (CRS) production function, capital Investments yields return R at next period t+1 $R_{t+1} = f'(k_{t+1}) + 1 - \delta$, where $f'(k_{t+1})$ is the marginal product of capital per young agent, and δ is the rate of depreciation of Capital (k_{t+1}).

b. **Speculative/Bubbly Asset Investment** with no intrinsic value and constant supply of units of the bubbly asset $B_t = B_{t+1} = B = const.$ where the per young agent value of the bubbly asset b_t equals to:

$b_t = \frac{B\Pi_t}{N_t}$. Here, Π_t represents the price of the unit of the bubbly asset at time t, and N_t represents the number of young agents at t.

II) Consumers' Agents and DC pension Fund

Similar to Tirole's model (Tirole, 1985) this model has Consumer Agents, who rationally decide on their inter-temporal savings and we also added the second savings market participant, The DC Pension Fund.

a) **Consumer agents**, they live two periods and two generations overlap each other. When young, they consume "c_{1t} "and when old "c_{2t+1}". They are also endowed with a one unit of labor when young, which they transform in to wage "w_t". Part of it is consumed when they are young at t, part of it is taxed by the DC pension fund at an absolute amount "τ", and the rest they invest in the two available assets in order to provide consumption and utility in the second period. Their utility maximization gives the following results (following Tirole, 1985, augmented by the pension tax "τ" and the price of the bubbly asset "∏"):

$$\max U\{c_{1t}, c_{2t+1}\} s.t. \left\{ \begin{array}{c} w_t = c_{1t} + \tau_t + k_{t+1} + b_t \\ R_{t+1}k_{t+1} + \frac{\Pi_{t+1}}{\Pi_t}[b_t + \tau_t] = c_{2t+1} \end{array} \right\}$$

$$\max \Lambda = U(c_{1t}, c_{2t+1}) + \lambda_1[w_t - c_{1t} - \tau_t - k_{t+1} - b_t] + \lambda_2\left[R_{t+1}k_{t+1} + \frac{\Pi_{t+1}}{\Pi_t}[b_t + \tau_t] - c_{2t+1}\right]$$

$$w.r.t : \{c_{1t}, c_{2t+1}, k_{t+1}, b_t\}$$

First Order Conditions (FOC):

$$c_{1t} : U_1(.) = \lambda_1$$
$$c_{2t+1} : U_2(.) = \lambda_2$$
$$k_{t+1} : -\lambda_1 + \lambda_2 R_{t+1} = 0$$
$$b_t : -\lambda_1 + \lambda_2 \frac{\Pi_{t+1}}{\Pi_t} = 0$$

Solving consumers utility maximization problem, we arrive to the interior solution condition called the NON-ARBITRAGE CONDITION, which would play its role in the markets for productive and speculative/bubbly investments k_{t+1} and b_t.

$$R_{t+1} = \frac{\Pi_{t+1}}{\Pi_t}$$ - NON ARBIRTAGE CONDITION

b) DC Pension Funds

We introduced a DC pension fund in the model, which by law, taxes every young agent the amount τ_t at time t. It immediately invests the total collected amount in the bubbly asset by the rule, and provides the amount $\tau_t \frac{\Pi_{t+1}}{\Pi_t}$ as pension to today's young agent in the next period by selling previously purchased amounts of the bubbly asset units at the current price.

The impact of the existence of the pension fund on the financial market has two sides, direct and indirect. The Direct effect is described by the investment formula which directly impacts the market of the bubbly asset. By simplicity the DC pension fund invests only in the bubbly asset. The Indirect effect has its effect through capturing part of individual agent's first period gross income and adding to the second period income, which affects its saving decision of each individual both for the bubbly and productive investment assets.

Now that we described the agents and the institution in the model, we try to define the dynamics of the two in the market, by equalling their supply with the demand:

III) Financial market (dynamics of productive capital and speculative/bubbly asset)

a) Evolution of the value of the Bubble Asset b_t

$b_t = \frac{\{B\}\Pi_t}{N_t}$, here the value of the bubbly asset per young agent b_t equals the supplied amount of units of bubbly asset B which is constant, multiplied by their current price Π_t. The factor which would define the value of the bubble per young agent will be its price Π_t. To get to the Equilibrium Law of Motion of the value of the bubbly asset per young agent, we first should apply the effect of the DC pension fund investments and disinvestments at each period. The DC Pension fund buys the number of units $\frac{\tau}{\Pi_t}N_t$ (negative in sign because it lowers market supply) and sells $\frac{\tau}{\Pi_{t-1}}N_{t-1}$

$$b_t = \frac{\left\{B - \frac{\tau}{\Pi_t}N_t + \frac{\tau}{\Pi_{t-1}}N_{t-1}\right\}\Pi_t}{N_t}$$

$$b_t = \frac{\left\{B + \frac{\tau}{\Pi_{t-1}}N_{t-1} - \frac{\tau}{\Pi_t}N_{t-1}(1+n)\right\}\Pi_t}{N_t}$$

$$b_t = \frac{B\Pi_t + \tau\frac{\Pi_t}{\Pi_{t-1}}N_{t-1} - \tau N_{t-1}(1+n)}{N_t}$$

Now, we plug-in the Consumer agent's solution, i.e. the Non-Arbitrage condition, and we get:

$$b_t = \frac{B\Pi_t + \tau R_t N_{t-1} - \tau N_{t-1}(1+n)}{N_t}$$

$$b_t = \frac{B\Pi_t}{N_t} + \left\{\frac{\tau N_{t-1}\left(R_t - (1+n)\right)}{N_{t-1}(1+n)}\right\}$$

$$b_t = \frac{B\Pi_t}{N_t} + \left\{\frac{\tau\left(R_t - (1+n)\right)}{(1+n)}\right\}$$, from here for the two consecutive periods we have:

$$\begin{cases} b_t = \dfrac{B\Pi_t}{N_t} + \tau\dfrac{\left[R_t - (1+n)\right]}{1+n} \\ b_{t+1} = \dfrac{B\Pi_{t+1}}{N_{t+1}} + \tau\dfrac{\left[R_{t+1} - (1+n)\right]}{1+n} \end{cases}$$

From this system, the dynamics of the price of the Bubbly Asset could be expressed as:

$$\begin{cases} \Pi_t = \dfrac{N_t}{B}\left\{b_t - \tau\dfrac{\left[R_t - (1+n)\right]}{1+n}\right\} \\ \Pi_{t+1} = \dfrac{N_t(1+n)}{B}\left\{b_{t+1} - \tau\dfrac{\left[R_{t+1} - (1+n)\right]}{1+n}\right\} \end{cases}$$

Finally, dividing the two equations, the price change of the bubbly asset could be represented by:

$$\frac{\Pi_{t+1}}{\Pi_t}=(1+n)\left\{\frac{b_{t+1}-\tau\dfrac{[R_{t+1}-(1+n)]}{1+n}}{b_t-\tau\dfrac{[R_t-(1+n)]}{1+n}}\right\}$$

Once again, since the consumer agent could invest into both investment assets, the Non-Arbitrage condition has to hold $\frac{\Pi_{t+1}}{\Pi_t}=R_{t+1}$, and so we transform this equation to:

$$R_{t+1}=(1+n)\left\{\frac{b_{t+1}-\tau\dfrac{[R_{t+1}-(1+n)]}{1+n}}{b_t-\tau\dfrac{[R_t-(1+n)]}{1+n}}\right\}$$

$$R_{t+1}=(1+n)\left\{\frac{b_{t+1}(1+n)-\tau[R_{t+1}-(1+n)]}{b_t(1+n)-\tau[R_t-(1+n)]}\right\}$$

$$R_{t+1}\{b_t(1+n)-\tau[R_t-(1+n)]\}=(1+n)\{b_{t+1}(1+n)-\tau[R_{t+1}(1+n)]\}$$

$$R_{t+1}\{b_t(1+n)-\tau[R_t-(1+n)]\}+(1+n)\tau[R_{t+1}-(1+n)]=(1+n)^2 b_{t+1}$$

$$b_{t+1}=\frac{R_{t+1}[b_t(1+n)+\tau[R_t-(1+n)]]}{(1+n)^2}+\frac{\tau[R_{t+1}-(1+n)]}{1+n}$$

Finally, exchanging $R_{t+1}=f'(k_{t+1})+1-\delta$ in this equation, we get the Equilibrium Law of Motion (ELM) equation for the dynamics of the bubbly asset in the economy:

$$b_{t+1}=\frac{[f'(k_{t+1})+1-\delta][b_t(1+n)+\tau[(f'(k_t)+1-\delta)-(1+n)]]}{(1+n)^2}+\frac{\tau[(f'(k_{t+1})+1-\delta-(1+n)]}{1+n}$$

Now in order to find the locus of steady state points, we erase the time subscripts and solve the equation. Erasing subscripts, we get the following:

$$b=\frac{[f'(k)+1-\delta][b(1+n)+\tau[(f'(k)+1-\delta)-(1+n)]]}{(1+n)^2}+\frac{\tau[(f'(k)+1-\delta-(1+n)]}{1+n}$$

Or.

$$b = \frac{R\left[b(1+n) + \tau\left[R-(1+n)\right]\right]}{(1+n)^2} + \frac{\tau\left[R-(1+n)\right]}{1+n}$$

We see that only when the interest rate R is equal to 1+n, we do get steady state solutions for b:

$$b = \frac{(1+n)\left[b(1+n) + \tau\left[(1+n)-(1+n)\right]\right]}{(1+n)^2} + \frac{\tau\left[(1+n)-(1+n)\right]}{1+n}$$

$$b = \frac{(1+n)\left[b(1+n) + \tau \bullet 0\right]}{(1+n)^2} + \frac{\tau \bullet 0}{1+n}$$

$$b = \frac{b(1+n)^2}{(1+n)^2} = b$$

We conclude that for $R_t=R_{t+1}=R=(1+n)$ =>$b_t=b_{t+1}=b$, which determines a vertical loci of SS point from k=f'(n)

b) Evolution of the value of the Productive Capital k_t

We define the savings market as place where the savings demand of every young agent

$N_t S_t^*(w(k_t)-\tau;\tau;\ f'(k_t)+1-\delta)$, meets with the supply of available investment assets

$K_{t+1} + b_t N_t$, and the savings market clears:

$$N_t S_t^*(w(k_t)-\tau;\tau;\ f'(k_t)+1-\delta) = K_{t+1} + b_t N_t$$

$N_t S_t^*(w(k_t)-\tau;\tau;\ f'(k_t)+1-\delta) = K_{t+1} + b_t N_t$, transforming in per young agent terms we get

$$S_t^*(w(k_t)-\tau;\tau;\ f'(k_t)+1-\delta) = k_{t+1}(1+n) + b_t$$

And from here the Equilibrium Law of Motion (ELM) for the productive capital asset:

$$k_{t+1} = \frac{S_t^*(w(k_t)-\tau;\tau;\ f'(k_t)+1-\delta) - b_t}{(1+n)}$$

APPENDIX III.1 Data set

Date	dummy1	dummy2	dummy3	dummy4	dummyMo	PFInvTotalM	PFInvDirect	PFInvOIFM	OMF_AUM	LTinterestC	Subscriberf	DYNEF	B_Croatia	B_US
Oct-97	0	0	0	0	0				100	8.84	0.3	8.54	14.8838	12.2988
Nov-97	0	0	0	0	0				100	8.63	0.3	8.33	7.80169	13.5536
Dec-97	0	0	0	0	0	0	0	0	100	9.1	0.3	8.8	15.1233	14.1937
Jan-98	0	0	0	0	0	7760.91	7569.357	191.5531	100	9.54	0.3	9.24	3.99428	21.6949
Feb-98	0	0	0	0	1	15196.19	14821.12	375.069	100	8.59	0.3	8.29	15.621	27.2072
Mar-98	0	0	0	0	0	21980.2	21437.69	542.5104	100	8.93	0.3	8.63	14.8539	27.7959
Apr-98	0	0	0	0	0	27787.31	27101.47	685.8404	100	9.94	0.3	9.639999	3.30514	24.8397
May-98	0	0	0	0	0	32291.9	31494.88	797.0215	100	9.57	0.3	9.27	-9.56054	29.1961
Jun-98	0	0	0	0	0	35168.32	34300.3	868.0167	100	9.51	0.3	9.21	-9.56164	27.0213
Jul-98	0	0	0	0	0	36090.95	35200.16	890.7888	100	9.55	0.3	9.25	-14.8502	7.93221
Aug-98	0	0	0	0	0	34734.14	33876.84	857.3005	100	9.8	0.3	9.5	-51.4306	14.0677
Sep-98	0	0	0	0	0	30772.28	30012.77	759.5146	100	9.88	0.3	9.58	-42.3587	22.5863
Oct-98	0	0	0	0	0	23879.72	23290.33	589.3941	100	9.16	0.3	8.86	-39.5435	29.163
Nov-98	0	0	0	0	0	13730.84	13391.94	338.9016	100	9.07	0.3	8.77	-30.4122	35.7429
Dec-98	0	0	0	0	0	0	0	0	100	7.73	0.3	7.43	-31.2313	40.5865
Jan-99	0	0	0	0	0	-17312.8	-16885.5	-427.311	100	8.66	0.3	8.36	-26.8115	35.3554
Feb-99	0	0	0	0	1	-36905.03	-35994.1	-910.882	100	8.73	0.3	8.429999	-32.0015	39.8942
Mar-99	0	0	0	0	0	-57148.52	-55738	-1410.53	100	8.92	0.3	8.62	-35.3192	44.4705
Apr-99	0	0	0	0	0	-76415.12	-74529.1	-1886.06	100	8.53	0.3	8.23	-36.5757	40.1567
May-99	0	0	0	0	0	-93076.65	-90779.4	-2297.3	100	9.14	0.3	8.84	-34.3785	47.0492
Jun-99	0	0	0	0	0	-105505	-102901	-2604.05	100	8.66	0.3	8.36	-36.2797	41.5826
Jul-99	0	0	0	0	0	-112071.9	-109306	-2766.13	100	8.6	0.3	8.3	-40.3038	39.9554
Aug-99	0	0	0	0	0	-111149.3	-108406	-2743.36	100	8.88	0.3	8.58	-42.4854	35.2465
Sep-99	0	0	0	0	0	-101108.9	-98613.4	-2495.55	100	8.89	0.3	8.59	-55.7304	42.9552
Oct-99	0	0	0	0	0	-80322.7	-78340.2	-1982.51	100	9.03	0.3	8.73	-53.4748	44.9242
Nov-99	0	0	0	0	0	-47162.45	-45998.4	-1164.05	100	8.48	0.3	8.179999	-44.1725	52.5157
Dec-99	0	0	0	0	0	0	0	0	100	8.87	0.3	8.57	-38.3791	44.0087
Jan-00	0	0	0	0	0	61490.29	59972.6	1517.69	100	8.88	0.3	8.58	-33.0937	40.3919
Feb-00	0	0	0	0	1	132423.9	129155.5	3268.458	100	8.71	0.3	8.41	-27.3408	53.1878
Mar-00	0	0	0	0	0	206613.9	201514.3	5099.598	100	8.64	0.3	8.34	-18.791	47.7193
Apr-00	0	0	0	0	0	277873.2	271014.8	6858.404	100	8.55	0.3	8.25	-29.075	43.7549
May-00	0	0	0	0	0	340014.7	331622.5	8392.168	100	8.48	0.3	8.179999	-25.7243	46.4584
Jun-00	0	0	0	0	0	386851.5	377303.3	9548.184	100	7.54	0.3	7.24	-29.2947	43.6624
Jul-00	0	0	0	0	0	412196.6	402022.8	10173.75	100	7.47	0.3	7.17	-33.3473	51.9579
Aug-00	0	0	0	0	0	409862.9	399746.8	10116.15	100	7.61	0.3	7.31	-30.7664	43.431
Sep-00	0	0	0	0	0	373663.4	364440.7	9222.678	100	7.37	0.3	7.07	-31.7093	42.3255
Oct-00	0	0	0	0	0	297411.1	290070.5	7340.635	100	7.18	0.3	6.88	-30.0365	30.5678
Nov-00	0	0	0	0	0	174919	170601.7	4317.312	100	7.38	0.3	7.08	-26.0294	30.7357
Dec-00	0	0	0	0	0	0	0	0	100	7.2	0.3	6.9	-27.7445	34.8921
Jan-01	0	0	0	0	0	-228648.3	-223005	-5643.45	100	7.17	0.3	6.87	-28.4806	22.1071
Feb-01	0	0	0	0	1	-492790.7	-480628	-12163	100	7.36	0.3	7.06	-22.9832	13.9549
Mar-01	0	0	0	0	0	-769307	-750319	-18987.9	100	7.07	0.3	6.77	-25.6735	22.3737
Apr-01	0	0	0	0	0	-1035077	-1009530	-25547.6	100	7.15	0.3	6.85	-22.5087	22.6622
May-01	0	0	0	0	0	-1266982	-1235711	-31271.4	100	6.72	0.3	6.42	-26.3549	19.267
Jun-01	0	0	0	0	0	-1441901	-1406312	-35588.7	100	6.26	0.3	5.96	-22.8671	17.6467
Jul-01	0	0	0	0	0	-1536715	-1498786	-37928.9	100	6.07	0.3	5.77	-21.5249	9.78868
Aug-01	0	0	0	0	0	-1528302	-1490581	-37721.2	100	6.28	0.3	5.98	-22.0468	0.528004
Sep-01	0	0	0	0	0	-1393545	-1359150	-34395.2	100	6.44	0.3	6.14	-28.1531	2.05555
Oct-01	0	0	0	0	0	-1109322	-1081942	-27380	100	6.24	0.3	5.94	-27.9572	9.41559
Nov-01	0	0	0	0	0	-652513.4	-636408	-16105.2	100	5.93	0.3	5.63	-23.1954	9.93163
Dec-01	0	0	0	0	0	0	0	0	100	5.68	0.3	5.38	-22.442	7.91353
Jan-02	1	0	0	0	0	859587.1	838530.9	21056.1	100	5.79	10	-4.21	-13.1638	5.37457
Feb-02	1	0	0	0	1	1890610	1845227	45383.34	100	5.47	10	-4.53	-11.5875	8.93869
Mar-02	1	0	0	0	0	3045681	2974829	70851.87	100	5.06	10	-4.94	-6.16309	1.96121
Apr-02	1	0	0	0	0	4277410	4182078	95331.81	100	4.58	10	-5.42	-10.3826	0.752717
May-02	1	0	0	0	0	5538408	5421715	116693.3	100	4.6	10	-5.4	-11.3837	-6.80796
Jun-02	1	0	0	0	0	6781287	6648480	132806.6	100	4.51	10	-5.49	-16.3257	-14.5052
Jul-02	1	0	0	0	0	7958657	7817115	141541.6	100	4.25	10	-5.75	-22.1719	-14.4216
Aug-02	1	0	0	0	0	9023129	8882360	140768.7	100	4.05	10	-5.95	-21.1579	-24.132
Sep-02	1	0	0	0	0	9927315	9798957	128358	100	4.18	10	-5.82	-21.8371	-17.891
Oct-02	1	0	0	0	0	1.06E+07	1.05E+07	102179.5	100	3.75	10	-6.25	-23.4688	-13.5383
Nov-02	1	0	0	0	0	1.11E+07	1.10E+07	60103.46	100	3.52	10	-6.48	-19.2352	-19.0655
Dec-02	1	0	0	0	0	1.12E+07	1.12E+07	0	100	3.64	10	-6.36	-19.5015	-21.5842
Jan-03	1	0	0	0	0	1.10E+07	1.11E+07	-78581	100	3.46	9.65313	-6.19313	-25.3708	-23.2101
Feb-03	1	0	0	0	1	1.05E+07	1.07E+07	-169370	100	3.51	9.65313	-6.14313	-28.9675	-22.8611
Mar-03	1	0	0	0	0	9818082	1.01E+07	-264420	100	3.36	9.65313	-6.29313	-29.2527	-16.9236
Apr-03	1	0	0	0	0	8938610	9294390	-355780	100	3.16	9.65313	-6.49313	-25.5729	-13.0228
May-03	1	0	0	0	0	7951245	8386747	-435502	100	3.13	9.65313	-6.52313	-24.3886	-12.3669
Jun-03	1	0	0	0	0	6916474	7412111	-495638	100	3.37	9.65313	-6.28313	-23.4732	-11.4163
Jul-03	1	1	0	0	0	5894783	6423021	-528238	100	3.28	9.65313	-6.37313	-25.5889	-10.3074
Aug-03	1	1	0	0	0	4946661	5472015	-525354	100	3.83	9.65313	-5.82313	-24.064	-11.8426
Sep-03	1	1	0	0	0	4132595	4611632	-479037	100	5.25	9.65313	-4.40313	-29.506	-7.48163
Oct-03	1	1	0	0	0	3513071	3894409	-381338	100	4.71	9.65313	-4.94313	-26.9242	-7.30478
Nov-03	1	1	0	0	0	3148577	3372886	-224309	100	3.57	9.65313	-6.08313	-24.503	-3.10101
Dec-03	1	1	0	0	0	3099600	3099600	0	100	4.46	9.65313	-5.19313	-25.1894	-1.93232
Jan-04	1	1	0	0	0	3402937	3109669	293267.8	100	4.96	10.6578	-5.69782	-24.9296	-1.24134
Feb-04	1	1	0	0	1	4000620	3368521	632098.4	100	4.94	10.6578	-5.71782	-26.7153	-3.34992
Mar-04	1	1	0	0	0	4810991	3824164	986826.5	100	5.36	10.6578	-5.29782	-28.2377	-5.45257
Apr-04	1	1	0	0	0	5752393	4424606	1327787	100	4.31	10.6578	-6.34782	-27.8286	-4.79085
May-04	1	1	0	0	0	6743166	5117852	1625315	100	3.42	10.6578	-7.23782	-30.6985	-3.56262
Jun-04	1	1	0	0	0	7701654	5851911	1849744	100	3.34	10.6578	-7.31782	-30.0856	-7.37076
Jul-04	1	1	0	0	0	8546198	6574789	1971409	100	5	10.6578	-5.65782	-28.0891	-7.65589
Aug-04	1	1	0	0	0	9195141	7234495	1960646	100	6.25	10.6578	-4.40782	-26.2371	-7.28752
Sep-04	1	1	0	0	0	9566824	7779035	1787789	100	5.93	10.6578	-4.72782	-17.1029	-6.48617
Oct-04	1	1	0	0	0	9579589	8156417	1423172	100	4.66	10.6578	-5.99782	-13.9801	-3.38873
Nov-04	1	1	0	0	0	9151778	8314647	837131.1	100	3.36	10.6578	-7.29782	-5.7191	-0.7757
Dec-04	1	1	0	0	0	8201734	8201734	0	100	4.12	10.6578	-6.53782	-8.39965	-3.78938
Jan-05	1	1	0	0	0	6692990	7784482	-1091492		4.13	9.25779	-5.12779	11.7674	-2.47913
Feb-05	1	1	0	0	1	4769852	7104888	-2335037		4.05	9.25779	-5.20779	20.3716	-4.83709

Date	dummy1	dummy2	dummy3	dummy4	dummyMo	PFInvTotalM	PFInvDirect	PFInvOIFM	OMF_AUM	LTinterestC	Subscriber	DYNEF	B_Croatia	B_US
Mar-05	1	1	0	0	0	2621815	6223748	-3601932		3.88	9.25779	-5.37779	5.67191	-7.22938
Apr-05	1	1	0	0	0	438379.1	5201856	-4763477		3.47	9.25779	-5.78779	1.86833	-4.93871
May-05	1	1	0	0	0	-1590960	4100008	-5690968		3.34	9.25779	-5.91779	-1.79147	-5.43524
Jun-05	1	1	0	0	0	-3276706	2978998	-6255704		3.55	9.25779	-5.70779	2.2763	-2.51931
Jul-05	1	1	0	0	0	-4429361	1899621	-6328982		3.92	9.25779	-5.33779	3.24086	-4.08845
Aug-05	1	1	0	0	0	-4859427	922673.2	-5782100		4.79	9.25779	-4.46779	7.63874	-3.89579
Sep-05	1	1	0	0	0	-4377408	108948.8	-4486357		3.93	9.25779	-5.32779	12.6189	-6.06161
Oct-05	1	1	0	0	0	-2793806	-480757	-2313049		3.66	9.25779	-5.59779	10.925	-3.22873
Nov-05	1	1	0	0	0	80875.38	-785650	866524.6		3.6	9.25779	-5.65779	10.022	-3.78833
Dec-05	1	1	0	0	0	4436134	-744933	5181067	231.132	3.37	9.25779	-5.88779	7.65503	-1.81285
Jan-06	1	1	0	0	0	1.04E+07	-302974	1.07E+07		3.2	7.84564	-4.64564	11.8016	-2.23876
Feb-06	1	1	0	0	1	1.80E+07	575225.3	1.75E+07		3.38	7.84564	-4.46564	13.9518	-1.62509
Mar-06	1	1	0	0	0	2.72E+07	1919498	2.53E+07		3.42	7.84564	-4.42564	28.0813	-0.90156
Apr-06	1	1	0	0	0	3.79E+07	3759681	3.41E+07		3.34	7.84564	-4.50564	31.3085	-4.41874
May-06	1	1	0	0	0	5.01E+07	6125609	4.39E+07		3.23	7.84564	-4.61564	29.2982	-4.8596
Jun-06	1	1	0	0	0	6.36E+07	9047117	5.46E+07		3.28	7.84564	-4.56564	40.6705	-4.76224
Jul-06	1	1	0	0	0	7.86E+07	1.26E+07	6.60E+07		3.16	7.84564	-4.68564	47.609	-3.12768
Aug-06	1	1	0	0	0	9.48E+07	1.67E+07	7.81E+07		3.12	7.84564	-4.72564	57.9917	-1.14584
Sep-06	1	1	0	0	0	1.12E+08	2.14E+07	9.09E+07		3.46	7.84564	-4.38564	59.9285	1.56164
Oct-06	1	1	0	0	0	1.31E+08	2.69E+07	1.04E+08		3.53	7.84564	-4.31564	63.3825	2.82338
Nov-06	1	1	0	0	0	1.51E+08	3.30E+07	1.18E+08		3.1	7.84564	-4.74564	61.8843	3.70806
Dec-06	1	1	0	0	0	1.72E+08	3.99E+07	1.32E+08	1375.68	2.98	7.84564	-4.86564	58.2941	4.75109
Jan-07	1	1	0	0	0	1.94E+08	4.75E+07	1.46E+08	1398.17	3.56	8.62193	-5.06194	84.1299	2.05993
Feb-07	1	1	0	0	1	2.16E+08	5.59E+07	1.61E+08	1527.96	3.46	8.17591	-4.71591	84.2877	2.67489
Mar-07	1	1	0	0	0	2.39E+08	6.48E+07	1.74E+08	1630.8	3.49	7.69047	-4.20048	104.045	6.70198
Apr-07	1	1	0	0	0	2.60E+08	7.43E+07	1.86E+08	1670.61	3.58	5.40612	-1.82613	120.38	9.74703
May-07	1	1	0	0	0	2.80E+08	8.43E+07	1.96E+08	1669.75	3.75	6.32435	-2.57435	135.902	7.3746
Jun-07	1	1	0	0	0	2.98E+08	9.46E+07	2.03E+08	1637.62	4.55	6.08563	-1.53563	129.195	3.7785
Jul-07	1	1	0	0	0	3.13E+08	1.05E+08	2.07E+08	1590.75	4.95	6.41371	-1.46372	137.193	4.94984
Aug-07	1	1	0	0	0	3.24E+08	1.16E+08	2.08E+08	1503.72	4.25	6.08681	-1.83681	122.455	8.53745
Sep-07	1	1	0	0	0	3.32E+08	1.27E+08	2.05E+08	1408.28	4.82	7.70663	-2.88663	133.738	9.9753
Oct-07	1	1	0	0	0	3.35E+08	1.39E+08	1.97E+08	1360.71	5.65	7.91186	-2.26186	136.092	4.96872
Nov-07	1	1	0	0	0	3.33E+08	1.50E+08	1.83E+08	1083.22	5.94	9.79835	-3.85835	113.128	3.90201
Dec-07	1	1	0	0	0	3.25E+08	1.61E+08	1.64E+08	1141.59	5.42	8.30486	-2.88486	138.091	-2.60367
Jan-08	1	1	1	0	0	3.11E+08	1.72E+08	1.38E+08	991.404	5.79	7.80047	-2.01047	102.982	-6.13427
Feb-08	1	1	1	0	1	2.91E+08	1.83E+08	1.08E+08	822.379	5.01	8.03177	-3.02177	89.7297	-6.83736
Mar-08	1	1	1	0	0	2.66E+08	1.93E+08	7.32E+07	695.914	4.48	7.76329	-3.28329	71.2118	-2.55783
Apr-08	1	1	1	1	0	2.39E+08	2.02E+08	3.61E+07	600.983	4.12	9.52513	-5.40513	67.7501	-1.66891
May-08	1	1	1	1	0	2.08E+08	2.10E+08	-2356313	541.397	4.2	6.18834	-1.98834	75.2204	-10.2594
Jun-08	1	1	1	1	0	1.76E+08	2.17E+08	-4.08E+07	501.956	4.16	8.23155	-4.07155	57.6229	-10.9792
Jul-08	1	1	1	1	0	1.44E+08	2.22E+08	-7.81E+07	422.205	4.23	6.2418	-2.0118	60.444	-9.72636
Aug-08	1	1	1	1	0	1.12E+08	2.25E+08	-1.13E+08	377.606	4.01	5.6532	-1.6432	54.7635	-17.7694
Sep-08	1	1	1	1	0	8.17E+07	2.26E+08	-1.44E+08	309.248	5.2	7.04557	-1.84557	32.9665	-31.5738
Oct-08	1	1	1	1	0	5.40E+07	2.24E+08	-1.70E+08	270.928	7.52	7.11181	0.408188	-2.17333	-36.577
Nov-08	1	1	1	1	0	2.97E+07	2.19E+08	-1.89E+08	267.904	10.24	8.42164	1.818356	-27.9776	-35.9611
Dec-08	1	1	1	1	0	1.01E+07	2.12E+08	-2.02E+08	226.072	5.65	4.7635	0.886502	-22.5179	-41.3365
Jan-09	1	1	1	1	0	-4345490	2.01E+08	-2.05E+08	210.082	7.81	5.44129	2.368709	-24.0355	-47.6872
Feb-09	1	1	1	1	1	-1.38E+07	1.87E+08	-2.01E+08	197.838	11.83	4.91034	6.919664	-37.247	-43.1124
Mar-09	1	1	1	1	0	-1.88E+07	1.71E+08	-1.90E+08	176.605	9.3	4.5388	4.761203	-33.9146	-37.6517
Apr-09	1	1	1	1	0	-2.01E+07	1.54E+08	-1.74E+08	165.084	6.5	3.00834	3.491663	-27.1427	-34.218
May-09	1	1	1	1	0	-1.81E+07	1.35E+08	-1.54E+08	205.299	6	3.68816	2.311841	-1.57116	-34.0803
Jun-09	1	1	1	1	0	-1.36E+07	1.16E+08	-1.30E+08	204.249	6.05	3.43048	2.619516	-12.9457	-29.4141
Jul-09	1	1	1	1	0	-6948422	9.73E+07	-1.04E+08	191.915	6.83	0.899607	5.930393	-13.6292	-27.2723
Aug-09	1	1	1	1	0	1101062	7.88E+07	-7.77E+07	191.508	6.48	2.09672	4.383281	-7.52505	-24.908
Sep-09	1	1	1	1	0	1.00E+07	6.14E+07	-5.14E+07	177.599	6.32	5.24091	1.079094	1.2809	-26.6197
Oct-09	1	1	1	1	0	1.92E+07	4.56E+07	-2.64E+07	166.209	5.05	5.75995	-0.70995	-1.00931	-22.6496
Nov-09	1	1	1	1	0	2.80E+07	3.21E+07	-4017184	165.406	2.69	5.92206	-3.23206	-4.47363	-21.5171
Dec-09	1	1	1	1	0	3.60E+07	2.13E+07	1.48E+07	165.123	2.52	3.71569	-1.19569	-7.25269	-24.6506
Jan-10	1	1	1	1	0	4.26E+07	1.36E+07	2.90E+07	161.287	2.7	3.18891	-0.48891	2.11125	-22.7389
Feb-10	1	1	1	1	1	4.78E+07	8993704	3.88E+07	162.678	2.64	4.29103	-1.65103	-0.77918	-18.4455
Mar-10	1	1	1	1	0	5.18E+07	7132510	4.47E+07	163.939	2.07	3.47162	-1.40162	-0.42559	-17.4932
Apr-10	1	1	1	1	0	5.47E+07	7750196	4.69E+07	168.83	2.15	3.19053	-1.04054	0.568308	-24.4861
May-10	1	1	1	1	0	5.65E+07	1.06E+07	4.60E+07	169.767	2.19	4.16946	-1.97946	-7.44205	-28.7707
Jun-10	1	1	1	1	0	5.75E+07	1.53E+07	4.22E+07	171.811	1.94	3.80555	-1.86555	-13.5559	-24.1232
Jul-10	1	1	1	1	0	5.78E+07	2.17E+07	3.61E+07	170.582	2.04	2.95349	-0.91349	-13.6468	-27.9615
Aug-10	1	1	1	1	0	5.74E+07	2.95E+07	2.79E+07	170.096	1.79	3.21433	-1.42433	-14.1948	-21.9116
Sep-10	1	1	1	1	0	5.65E+07	3.84E+07	1.82E+07	165.838	1.75	3.79346	-2.04346	-11.2179	-19.2985
Oct-10	1	1	1	1	0	5.53E+07	4.81E+07	7224198	168.793	1.95	3.53292	-1.58292	-13.5138	-19.7459
Nov-10	1	1	1	1	0	5.38E+07	5.83E+07	-4518547	169.837	1.91	5.21024	-3.30024	-17.4637	-14.7831
Dec-10	1	1	1	1	0	5.22E+07	6.89E+07	-1.67E+07	151.033	1.93	3.76844	-1.83844	-2.68289	-13.1357
Jan-11	1	1	1	1	0					1.89	3.21218	-1.32218	5.50491	-10.6493
Feb-11	1	1	1	1	1					1.47	3.57755	-2.10755	2.95133	-11.0301
Mar-11	1	1	1	1	0					1.66	3.88656	-2.22656	5.03613	-8.78852
Apr-11	1	1	1	1	0					1.37	2.78553	-1.41553	2.2662	-10.3077
May-11	1	1	1	1	0					1.32	2.03005	-0.71005	4.13925	-12.226
Jun-11	1	1	1	1	0					1.6	2.37079	-0.77079	1.9444	-14.3991
Jul-11	1	1	1	1	0					1.67	2.53065	-0.86065	-0.66754	-19.5305
Aug-11	1	1	1	1	0					2.07	3.41665	-1.34665	-7.058	-25.5541
Sep-11	1	1	1	1	0					2.2	3.235	-1.035	-15.2623	-17.8085
Oct-11	1	1	1	1	0					2.82	4.27371	-1.45371	-15.8021	-18.4951
Nov-11	1	1	1	1	0					2.5	4.8793	-2.3793	-20.5296	-18.0709
Dec-11	1	1	1	1	0					2.46	3.42229	-0.96229	-20.4848	-14.7814
Jan-12	1	1	1	1	0					3.1	3.4564	-0.3564	-21.077	-11.6131
Feb-12	1	1	1	1	1					3.24	3.59073	-0.35073	-18.3391	-9.14165
Mar-12	1	1	1	1	0					2.56	3.12873	-0.56873	-16.2246	-10.1166
Apr-12	1	1	1	1	0					2.09	5.19786	-3.10786	-17.7237	-16.0214
May-12	1	1	1	1	0					1.8	3.65501	-1.85501	-23.7698	-12.9821
Jun-12	1	1	1	1	0					1.85	3.16108	-1.31108	-22.6097	-12.144
Jul-12	1	1	1	1	0					2.12	2.69935	-0.57936	-22.5416	-10.6694
Aug-12	1	1	1	1	0					2.31	2.39094	-0.08094	-23.5055	-8.77081
Sep-12	1	1	1	1	0					2.27	3.3327	-1.0627	-22.0311	-10.8359
Oct-12	1	1	1	1	0					1.73	3.82788	-2.09788	-20.3669	-10.841
Nov-12	1	1	1	1	0					2.17	3.20918	-1.03918	-20.2596	-10.47
Dec-12	1	1	1	1	0					2.08	2.44938	-0.36938	-21.2883	-6.22591
Jan-13	1	1	1	1	0					1.87	2.33996	-0.46996	-14.7648	-5.46087
Feb-13	1	1	1	1	1					1.81	3.21953	-1.40953	-12.3131	-2.26267
Mar-13	1	1	1	1	0					1.65	2.80068	-1.15068		

APPENDIX III.2 Stata program for Cubic-Spline lower to higher frequency interpolation

We here present a modified version of a STATA-program taken from a free online sharing database provided by www.columbiaeconomics.com:

```
mata // This line launches the Mata system inside Stata

X = st_data((1,14),"x") // This pulls in the x quarterly markers data.

Y = st_data((1,14),"y") // This pulls in the quarterly y data we want to interpolate between.

XX = st_data(.,"xx") // This pulls in the xx monthly markers we want to interpolate at.

A = spline3(X,Y) // This generates the cubic spline coefficients matrix, and stores it in A.

B = spline3eval(A,XX) // This performs the interpolation, and store the values in B.

st_store(.,"yy",B) // This pushes the interpolated figures in B back into the yy variable in
Stata.

End
```

APPENDIX III.3 – Data set unit root tests of our endogenous variables in their level and first difference forms using ADF, UR with structural break tests and KPSS

1. Unit root tests of the Croatian equity bubble variable "B_Croatia" in its level form

1.1. Augmented Dickey Fuller test results for "B_Croatia" (lags=4); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      B_Croatia
sample range:             [1998 M3, 2013 M2], T = 180
lagged differences:       4
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -2.1133
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0334       -2.1133
dx(-1)         0.0699        0.9401
dx(-2)         0.1572        2.1181
dx(-3)         0.0970        1.3044
dx(-4)         0.0675        0.9046
RSS            13324.8013

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               15.1923
 p-Value (Chi^2):          0.2311
Ljung & Box:               15.9869
 p-Value (Chi^2):          0.1918

JARQUE-BERA TEST:

test statistic:            91.3276
 p-Value(Chi^2):           0.0000
skewness:                 -0.2736
```

```
kurtosis:                  6.4464
```

```
ADF Test for series:      B_Croatia
sample range:             [1998 M3, 2013 M2], T = 180
lagged differences:       4
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-3.43     -2.86     -2.57
value of test statistic: -2.1077
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0334       -2.1077
dx(-1)         0.0698        0.9365
dx(-2)         0.1570        2.1092
dx(-3)         0.0969        1.2986
dx(-4)         0.0673        0.8994
constant      -0.1017       -0.1558
RSS            13322.9419

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               15.1892
 p-Value (Chi^2):          0.2313
Ljung & Box:               15.9837
 p-Value (Chi^2):          0.1920

JARQUE-BERA TEST:

test statistic:            91.3947
 p-Value(Chi^2):           0.0000
skewness:                 -0.2739
kurtosis:                  6.4476
```

```
ADF Test for series:      B_Croatia
sample range:             [1998 M3, 2013 M2], T = 180
lagged differences:       4
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-3.96     -3.41     -3.13
value of test statistic: -2.2588
regression results:
---------------------------------------
variable      coefficient   t-statistic
```

```
---------------------------------------
 x(-1)        -0.0378       -2.2588
dx(-1)         0.0717        0.9603
dx(-2)         0.1590        2.1330
dx(-3)         0.1003        1.3420
dx(-4)         0.0706        0.9421
constant      -0.0963       -0.1474
trend          0.0109        0.8236
RSS            13270.9108

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               15.3451
 p-Value (Chi^2):          0.2231
Ljung & Box:               16.1499
 p-Value (Chi^2):          0.1845

JARQUE-BERA TEST:

test statistic:            84.9416
 p-Value(Chi^2):           0.0000
skewness:                 -0.2338
kurtosis:                  6.3327
```

1.2. Unit Root tests with structural break for “B_Croatia” (lags=4); with an impulse dummy (December 2007) and with a shift dummy (August 1998); with constant and no trend and with both constant and a trend

```
UR test with Structural Break
Break date search for series:  B_Croatia
sample range:                  [1998 M3, 2013 M2], T = 180
searched range:                [1998 M7, 2012 M12], T = 174
number of lags (1st diff):     4
suggested break date:          2007 M12
```

```
UR Test with structural break for series: B_Croatia
sample range:                  [1998 M3, 2013 M2], T = 180
number of lags (1st diff):     4
value of test statistic:      -2.3645
used break date:               2007 M12
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
```

```
variable      coefficient   t-statistic
---------------------------------------
d(const)       14.3558       24.1659
d(shiftfkt)    32.3548       38.5122
dx(-1)         0.1340        1.8429
dx(-2)         0.0663        0.9054
dx(-3)         0.0657        0.8978
dx(-4)         0.1482        2.0384

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags

Portmanteau:               4.5839
 p-Value (Chi^2):          0.9705
Ljung & Box:               4.8507
 p-Value (Chi^2):          0.9628

JARQUE-BERA TEST:

test statistic:            71.7519
 p-Value(Chi^2):           0.0000
skewness:                 -0.1698
kurtosis:                  6.0830
```

```
sample range:                  [1998 M3, 2013 M2], T = 180
number of lags (1st diff):     4
value of test statistic:      -2.1903
used break date:               2007 M12
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -0.1350       -0.0167
d(const)       14.4370       24.2373
d(shiftfkt)    32.3535       38.4072
dx(-1)         0.1339        1.8418
dx(-2)         0.0661        0.9029
dx(-3)         0.0657        0.8967
dx(-4)         0.1480        2.0357

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
```

```
Hannan-Quinn Criterion:   2
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags

Portmanteau:               4.5376
 p-Value (Chi^2):          0.9717
Ljung & Box:               4.7980
 p-Value (Chi^2):          0.9644

JARQUE-BERA TEST:

test statistic:            74.2149
 p-Value(Chi^2):           0.0000
skewness:                 -0.2125
kurtosis:                  6.1257
```

```
Break date search for series:  B_Croatia
sample range:                  [1998 M3, 2013 M2], T = 180
searched range:                [1998 M7, 2012 M12], T = 174
number of lags (1st diff):     4
suggested break date:          1998 M8
```

```
UR Test with structural break for series: B_Croatia
sample range:                  [1998 M3, 2013 M2], T = 180
number of lags (1st diff):     4
value of test statistic:      -1.9953
used break date:               1998 M8
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       14.5664       23.8488
d(shiftfkt)   -36.8880      -60.3948
dx(-1)         0.0566        0.7693
dx(-2)         0.1842        2.5060
dx(-3)         0.0633        0.8612
dx(-4)         0.0064        0.0865

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               21.6343
 p-Value (Chi^2):          0.0418
Ljung & Box:               22.7723
 p-Value (Chi^2):          0.0297
```

```
JARQUE-BERA TEST:

test statistic:            68.2562
 p-Value(Chi^2):           0.0000
skewness:                 -0.0305
kurtosis:                  6.0246
```

```
UR Test with structural break for series: B_Croatia
sample range:                  [1998 M3, 2013 M2], T = 180
number of lags (1st diff):     4
value of test statistic:      -2.0289
used break date:               1998 M8
shiftfunction:                 shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%        10%
---------------------------------------
1000      -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       0.0625        0.0075
d(const)       14.5228       23.7127
d(shiftfkt)   -36.9170      -60.2778
dx(-1)         0.0565        0.7688
dx(-2)         0.1842        2.5062
dx(-3)         0.0632        0.8606
dx(-4)         0.0063        0.0861

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M11, 2013 M2], T = 172

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               21.6831
 p-Value (Chi^2):          0.0412
Ljung & Box:               22.8239
 p-Value (Chi^2):          0.0293

JARQUE-BERA TEST:

test statistic:            68.1161
 p-Value(Chi^2):           0.0000
skewness:                 -0.0190
kurtosis:                  6.0218
```

1.3. Unit Root tests with structural break for “B_Croatia” (lags=8); with a shift dummy (October 2008); with constant and no trend and with both constant and a trend

*this additional test was performed because of the rejection of the residual no-autocorrelation tests in the previous section

```
Break date search for series:  B_Croatia
sample range:                  [1998 M7, 2013 M2], T = 176
searched range:                [1999 M3, 2012 M12], T = 166
number of lags (1st diff):     8
suggested break date:          2008 M10

UR Test with structural break for series: B_Croatia
sample range:                  [1998 M7, 2013 M2], T = 176
number of lags (1st diff):     8
value of test statistic:      -2.1565
used break date:               2008 M10
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%        10%
---------------------------------------
1000      -3.48     -2.88     -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       14.1070       23.7889
d(shiftfkt)   -34.3256      -57.8838
dx(-1)        -0.0258       -0.3523
dx(-2)         0.1739        2.3854
dx(-3)         0.1080        1.4814
dx(-4)         0.0049        0.0677
dx(-5)         0.0866        1.1862
dx(-6)        -0.1734       -2.3785
dx(-7)         0.1143        1.5681
dx(-8)         0.0697        0.9510

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M9, 2013 M2], T = 174

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               1.2989
 p-Value (Chi^2):          0.9999
Ljung & Box:               1.3660
 p-Value (Chi^2):          0.9999

ARCH-LM TEST with 2 lags:

test statistic:            24.5737
 p-Value(Chi^2):           0.0000
F statistic:               14.3211
 p-Value(F):               0.0000

UR Test with structural break for series: B_Croatia
sample range:                  [1998 M7, 2013 M2], T = 176
number of lags (1st diff):     8
value of test statistic:      -2.2120
used break date:               2008 M10
shiftfunction:                 shift dummy
```

```
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       0.0358        0.0044
d(const)       14.0837       23.6847
d(shiftfkt)   -34.3435      -57.7558
dx(-1)        -0.0259       -0.3532
dx(-2)         0.1738        2.3852
dx(-3)         0.1080        1.4816
dx(-4)         0.0049        0.0675
dx(-5)         0.0867        1.1865
dx(-6)        -0.1733       -2.3779
dx(-7)         0.1142        1.5675
dx(-8)         0.0697        0.9501

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M9, 2013 M2], T = 174

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    8
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               1.2677
 p-Value (Chi^2):          0.9999
Ljung & Box:               1.3328
 p-Value (Chi^2):          0.9999

ARCH-LM TEST with 2 lags:

test statistic:            24.6359
 p-Value(Chi^2):           0.0000
F statistic:               14.3633
 p-Value(F):               0.0000
```

1.4. KPSS test for stationarity of “B_Croatia” (lags=4) testing H0: “B_Croatia” is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  B_Croatia
sample range:          [1997 M10, 2013 M2], T = 185
number of lags:        4
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%         1%
0.347      0.463      0.739
value of test statistic: 0.7656
```

```
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  B_Croatia
sample range:          [1997 M10, 2013 M2], T = 185
number of lags:        4
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%        5%         1%
0.119      0.146      0.216
value of test statistic: 0.3778
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

2. Unit root tests of the first difference of Croatian equity bubble variable “B_Croatia_D1”

2.1. Augmented Dickey Fuller test results for the first difference transformation of “B_Croatia” (lags=3); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      B_Croatia_d1
sample range:             [1998 M3, 2013 M2], T = 180
lagged differences:       3
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -5.3135
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.6760       -5.3135
dx(-1)        -0.2642       -2.2515
dx(-2)        -0.1202       -1.1679
dx(-3)        -0.0441       -0.5915
RSS            13664.8608

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    7
Final Prediction Error:   7
Hannan-Quinn Criterion:   1
Schwarz Criterion:        1

PORTMANTEAU TEST with 12 lags
```

```
Portmanteau:                14.5850
 p-Value (Chi^2):           0.2649
Ljung & Box:                15.3465
 p-Value (Chi^2):           0.2230

JARQUE-BERA TEST:

test statistic:             116.0873
 p-Value(Chi^2):            0.0000
skewness:                  -0.7243
kurtosis:                   6.6579
```

```
ADF Test for series:       B_Croatia_d1
sample range:              [1998 M3, 2013 M2], T = 180
lagged differences:        3
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -5.3008
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.6766       -5.3008
dx(-1)        -0.2637       -2.2396
dx(-2)        -0.1199       -1.1611
dx(-3)        -0.0439       -0.5872
constant      -0.0994       -0.1509
RSS            13663.0829

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:              [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:     7
Final Prediction Error:    7
Hannan-Quinn Criterion:    1
Schwarz Criterion:         1

PORTMANTEAU TEST with 12 lags

Portmanteau:                14.5828
 p-Value (Chi^2):           0.2650
Ljung & Box:                15.3441
 p-Value (Chi^2):           0.2232

JARQUE-BERA TEST:

test statistic:             116.1746
 p-Value(Chi^2):            0.0000
skewness:                  -0.7246
kurtosis:                   6.6592
```

```
ADF Test for series:       B_Croatia_d1
sample range:              [1998 M3, 2013 M2], T = 180
lagged differences:        3
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
```

```
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -5.2836
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.6764       -5.2836
dx(-1)        -0.2638       -2.2343
dx(-2)        -0.1200       -1.1588
dx(-3)        -0.0439       -0.5860
constant      -0.0988       -0.1495
trend          0.0013        0.1007
RSS            13662.2862

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    7
Final Prediction Error:   7
Hannan-Quinn Criterion:   1
Schwarz Criterion:        1

PORTMANTEAU TEST with 12 lags

Portmanteau:               14.5824
 p-Value (Chi^2):          0.2651
Ljung & Box:               15.3438
 p-Value (Chi^2):          0.2232

JARQUE-BERA TEST:

test statistic:            115.8100
 p-Value(Chi^2):           0.0000
skewness:                 -0.7273
kurtosis:                  6.6504
```

2.2. KPSS test for stationarity of "B_Croatia_D1" (lags=3) testing H0: "B_Croatia_D1" is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  B_Croatia_d1
sample range:          [1997 M11, 2013 M2], T = 184
number of lags:        3
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%        1%
0.347      0.463      0.739
value of test statistic: 0.1053
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  B_Croatia_d1
```

```
sample range:          [1997 M11, 2013 M2], T = 184
number of lags:        3
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%       5%        1%
0.119     0.146     0.216
value of test statistic: 0.1063
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

3. Unit root tests of the Croatian financial market dynamic efficiency variable "DYNEF"

3.1. Augmented Dickey Fuller test results for "DYNEF" (lags=2); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      DYNEF
sample range:             [2007 M3, 2010 M12], T = 46
lagged differences:       2
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-2.56     -1.94     -1.62
value of test statistic: -2.0152
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.1744       -2.0152
dx(-1)        -0.0707       -0.4658
dx(-2)        -0.0397       -0.2633
RSS            111.5818

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               9.1759
 p-Value (Chi^2):          0.6878
Ljung & Box:               11.5594
 p-Value (Chi^2):          0.4817

JARQUE-BERA TEST:

test statistic:            9.3443
 p-Value(Chi^2):           0.0094
```

```
skewness:                  0.8200
kurtosis:                  4.4784
```

```
ADF Test for series:      DYNEF
sample range:             [2007 M3, 2010 M12], T = 46
lagged differences:       2
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -2.0086
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.1869       -2.0086
dx(-1)        -0.0629       -0.4068
dx(-2)        -0.0324       -0.2114
constant      -0.1000       -0.3911
RSS            111.1769

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               9.2236
 p-Value (Chi^2):          0.6837
Ljung & Box:               11.6170
 p-Value (Chi^2):          0.4769

JARQUE-BERA TEST:

test statistic:            10.2660
 p-Value(Chi^2):           0.0059
skewness:                  0.8503
kurtosis:                  4.5698
```

```
ADF Test for series:      DYNEF
sample range:             [2007 M3, 2010 M12], T = 46
lagged differences:       2
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -1.7334
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
```

```
 x(-1)          -0.1772        -1.7334
dx(-1)          -0.0743        -0.4548
dx(-2)          -0.0411        -0.2580
constant        -0.0928        -0.3563
trend           -0.0049        -0.2414
RSS              111.0190

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               9.3513
 p-Value (Chi^2):          0.6727
Ljung & Box:               11.7841
 p-Value (Chi^2):          0.4632

JARQUE-BERA TEST:

test statistic:            9.7481
 p-Value(Chi^2):           0.0076
skewness:                  0.8277
kurtosis:                  4.5316
```

3.2. Unit Root tests with structural break for "DYNEF" (lags=2); with an impulse dummy (September 2009) and with a shift dummy (February 2009); with constant and no trend and with both constant and a trend

```
Break date search for series:  DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
searched range:                [2007 M5, 2010 M10], T = 42
number of lags (1st diff):     2
suggested break date:          2009 M2

UR Test with structural break for series: DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
number of lags (1st diff):     2
value of test statistic:      -2.0988
used break date:               2009 M2
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%         10%
---------------------------------------
1000      -3.48     -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)      -4.8871       -23.3539
```

```
d(shiftfkt)    3.5784       12.0915
dx(-1)        -0.1624      -1.1372
dx(-2)        -0.0302      -0.2116

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    6
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               6.9753
 p-Value (Chi^2):          0.8592
Ljung & Box:               8.8454
 p-Value (Chi^2):          0.7161

JARQUE-BERA TEST:

test statistic:            0.5246
 p-Value(Chi^2):           0.7693
skewness:                  0.2643
kurtosis:                  2.9793
```

```
UR Test with structural break for series: DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
number of lags (1st diff):     2
value of test statistic:      -1.8894
used break date:               2009 M2
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%         5%         10%
---------------------------------------
1000      -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       0.0600        0.0406
d(const)      -4.9583      -23.4696
d(shiftfkt)    3.5784       11.9770
dx(-1)        -0.1640      -1.1486
dx(-2)        -0.0327      -0.2291

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    6
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               6.4544
 p-Value (Chi^2):          0.8915
Ljung & Box:               8.1399
```

```
 p-Value (Chi^2):          0.7741

JARQUE-BERA TEST:

test statistic:            0.5487
 p-Value(Chi^2):           0.7601
skewness:                  0.2656
kurtosis:                  2.8973
```

```
Break date search for series:  DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
searched range:                [2007 M5, 2010 M10], T = 42
number of lags (1st diff):     2
suggested break date:          2009 M2
```

```
UR Test with structural break for series: DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
number of lags (1st diff):     2
value of test statistic:      -1.7435
used break date:               2009 M2
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)      -4.8875       -23.0928
d(shiftfkt)    4.4177        20.8728
dx(-1)        -0.1422       -0.9952
dx(-2)        -0.0174       -0.1215

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               8.1054
 p-Value (Chi^2):          0.7768
Ljung & Box:               9.9537
 p-Value (Chi^2):          0.6200

JARQUE-BERA TEST:

test statistic:            0.7626
 p-Value(Chi^2):           0.6830
skewness:                  0.3128
kurtosis:                  2.8765
```

```
UR Test with structural break for series: DYNEF
sample range:                  [2007 M3, 2010 M12], T = 46
number of lags (1st diff):     2
value of test statistic:      -2.0812
used break date:               2009 M2
shiftfunction:                 shift dummy
```

```
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%         5%         10%
---------------------------------------
1000      -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -0.0333       -0.0223
d(const)      -4.8492       -22.6741
d(shiftfkt)    4.4620        20.8638
dx(-1)        -0.1428       -1.0000
dx(-2)        -0.0170       -0.1189

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               8.1850
 p-Value (Chi^2):          0.7705
Ljung & Box:               10.0624
 p-Value (Chi^2):          0.6105

JARQUE-BERA TEST:

test statistic:            1.0485
 p-Value(Chi^2):           0.5920
skewness:                  0.3737
kurtosis:                  2.9760
```

3.3. KPSS test for stationarity of “DYNEF” (lags=2) testing H0: “DYNEF” is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  DYNEF
sample range:          [2006 M12, 2010 M12], T = 49
number of lags:        2
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%       5%        1%
0.347      0.463     0.739
value of test statistic: 0.4778
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  DYNEF
sample range:          [2006 M12, 2010 M12], T = 49
```

```
number of lags:        2
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%       5%        1%
0.119     0.146     0.216
value of test statistic: 0.2501
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

4. Unit root tests of the first difference transformation of Croatian financial market dynamic efficiency variable “DYNEF_D1”

4.1. Augmented Dickey Fuller test results for the “DYNEF_D1” (lags=1); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      DYNEF_d1
sample range:             [1998 M1, 2013 M2], T = 182
lagged differences:       1
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-2.56     -1.94     -1.62
value of test statistic: -10.4827
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -1.1257       -10.4827
dx(-1)         0.0809        1.0877
RSS            266.4609

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags
```

```
Portmanteau:                 10.4688
 p-Value (Chi^2):            0.5749
Ljung & Box:                 10.9647
 p-Value (Chi^2):            0.5319

JARQUE-BERA TEST:

test statistic:              3678.4803
 p-Value(Chi^2):             0.0000
skewness:                   -2.3146
kurtosis:                    24.5324
```

```
ADF Test for series:        DYNEF_d1
sample range:               [1998 M1, 2013 M2], T = 182
lagged differences:         1
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -10.4898
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -1.1300       -10.4898
dx(-1)         0.0831        1.1142
constant      -0.0624       -0.6901
RSS            265.7539

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:               [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:      0
Final Prediction Error:     0
Hannan-Quinn Criterion:     0
Schwarz Criterion:          0

PORTMANTEAU TEST with 12 lags

Portmanteau:                 10.4510
 p-Value (Chi^2):            0.5765
Ljung & Box:                 10.9464
 p-Value (Chi^2):            0.5335

JARQUE-BERA TEST:

test statistic:              3682.0871
 p-Value(Chi^2):             0.0000
skewness:                   -2.3165
kurtosis:                    24.5426
```

```
ADF Test for series:        DYNEF_d1
sample range:               [1998 M1, 2013 M2], T = 182
lagged differences:         1
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
```

```
value of test statistic: -10.5266
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -1.1393       -10.5266
dx(-1)         0.0878        1.1742
constant      -0.0621       -0.6860
trend          0.0016        0.9278
RSS            264.4749

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M10, 2013 M2], T = 173

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    0
Final Prediction Error:   0
Hannan-Quinn Criterion:   0
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               10.6491
 p-Value (Chi^2):          0.5592
Ljung & Box:               11.1599
 p-Value (Chi^2):          0.5153

JARQUE-BERA TEST:

test statistic:            3557.7863
 p-Value(Chi^2):           0.0000
skewness:                 -2.3651
kurtosis:                  24.1373
```

4.2. KPSS test for stationarity of the first difference of “DYNEF” (lags=1) testing H0: “DYNEF” is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  DYNEF_d1
sample range:          [1997 M11, 2013 M2], T = 184
number of lags:        1
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%       5%        1%
0.347     0.463     0.739
value of test statistic: 0.1381
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  DYNEF_d1
sample range:          [1997 M11, 2013 M2], T = 184
number of lags:        1
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%       5%        1%
```

```
0.119      0.146      0.216
value of test statistic: 0.0542
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

5. Unit root tests of the Croatian DC pension fund direct equity investment variable “PFInvDirect”

5.1. Augmented Dickey Fuller test results for “PFInvDirect” (lags=4); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      PFInvDirect
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       4
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -0.6932
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0000       -0.6932
dx(-1)         3.1517        41.1263
dx(-2)        -3.7065       -16.7978
dx(-3)         1.9261         8.6908
dx(-4)        -0.3759       -4.8374
RSS            420191151618.8523

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4

PORTMANTEAU TEST with 12 lags

Portmanteau:               52.2988
 p-Value (Chi^2):          0.0000
Ljung & Box:               57.1959
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7697.9089
 p-Value(Chi^2):           0.0000
```

```
skewness:                  -0.7310
kurtosis:                   37.8328
```

```
ADF Test for series:       PFInvDirect
sample range:              [1998 M5, 2010 M12], T = 152
lagged differences:        4
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -1.2978
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0001       -1.2978
dx(-1)         3.1402        40.7538
dx(-2)        -3.6792       -16.6217
dx(-3)         1.9031        8.5723
dx(-4)        -0.3686       -4.7396
constant       6749.4925     1.2406
RSS            415807705551.7676

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:              [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4

PORTMANTEAU TEST with 12 lags

Portmanteau:                50.9722
 p-Value (Chi^2):           0.0000
Ljung & Box:                55.7248
 p-Value (Chi^2):           0.0000

JARQUE-BERA TEST:

test statistic:             7252.7767
 p-Value(Chi^2):            0.0000
skewness:                  -0.4967
kurtosis:                   36.8259
```

```
ADF Test for series:       PFInvDirect
sample range:              [1998 M5, 2010 M12], T = 152
lagged differences:        4
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -2.3181
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0003       -2.3181
```

```
dx(-1)          3.1060        39.6005
dx(-2)         -3.5847       -15.9424
dx(-3)          1.8061         7.9994
dx(-4)         -0.3311        -4.1636
constant        16117.3494    2.2128
trend           338.4085      1.9129
RSS             405572312768.3825
```

```
OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               55.5405
 p-Value (Chi^2):          0.0000
Ljung & Box:               60.6720
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            6924.5482
 p-Value(Chi^2):           0.0000
skewness:                 -0.5010
kurtosis:                  36.0507
```

5.2. Unit Root tests with structural break for "PFInvDirect" (lags=4); with an impulse dummy (November 1998) and with a shift dummy (September 1998); with constant and no trend and with both constant and a trend

```
Break date search for series:  PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M11, 2010 M10], T = 144
number of lags (1st diff):     4
suggested break date:          1998 M11
```

```
UR Test with structural break for series: PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -1.3254
used break date:               1998 M11
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%         10%
---------------------------------------
1000      -3.48     -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)      -7595.2602    -1.8229
```

```
d(shiftfkt)   -42.6686      -0.0072
dx(-1)         3.1389        42.3961
dx(-2)        -3.6790       -17.3367
dx(-3)         1.9088        8.9947
dx(-4)        -0.3733       -5.0426

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4

PORTMANTEAU TEST with 12 lags

Portmanteau:               50.6436
 p-Value (Chi^2):          0.0000
Ljung & Box:               55.3994
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7100.9092
 p-Value(Chi^2):           0.0000
skewness:                 -0.4953
kurtosis:                  36.5804
```

```
UR Test with structural break for series: PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -1.9431
used break date:               1998 M11
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%        10%
---------------------------------------
1000      -3.55     -3.03     -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       536510.8911   10.2546
d(const)      -10142.2249   -2.4290
d(shiftfkt)   -42.6731      -0.0072
dx(-1)         3.1375        42.3964
dx(-2)        -3.6774       -17.3446
dx(-3)         1.9095        9.0063
dx(-4)        -0.3744       -5.0589

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags
```

```
Portmanteau:                 51.9750
 p-Value (Chi^2):            0.0000
Ljung & Box:                 56.8250
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              6946.9965
 p-Value(Chi^2):             0.0000
skewness:                   -0.3320
kurtosis:                    36.2222
```

```
Break date search for series:  PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M9, 2010 M10], T = 146
number of lags (1st diff):     4
suggested break date:          1998 M9
```

```
UR Test with structural break for series: PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -1.3254
used break date:               1998 M9
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)      -7595.2601    -1.8229
d(shiftfkt)    4.1850        0.0010
dx(-1)         3.1389        42.3961
dx(-2)        -3.6790       -17.3367
dx(-3)         1.9088        8.9947
dx(-4)        -0.3733       -5.0425

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        4

PORTMANTEAU TEST with 12 lags

Portmanteau:                 50.6438
 p-Value (Chi^2):            0.0000
Ljung & Box:                 55.3996
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              7100.8968
 p-Value(Chi^2):             0.0000
skewness:                   -0.4953
kurtosis:                    36.5803
```

```
UR Test with structural break for series: PFInvDirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):      4
value of test statistic:       -1.9431
used break date:                1998 M9
shiftfunction:                  shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       536510.4924   10.2545
d(const)      -10142.2245   -2.4290
d(shiftfkt)    3.7690        0.0009
dx(-1)         3.1375        42.3964
dx(-2)        -3.6774       -17.3446
dx(-3)         1.9095        9.0063
dx(-4)        -0.3744       -5.0589

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.9752
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.8252
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            6946.9845
 p-Value(Chi^2):           0.0000
skewness:                 -0.3320
kurtosis:                  36.2222
```

5.3. KPSS test for stationarity of “PFInvDirect” (lags=4) testing H0: “PFInvDirect” is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  PFInvDirect
sample range:          [1997 M12, 2010 M12], T = 157
number of lags:        4
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%         1%
0.347      0.463      0.739
```

```
value of test statistic: 1.3816
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  PFInvDirect
sample range:          [1997 M12, 2010 M12], T = 157
number of lags:        4
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%       5%        1%
0.119     0.146     0.216
value of test statistic: 0.2079
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

6. Unit root tests of the first difference transformation of Croatian DC pension fund direct equity investment variable "PFInvDirect_D1"

6.1. Augmented Dickey Fuller test results for "PFInvDirect_D1" (lags=3); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      PFInvDirect_d1
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-2.56     -1.94     -1.62
value of test statistic: -3.8384
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0046       -3.8384
dx(-1)         2.1618        28.5035
dx(-2)        -1.5609       -10.7816
dx(-3)         0.3829         4.9804
RSS            421564895072.5768

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
```

```
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.8774
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.7453
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7599.7634
 p-Value(Chi^2):           0.0000
skewness:                 -0.9356
kurtosis:                  37.5899
```

```
ADF Test for series:      PFInvDirect_d1
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -3.8698
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0047       -3.8698
dx(-1)         2.1603        28.4033
dx(-2)        -1.5603       -10.7527
dx(-3)         0.3838        4.9789
constant       2534.8469     0.5794
RSS            420604352473.0487

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.5768
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.4170
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7452.5733
 p-Value(Chi^2):           0.0000
skewness:                 -0.9518
kurtosis:                  37.2505
```

```
ADF Test for series:      PFInvDirect_d1
```

```
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -3.8509
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0047       -3.8509
dx(-1)         2.1603        28.3065
dx(-2)        -1.5603       -10.7160
dx(-3)         0.3838        4.9616
constant       2533.6395     0.5771
trend         -2.0830       -0.0208
RSS            420603104088.6884

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.5510
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.3888
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7451.1241
 p-Value(Chi^2):           0.0000
skewness:                 -0.9456
kurtosis:                  37.2479
```

6.2. Unit Root tests with structural break for “PFInvDirect_D1” (lags=3); with an impulse dummy (August 1998) and with a shift dummy (August 1998); with constant and no trend and with both constant and a trend

```
Break date search for series:  PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M8, 2010 M10], T = 147
number of lags (1st diff):     3
suggested break date:          1998 M8

*** Fri, 25 Sep 2015 12:41:22 ***
UR Test with structural break for series: PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
```

```
value of test statistic:      -3.9224
used break date:               1998 M8
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%        10%
---------------------------------------
1000      -3.48     -2.88     -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       7587.6462     1.7267
d(shiftfkt)    0.0118        0.0000
dx(-1)         2.2055        29.0092
dx(-2)        -1.5443       -10.4896
dx(-3)         0.3135        4.1242

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.2409
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.0841
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7298.2282
 p-Value(Chi^2):           0.0000
skewness:                 -0.9498
kurtosis:                  37.0055
```

```
UR Test with structural break for series: PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
value of test statistic:      -3.6846
used break date:               1998 M8
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%        10%
---------------------------------------
1000      -3.55     -3.03     -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       28745.8878    0.5221
d(const)       6860.4920     1.5563
d(shiftfkt)    0.0119        0.0000
dx(-1)         2.2053        29.0063
dx(-2)        -1.5441       -10.4885
dx(-3)         0.3135        4.1232
```

```
OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               52.7405
 p-Value (Chi^2):          0.0000
Ljung & Box:               57.7007
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7386.4441
 p-Value(Chi^2):           0.0000
skewness:                 -1.1702
kurtosis:                  37.1837
```

```
Break date search for series:  PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M8, 2010 M10], T = 147
number of lags (1st diff):     3
suggested break date:          1998 M8
```

```
UR Test with structural break for series: PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
value of test statistic:      -3.9224
used break date:               1998 M8
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       7587.6462     1.7267
d(shiftfkt)   -0.1604       -0.0000
dx(-1)         2.2055        29.0092
dx(-2)        -1.5443       -10.4896
dx(-3)         0.3135        4.1242

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               51.2409
```

```
 p-Value (Chi^2):          0.0000
Ljung & Box:               56.0841
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7298.2282
 p-Value(Chi^2):           0.0000
skewness:                 -0.9498
kurtosis:                  37.0055
```

```
UR Test with structural break for series: PFInvDirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
value of test statistic:      -3.6846
used break date:               1998 M8
shiftfunction:                 shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       28745.9090    0.5221
d(const)       6860.4917     1.5563
d(shiftfkt)   -2.3651       -0.0005
dx(-1)         2.2053        29.0063
dx(-2)        -1.5441       -10.4885
dx(-3)         0.3135        4.1232

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               52.7405
 p-Value (Chi^2):          0.0000
Ljung & Box:               57.7007
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            7386.4442
 p-Value(Chi^2):           0.0000
skewness:                 -1.1702
kurtosis:                  37.1837
```

6.3. KPSS test for stationarity of "PFInvDirect_D1" (lags=3) testing H0: "PFInvDirect_D1" is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  PFInvDirect_d1
sample range:          [1998 M1, 2010 M12], T = 156
number of lags:        3
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%         1%
0.347      0.463      0.739
value of test statistic: 0.1382
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  PFInvDirect_d1
sample range:          [1998 M1, 2010 M12], T = 156
number of lags:        3
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%        5%         1%
0.119      0.146      0.216
value of test statistic: 0.1389
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

7. Unit root tests of the Croatian DC pension fund investment into Croatian and SEE equity focused open-end investment funds variable "PFInvIndirect"

7.1. Augmented Dickey Fuller test results for "PFInvIndirect" (lags=4); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      PFInvIndirect
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       4
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -2.1738
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
```

```
 x(-1)         -0.0006       -2.1738
dx(-1)          3.0286        38.8410
dx(-2)         -3.4386       -15.6462
dx(-3)          1.7283        7.8567
dx(-4)         -0.3261       -4.1620
RSS             3064950066013.2510

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               81.0214
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.7198
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4295.3597
 p-Value(Chi^2):           0.0000
skewness:                 -1.4755
kurtosis:                  28.8748
```

```
ADF Test for series:      PFInvIndirect
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       4
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -2.1727
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)         -0.0006       -2.1727
dx(-1)          3.0280        38.6886
dx(-2)         -3.4371       -15.5805
dx(-3)          1.7267        7.8190
dx(-4)         -0.3254       -4.1366
constant        2562.9243     0.2150
RSS             3063979544191.3853

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags
```

```
Portmanteau:                 80.9589
 p-Value (Chi^2):            0.0000
Ljung & Box:                 88.6486
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              4299.2680
 p-Value(Chi^2):             0.0000
skewness:                   -1.4764
kurtosis:                    28.8865
```

```
ADF Test for series:        PFInvIndirect
sample range:               [1998 M5, 2010 M12], T = 152
lagged differences:         4
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-3.96     -3.41     -3.13
value of test statistic: -2.1757
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0006       -2.1757
dx(-1)         3.0271        38.5509
dx(-2)        -3.4353       -15.5258
dx(-3)         1.7258        7.7926
dx(-4)        -0.3253       -4.1240
constant       2513.2528     0.2103
trend         -112.4831     -0.4176
RSS            3060299395528.2163

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:               [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:      5
Final Prediction Error:     5
Hannan-Quinn Criterion:     5
Schwarz Criterion:          5

PORTMANTEAU TEST with 12 lags

Portmanteau:                 81.3930
 p-Value (Chi^2):            0.0000
Ljung & Box:                 89.1281
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              4185.1448
 p-Value(Chi^2):             0.0000
skewness:                   -1.3603
kurtosis:                    28.5619
```

7.2. Unit Root tests with structural break for "PFInvDirect" (lags=4); with an impulse dummy (August 1998) and with a shift dummy (August 1998); with constant and no trend and with both constant and a trend

```
Break date search for series:  PFInvIndirect
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M9, 2010 M10], T = 146
number of lags (1st diff):     4
suggested break date:          1998 M9
```

```
UR Test with structural break for series: PFInvIndirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -2.2097
used break date:               1998 M9
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)      -191.8316     -0.0168
d(shiftfkt)    0.0640        0.0000
dx(-1)         3.0531        41.0171
dx(-2)        -3.5089       -16.7090
dx(-3)         1.8075        8.6072
dx(-4)        -0.3608       -4.8466

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               80.4259
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.1199
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4209.2644
 p-Value(Chi^2):           0.0000
skewness:                 -1.4719
kurtosis:                  28.6974
```

```
UR Test with structural break for series: PFInvIndirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -2.2530
used break date:               1998 M9
```

```
shiftfunction:                  impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -197825.9292  -1.3791
d(const)       1602.9377     0.1400
d(shiftfkt)    0.0640        0.0000
dx(-1)         3.0530        41.0158
dx(-2)        -3.5088       -16.7085
dx(-3)         1.8075        8.6070
dx(-4)        -0.3608       -4.8466

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               80.8671
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.6074
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4095.7423
 p-Value(Chi^2):           0.0000
skewness:                 -1.3546
kurtosis:                  28.3700
```

```
Break date search for series:  PFInvIndirect
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M9, 2010 M10], T = 146
number of lags (1st diff):     4
suggested break date:          1998 M9
```

```
UR Test with structural break for series: PFInvIndirect
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     4
value of test statistic:      -2.2097
used break date:               1998 M9
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
```

```
d(const)      -191.8316     -0.0168
d(shiftfkt)    0.1097        0.0000
dx(-1)         3.0531        41.0171
dx(-2)        -3.5089       -16.7090
dx(-3)         1.8075        8.6072
dx(-4)        -0.3608       -4.8466

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5

PORTMANTEAU TEST with 12 lags

Portmanteau:               80.4259
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.1199
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4209.2644
 p-Value(Chi^2):           0.0000
skewness:                 -1.4719
kurtosis:                  28.6974
```

```
UR Test with structural break for series: PFInvIndirect
sample range:                   [1998 M5, 2010 M12], T = 152
number of lags (1st diff):       4
value of test statistic:        -2.2530
used break date:                 1998 M9
shiftfunction:                   shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -197826.0580  -1.3791
d(const)       1602.9386     0.1400
d(shiftfkt)    0.7353        0.0001
dx(-1)         3.0530        41.0158
dx(-2)        -3.5088       -16.7085
dx(-3)         1.8075        8.6070
dx(-4)        -0.3608       -4.8466

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1999 M1, 2010 M12], T = 144

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    5
Final Prediction Error:   5
Hannan-Quinn Criterion:   5
Schwarz Criterion:        5
```

```
PORTMANTEAU TEST with 12 lags

Portmanteau:                 80.8671
 p-Value (Chi^2):            0.0000
Ljung & Box:                 88.6074
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              4095.7422
 p-Value(Chi^2):             0.0000
skewness:                   -1.3546
kurtosis:                    28.3700
```

7.3. KPSS test for stationarity of "PFInvDirect" (lags=4) testing H0: "PFInvDirect" is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  PFInvIndirect
sample range:          [1997 M12, 2010 M12], T = 157
number of lags:        4
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%        1%
0.347      0.463      0.739
value of test statistic: 0.1407
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  PFInvIndirect
sample range:          [1997 M12, 2010 M12], T = 157
number of lags:        4
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%        5%        1%
0.119      0.146      0.216
value of test statistic: 0.1338
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

8. Unit root tests of the first difference of the Croatian DC pension fund investment into Croatian and SEE equity focused open-end investment funds variable "PFInvIndirect"

8.1. Augmented Dickey Fuller test results for “PFInvIndirect_D1” (lags=3); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      PFInvIndirect_d1
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -4.5906
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0090       -4.5906
dx(-1)         2.0798        27.3762
dx(-2)        -1.4704       -10.3278
dx(-3)         0.3693        4.8134
RSS            3163472127225.8657

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               80.9775
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.7614
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4308.2959
 p-Value(Chi^2):           0.0000
skewness:                 -1.4226
kurtosis:                  28.9261
```

```
ADF Test for series:      PFInvIndirect_d1
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -4.5763
regression results:
---------------------------------------
```

```
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0090       -4.5763
dx(-1)         2.0797        27.2827
dx(-2)        -1.4703       -10.2927
dx(-3)         0.3693        4.7972
constant      -1664.7229    -0.1398
RSS            3163051448179.3896

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               80.9677
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.7506
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4300.5027
 p-Value(Chi^2):           0.0000
skewness:                 -1.4220
kurtosis:                  28.9025
```

```
ADF Test for series:      PFInvIndirect_d1
sample range:             [1998 M5, 2010 M12], T = 152
lagged differences:       3
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -4.5739
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0090       -4.5739
dx(-1)         2.0791        27.1866
dx(-2)        -1.4699       -10.2588
dx(-3)         0.3694        4.7844
constant      -1724.9908    -0.1444
trend         -98.9229      -0.3627
RSS            3160203599879.9570

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3
```

```
PORTMANTEAU TEST with 12 lags

Portmanteau:                81.5037
 p-Value (Chi^2):           0.0000
Ljung & Box:                89.3423
 p-Value (Chi^2):           0.0000

JARQUE-BERA TEST:

test statistic:             4210.5334
 p-Value(Chi^2):            0.0000
skewness:                  -1.3214
kurtosis:                   28.6483
```

8.2. Unit Root tests with structural break for "PFInvDirect_D1" (lags=3); with an impulse dummy (August 1998) and with a shift dummy (August 1998); with constant and no trend and with both constant and a trend

```
Break date search for series:  PFInvIndirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M8, 2010 M10], T = 147
number of lags (1st diff):     3
suggested break date:          1998 M8

*** Fri, 25 Sep 2015 12:44:04 ***
UR Test with structural break for series: PFInvIndirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
value of test statistic:      -4.6381
used break date:               1998 M8
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       191.9800      0.0156
d(shiftfkt)    0.0004        0.0000
dx(-1)         2.1476        27.8793
dx(-2)        -1.4561       -9.8382
dx(-3)         0.2726        3.5390

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:                80.4358
```

```
 p-Value (Chi^2):          0.0000
Ljung & Box:               88.2230
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            4209.9360
 p-Value(Chi^2):           0.0000
skewness:                 -1.4171
kurtosis:                  28.7118
```

```
UR Test with structural break for series: PFInvIndirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
number of lags (1st diff):     3
value of test statistic:      -4.5764
used break date:               1998 M8
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -35704.5912   -0.2322
d(const)       1476.0240     0.1199
d(shiftfkt)    0.0004        0.0000
dx(-1)         2.1475        27.8782
dx(-2)        -1.4561       -9.8377
dx(-3)         0.2726        3.5385

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               81.7970
 p-Value (Chi^2):          0.0000
Ljung & Box:               89.7110
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            3938.3013
 p-Value(Chi^2):           0.0000
skewness:                 -1.0674
kurtosis:                  27.9279
```

```
Break date search for series:  PFInvIndirect_d1
sample range:                  [1998 M5, 2010 M12], T = 152
searched range:                [1998 M8, 2010 M10], T = 147
number of lags (1st diff):     3
suggested break date:          1998 M8
```

```
UR Test with structural break for series: PFInvIndirect_d1
sample range:                   [1998 M5, 2010 M12], T = 152
number of lags (1st diff):      3
value of test statistic:       -4.6381
used break date:                1998 M8
shiftfunction:                  shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       191.9800      0.0156
d(shiftfkt)   -0.0093       -0.0000
dx(-1)         2.1476        27.8793
dx(-2)        -1.4561       -9.8382
dx(-3)         0.2726        3.5390

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:              [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:     4
Final Prediction Error:    4
Hannan-Quinn Criterion:    4
Schwarz Criterion:         3

PORTMANTEAU TEST with 12 lags

Portmanteau:                15 80.4358
 p-Value (Chi^2):            0.0000
Ljung & Box:                 88.2230
 p-Value (Chi^2):            0.0000

JARQUE-BERA TEST:

test statistic:              4209.9360
 p-Value(Chi^2):             0.0000
skewness:                   -1.4171
kurtosis:                    28.7118
```

```
UR Test with structural break for series: PFInvIndirect_d1
sample range:                   [1998 M5, 2010 M12], T = 152
number of lags (1st diff):      3
value of test statistic:       -4.5764
used break date:                1998 M8
shiftfunction:                  shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -35704.6309   -0.2322
d(const)       1476.0254     0.1199
```

```
d(shiftfkt)    5.9062        0.0005
dx(-1)         2.1475        27.8782
dx(-2)        -1.4561       -9.8377
dx(-3)         0.2726        3.5385

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [1998 M12, 2010 M12], T = 145

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    4
Final Prediction Error:   4
Hannan-Quinn Criterion:   4
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               81.7970
 p-Value (Chi^2):          0.0000
Ljung & Box:               89.7110
 p-Value (Chi^2):          0.0000

JARQUE-BERA TEST:

test statistic:            3938.3014
 p-Value(Chi^2):           0.0000
skewness:                 -1.0674
kurtosis:                  27.9279
```

8.3. KPSS test for stationarity of "PFInvIndirect_D1" (lags=3) testing H0: "PFInvIndirect_D1" is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  PFInvIndirect_d1
sample range:          [1998 M1, 2010 M12], T = 156
number of lags:        3
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%        1%
0.347      0.463      0.739
value of test statistic: 0.0759
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  PFInvIndirect_d1
sample range:          [1998 M1, 2010 M12], T = 156
number of lags:        3
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%        5%        1%
0.119      0.146      0.216
value of test statistic: 0.0701
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

9. Unit root tests of the index variable representing the assets under management of the Croatian and SEE equity focused open-end investment funds weighted by their presence in the DC pension fund portfolios “OMF_AUMIndex”

9.1. Augmented Dickey Fuller test results for “OMF_AUMIndex” (lags=1); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      OMF_AUMIndex
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       1
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -2.2087
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0254       -2.2087
dx(-1)         0.3728        2.8277
RSS            165096.7638

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   1
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               12.5813
 p-Value (Chi^2):          0.4002
Ljung & Box:               15.0189
 p-Value (Chi^2):          0.2404

JARQUE-BERA TEST:

test statistic:            63.6528
 p-Value(Chi^2):           0.0000
skewness:                 -0.2822
kurtosis:                  8.6732
```

```
ADF Test for series:      OMF_AUMIndex
```

```
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       1
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-3.43     -2.86     -2.57
value of test statistic: -1.3310
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0215       -1.3310
dx(-1)         0.3627        2.6584
constant      -4.5359       -0.3393
RSS            164665.9561

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    11
Final Prediction Error:   8
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               12.4988
 p-Value (Chi^2):          0.4065
Ljung & Box:               14.9253
 p-Value (Chi^2):          0.2455

JARQUE-BERA TEST:

test statistic:            64.9714
 p-Value(Chi^2):           0.0000
skewness:                 -0.4242
kurtosis:                  8.6971
```

```
ADF Test for series:      OMF_AUMIndex
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       1
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%        5%        10%
-3.96     -3.41     -3.13
value of test statistic: -0.6963
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.0242       -0.6963
dx(-1)         0.3663        2.5431
constant      -2.8511       -0.1215
trend         -0.1267       -0.0879
RSS            164636.3729

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
```

```
sample range:             [2008 M1, 2010 M12], T = 36

optimal number of lags (searched up to 12 lags of 1. differences):
Akaike Info Criterion:    11
Final Prediction Error:   11
Hannan-Quinn Criterion:   2
Schwarz Criterion:        0

PORTMANTEAU TEST with 12 lags

Portmanteau:               12.5163
 p-Value (Chi^2):          0.4052
Ljung & Box:               14.9171
 p-Value (Chi^2):          0.2460

JARQUE-BERA TEST:

test statistic:            64.4759
 p-Value(Chi^2):           0.0000
skewness:                 -0.4244
kurtosis:                  8.6748
```

9.2. Unit Root tests with structural break for "OMF_AUMIndex" (lags=1); with an impulse dummy (November 2007) and with a shift dummy (December 2007); with constant and no trend and with both constant and a trend

```
Break date search for series:  OMF_AUMIndex
sample range:                  [2007 M2, 2010 M12], T = 47
searched range:                [2007 M3, 2010 M10], T = 44
number of lags (1st diff):     1
suggested break date:          2007 M12

UR Test with structural break for series: OMF_AUMIndex
sample range:                  [2007 M2, 2010 M12], T = 47
number of lags (1st diff):     1
value of test statistic:      -2.3340
used break date:               2007 M12
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T         1%        5%        10%
---------------------------------------
1000      -3.48     -2.88     -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       1361.9755     180.9782
d(shiftfkt)    120.9615      11.3655
dx(-1)         0.6092        5.3780

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M11, 2010 M12], T = 38

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
```

```
Final Prediction Error:   10
Hannan-Quinn Criterion:   2
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags

Portmanteau:               11.7391
 p-Value (Chi^2):          0.4669
Ljung & Box:               13.3822
 p-Value (Chi^2):          0.3419

JARQUE-BERA TEST:

test statistic:            230.7054
 p-Value(Chi^2):           0.0000
skewness:                 -2.2385
kurtosis:                  13.0162

UR Test with structural break for series: OMF_AUMIndex
sample range:                  [2007 M2, 2010 M12], T = 47
number of lags (1st diff):     1
value of test statistic:      -1.9250
used break date:               2007 M12
shiftfunction:                 impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -24.2128      -0.4635
d(const)       1374.4356     184.1849
d(shiftfkt)    120.3239      11.4016
dx(-1)         0.5450        4.5505

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M11, 2010 M12], T = 38

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   4
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags

Portmanteau:               12.3762
 p-Value (Chi^2):          0.4160
Ljung & Box:               14.0142
 p-Value (Chi^2):          0.2998

JARQUE-BERA TEST:

test statistic:            234.0094
 p-Value(Chi^2):           0.0000
skewness:                 -2.4139
kurtosis:                  12.9391

UR Test with structural break for series: OMF_AUMIndex
sample range:                  [2007 M2, 2010 M12], T = 47
```

```
number of lags (1st diff):     1
value of test statistic:      -2.9504
used break date:               2007 M12
shiftfunction:                 shift dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       1357.7212     213.7379
d(shiftfkt)    266.9013      42.0167
dx(-1)         0.7984        9.2812

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M11, 2010 M12], T = 38

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    6
Final Prediction Error:   6
Hannan-Quinn Criterion:   6
Schwarz Criterion:        1

PORTMANTEAU TEST with 12 lags

Portmanteau:               11.2449
 p-Value (Chi^2):          0.5081
Ljung & Box:               12.7671
 p-Value (Chi^2):          0.3862

JARQUE-BERA TEST:

test statistic:            738.5904
 p-Value(Chi^2):           0.0000
skewness:                 -3.6100
kurtosis:                  21.2544

UR Test with structural break for series: OMF_AUMIndex
sample range:                  [2007 M2, 2010 M12], T = 47
number of lags (1st diff):     1
value of test statistic:      -2.8057
used break date:               2007 M12
shiftfunction:                 shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -27.1631      -0.6097
d(const)       1364.9736     214.4621
d(shiftfkt)    265.5588      41.7241
dx(-1)         0.7626        8.2529

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
```

```
sample range:             [2007 M11, 2010 M12], T = 38

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    6
Final Prediction Error:   6
Hannan-Quinn Criterion:   6
Schwarz Criterion:        1

PORTMANTEAU TEST with 12 lags

Portmanteau:               11.3003
 p-Value (Chi^2):          0.5034
Ljung & Box:               12.8039
 p-Value (Chi^2):          0.3835

JARQUE-BERA TEST:

test statistic:            704.5319
 p-Value(Chi^2):           0.0000
skewness:                 -3.6485
kurtosis:                  20.7295
```

9.3. KPSS test for stationarity of "OMF_AUMIndex" (lags=1) testing H0: "OMF_AUMIndex" is stationary I(0) variable against the alternative of the variable being integrated at a level higher than 0 (test for both level and trend stationarity)

```
KPSS test for series:  OMF_AUMIndex
sample range:          [2006 M12, 2010 M12], T = 49
number of lags:        1
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%        5%         1%
0.347      0.463      0.739
value of test statistic: 2.0805
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  OMF_AUMIndex
sample range:          [2006 M12, 2010 M12], T = 49
number of lags:        1
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%        5%         1%
0.119      0.146      0.216
value of test statistic: 0.5440
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

10. Unit root tests of the first difference of the index variable representing the assets under management of the Croatian and SEE equity focused open-end investment funds weighted by their presence in the DC pension fund portfolios "OMF_AUMIndex_D1"

10.1. Augmented Dickey Fuller test results for "OMF_AUMIndex_D1" (lags=0); no constant or trend; constant and no trend and both with trend and constant

```
ADF Test for series:      OMF_AUMIndex_d1
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       0
no intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-2.56      -1.94      -1.62
value of test statistic: -4.1180
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.5384       -4.1180
RSS            182994.8894

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               13.7511
 p-Value (Chi^2):          0.3169
Ljung & Box:               15.9365
 p-Value (Chi^2):          0.1942

JARQUE-BERA TEST:

test statistic:            90.7487
 p-Value(Chi^2):           0.0000
skewness:                 -0.7847
kurtosis:                  9.6239
```

```
ADF Test for series:      OMF_AUMIndex_d1
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       0
intercept, no time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
```

```
 1%         5%         10%
-3.43      -2.86      -2.57
value of test statistic: -4.5591
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.6262       -4.5591
constant      -16.9434      -1.7531
RSS            171295.6975

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               12.7590
 p-Value (Chi^2):          0.3868
Ljung & Box:               14.9646
 p-Value (Chi^2):          0.2434

JARQUE-BERA TEST:

test statistic:            77.5849
 p-Value(Chi^2):           0.0000
skewness:                 -1.0021
kurtosis:                  8.9667
```

```
ADF Test for series:      OMF_AUMIndex_d1
sample range:             [2007 M2, 2010 M12], T = 47
lagged differences:       0
intercept, time trend
asymptotic critical values
reference: Davidson, R. and MacKinnon, J. (1993),
"Estimation and Inference in Econometrics" p 708, table 20.1,
Oxford University Press, London
 1%         5%         10%
-3.96      -3.41      -3.13
value of test statistic: -4.7039
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
 x(-1)        -0.6563       -4.7039
constant      -17.7162      -1.8339
trend          0.7593        1.1267
RSS            166492.4612

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags
```

```
Portmanteau:                12.4927
 p-Value (Chi^2):           0.4070
Ljung & Box:                15.0463
 p-Value (Chi^2):           0.2389

JARQUE-BERA TEST:

test statistic:             69.8343
 p-Value(Chi^2):            0.0000
skewness:                  -0.5770
kurtosis:                   8.8591
```

10.2. Unit Root tests with structural break for "OMF_AUMIndex_D1" (lags=0); with an impulse dummy (April 2007) and with a shift dummy (April 2007); with constant and no trend and with both constant and a trend

```
Break date search for series:  OMF_AUMIndex_d1
sample range:                  [2007 M2, 2010 M12], T = 47
searched range:                [2007 M2, 2010 M10], T = 45
number of lags (1st diff):     0
suggested break date:          2007 M4
```

```
UR Test with structural break for series: OMF_AUMIndex_d1
sample range:                  [2007 M2, 2010 M12], T = 47
number of lags (1st diff):     0
value of test statistic:      -5.1269
used break date:               2007 M4
shiftfunction:                 impulse dummy
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       22.4919       0.3050
d(shiftfkt)   -11.1674      -0.1071

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:                12.8362
 p-Value (Chi^2):           0.3811
Ljung & Box:                14.3354
 p-Value (Chi^2):           0.2798
```

```
JARQUE-BERA TEST:

test statistic:            106.4076
 p-Value(Chi^2):           0.0000
skewness:                 -1.4323
kurtosis:                  9.8783
```

```
UR Test with structural break for series: OMF_AUMIndex_d1
sample range:                   [2007 M2, 2010 M12], T = 47
number of lags (1st diff):      0
value of test statistic:       -4.5567
used break date:                2007 M4
shiftfunction:                  impulse dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)      -0.8787       -0.0017
d(const)       23.3706       0.3134
d(shiftfkt)   -11.1674      -0.1059

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        2

PORTMANTEAU TEST with 12 lags

Portmanteau:               15.8122
 p-Value (Chi^2):          0.2000
Ljung & Box:               17.7338
 p-Value (Chi^2):          0.1240

JARQUE-BERA TEST:

test statistic:            102.8845
 p-Value(Chi^2):           0.0000
skewness:                 -1.3559
kurtosis:                  9.8062
```

```
Break date search for series:  OMF_AUMIndex_d1
sample range:                   [2007 M2, 2010 M12], T = 47
searched range:                 [2007 M2, 2010 M10], T = 45
number of lags (1st diff):      0
suggested break date:           2007 M4
```

```
UR Test with structural break for series: OMF_AUMIndex_d1
sample range:                   [2007 M2, 2010 M12], T = 47
number of lags (1st diff):      0
value of test statistic:       -5.3419
used break date:                2007 M4
shiftfunction:                  shift dummy
critical values (Lanne et al. 2002):
```

```
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.48      -2.88      -2.58
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(const)       22.4919       0.3073
d(shiftfkt)   -63.0208      -0.8609

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        3

PORTMANTEAU TEST with 12 lags

Portmanteau:               16.6622
 p-Value (Chi^2):          0.1628
Ljung & Box:               18.8036
 p-Value (Chi^2):          0.0934

JARQUE-BERA TEST:

test statistic:            112.7525
 p-Value(Chi^2):           0.0000
skewness:                 -1.7047
kurtosis:                  9.8704
```

```
UR Test with structural break for series: OMF_AUMIndex_d1
sample range:                  [2007 M2, 2010 M12], T = 47
number of lags (1st diff):     0
value of test statistic:      -5.7508
used break date:               2007 M4
shiftfunction:                 shift dummy
time trend included
critical values (Lanne et al. 2002):
---------------------------------------
 T          1%         5%         10%
---------------------------------------
1000       -3.55      -3.03      -2.76
---------------------------------------
regression results:
---------------------------------------
variable      coefficient   t-statistic
---------------------------------------
d(trend)       0.4723        0.0009
d(const)       22.0197       0.2975
d(shiftfkt)   -63.4930      -0.8579

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA

sample range:             [2007 M12, 2010 M12], T = 37

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion:    10
Final Prediction Error:   10
Hannan-Quinn Criterion:   3
Schwarz Criterion:        2
```

```
PORTMANTEAU TEST with 12 lags

Portmanteau:                 14.6843
 p-Value (Chi^2):            0.2592
Ljung & Box:                 16.5445
 p-Value (Chi^2):            0.1676

JARQUE-BERA TEST:

test statistic:              116.7432
 p-Value(Chi^2):             0.0000
skewness:                   -1.7703
kurtosis:                    9.9551
```

```
KPSS test for series:  OMF_AUMIndex_d1
sample range:          [2007 M1, 2010 M12], T = 48
number of lags:        0
KPSS test based on y(t)=a+e(t) (level stationarity)
asymptotic critical values:
 10%       5%        1%
0.347     0.463     0.739
value of test statistic: 0.0000
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
with permission from Elsevier Science
```

```
KPSS test for series:  OMF_AUMIndex_d1
sample range:          [2007 M1, 2010 M12], T = 48
number of lags:        0
KPSS test based on y(t)=a+bt+e(t) (trend stationarity)
asymptotic critical values:
 10%       5%        1%
0.119     0.146     0.216
value of test statistic: 0.0000
reference: reprinted from JOURNAL OF ECONOMETRICS,
Vol 54, No 1, 1992, pp 159-178, Kwiatkowski et al:
"Testing the null hypothesis of stationarity ...",
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```