Natural convection heat transfer from horizontal annular finned tubes based on modified Rayleigh

H. Nemati1, M. Moradagha2, S.A. Shekoohi3, M.A. Moghimi4, J.P. Meyer4

1Department of Mechanics, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran

2MSc graduate student of Mech. Eng., Islamic Azad University, Shiraz branch, Shiraz, Iran

*3HAMPA Energy Engineering and Design Company, Shiraz, Iran*

*4Clean Energy Research Group, Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, South Africa*

The modification of the *Rayleigh*, *Ra*, number has been studied in the past. These studies have shown that the physical interpretation of the modified *Rayleigh* number which is commonly used is questionable. Moreover, although most of the available correlations work well, their behaviours in extremes (low or high *Ra*) are not correct where only a few correlations are available in low *Ra*. Therefore, a numerical simulation of heat transfer from annular circular finned tubes was conducted to present a comprehensive correlation for the modified *Rayleigh* number. As shown in this study, the flow forms a circular path around the tube. Based on this fact, a new modified *Rayleigh* number definition and correlation are proposed, which are valid for Nusselt numbers from 1 to 20 000. This range of *Ra* provides a complete picture of natural convection over circular annular finned tubes, especially at relatively low *Ra*. Finally, the end behaviours of the proposed correlation were compared with those of existing correlations.

Keywords: Natural convection, annular finned tube, modified *Rayleigh*, flow pattern, CFD

# Introduction

Natural convection heat transfer plays an important role in air heat exchangers used in many industries [1-3]. Air is an available, non-corrosive fluid that cannot be replaced easily by other fluids. Designing an air heater based on natural convection turns it into an unrivalled heat exchanger. No rotary equipment such as a fan or blower is required in this type of heat exchanger. Therefore, it is cost effective and also suitable for novel applications such as solar applications [4-6].

Heat transfer in natural convection is considerably less in comparison with forced convection heat transfer. Using an extended surface to enhance natural convection heat transfer from a horizontal tube array is a common method in engineering applications. Therefore, many researchers have studied natural convection heat transfer both numerically and experimentally [7-17] and are still working on it.

The study of natural convection from vertical parallel plates can be traced back to the analytical and experimental works of Elenbaas [18]. He used embedded resistors in the plates and measured voltage and current values to determine the convective heat transfer under steady-state condition. He also modified the *Rayleigh* number by multiplying it by plate spacing to plate length ratio, $S/H$ (Fig. 1).

|  |  |
| --- | --- |
|  | (1) |

 

*S*

*H*

Fig. 1: A schematic view of a plate fin.

The main reason for this modification is that the fluid flow in these channels is confined and is categorised as internal flow. For the internal flow, the developing flow is mainly different from a fully developed flow. It is known that the degree of development of the boundary layer depends on channel diameter to channel length ratio, i.e*.* ($S/H$) [19].

Elenbaas’s experimental results were correlated with [18]:

|  |  |
| --- | --- |
|  | (2) |

Eq.(2) reflects some critical aspects associated with the two limiting cases:

* As  Eq.(2) approaches to *Nu* for fully developed channel flow, i.e*.*

|  |  |
| --- | --- |
|  | (3) |

* On the contrary, as, Eq.(2) predicts *Nu* for a single vertical plate, i.e*.*

|  |  |
| --- | --- |
|  | (4) |

Eq.(2) is historically important because it is the base of other equations proposed to predicted natural convection from the horizontal annular finned tube.

In an annular finned tube, circular aligned discs are attached near each other on a horizontal tube as shown in Fig. 2.

*S*

*D*

*d*

Fig. 2: A schematic view of an annular circular finned tube.

Three main geometrical parameters, *D*, *d* and *S,* play important roles in the estimation of natural convection from an annular finned tube. In the above figure, *D*, *d* and *S* are the fin diameter, outer tube diameter and the space between two adjacent fins respectively. The heat transfer coefficient, *h,* or its dimensionless form, *Nu*,is a strong function of *Rayleigh* number, *Ra*, which is defined for a horizontal tube as the following [20, 21]:

|  |  |
| --- | --- |
|  | (5) |

where *β* is the air thermal expansion coefficient, *α* and *ν* are air thermal diffusivity and kinematic viscosity respectively and the *Nusselt* number based on fin spacing is:

|  |  |
| --- | --- |
|  | (6) |

Edwards and Chaddock [22] performed a series of experiments on horizontal annular finned tubes. Similar to a parallel plate-fin heat sink (Fig. 1), the flow field on the annular finned tube is internal flow [11]. In this regard, the authors presented the following form for the *Rayleigh* number:

|  |  |
| --- | --- |
|  | (7) |

Moreover, to specify the degree of development of the boundary layer, they followed the idea of Elenbaas [18] to modify the *Rayleigh* number. In this way:

|  |  |
| --- | --- |
|  | (8) |

 Comparing Eq.(8) and Eq.(1) shows that Edwards and Chaddock [22] simply replaced fin length in Eq.(1) by fin diameter. However, presents nothing about tube diameter, *d*. They presented a correlation for *Nu* in the range of .

Jones and Nwizu [23] used the same parameters as Edwards and Chaddock [22] did for . However, they reported that their results were slightly conservative. Kayansayan and Karabacak [9] used tube diameter as characteristic length. The reason for their selection was ability in direct comparison of the results with the literature data for a single cylinder. They did not modify the *Ra* definition since they focused on a range of data where the interaction of the fin boundary layer with the cylinder was weak.

Hahne and Zhu [10] used the mean diameter of fin and tube, $d\_{e}=(D+d)/2$, as the characteristic length in *Ra* to present their experimental results. They also modified *Ra* by the factor *S*/*d*. Similar to Hahne and Zhu [10], Knudsen and Pan [24] used the mean diameter as the characteristic length in their experimental correlation with *S*/*d* correction factor. Their correlation is a straight line in a log-log plot and is not applicable for fully developed flow.

The work of Wang et al. [11] was based on body gravity function and they used $\sqrt{A}$ as the characteristic length. However, their proposed equation seems very complicated and confusing. They also proposed a simplified model in their paper, but their simplified model is still lengthy.

The most important study in this field was undertaken by Tsubouchi and Masuda [7], which was reported in some handbooks or text books [21, 25]. Tsubouchi and Masuda [7] performed a large number of experiments on natural convection from annular finned tubes for . First of all, they performed a series of experiments on parallel isothermal discs and as a result, proposed the following equation for parallel isothermal circular discs:

|  |  |
| --- | --- |
|  | (9) |

where fluid properties are evaluated at the tube temperature. For a large range of , the correlation above approaches to Eq.(2) where the characteristic length, *H*, leads to (π*D*)/4. Finally, they proposed voluminousEq.(10) for isothermal circular fins around the tube with :

|  |  |
| --- | --- |
| , for  | (10) |

where

|  |  |
| --- | --- |
| ,, |  |

The traditional form of modified *Rayleigh* number (Eq.(8)), was later used by Senapati et al*.* [26] in their numerical simulations. They simulated natural convection heat transfer numerically and presented the following correlation:

|  |  |  |
| --- | --- | --- |
|   | for  | (11) |

 where

|  |  |  |  |
| --- | --- | --- | --- |
| , | , | , | , |
| , | , | , | , |

Their results slightly overshot the experimental correlation of Tsubouchi and Masuda. They claimed that considering heat transfer from the fin rim was the reason for overshooting in their work.

The above discussion is summarised in Table 1. As shown, most of these studies used the introduced *Ra* modification factor of Ref. [22]. However, all three main basic geometrical parameters shown in Fig. 2 are not included in this format of the modified *Ra*; indeed, the diameter of the central tube, *d*, was excluded in the modified *Rayleigh* number definition.

Moreover, the authors believe that although all these equations may predict the heat transfer coefficient with acceptable accuracy, the whole aspect of applying the *S*/*H* concept proposed by Elenbaas [18] has been missed in using *S*/*D* for annular fins. Actually, in a plate fin, *H* is the length passed by a developing flow, while in annular fin, *D* is nothing but a geometrical parameter. Therefore, the physical interpretation of using *S*/*D* might cast doubt. Although using “*D*” as an index for channel length in *S*/*D* ratio is correct for parallel discs (parallel fins with no central tube), it is not true for an annular finned tube. It does not make sense to the authors of this study that the same expression can be used in the definition of modified *Ra* in both parallel discs (without a tube in the centre) and annular fins (with a tube in the centre).

The other objective of this study is extending the applicable range of *Ras* in particular in lower range. As shown in Table 1, most of the proposed equations were done in relatively high *Ras*. However, at the lower range of *Ras* (for ), a unanimous agreement may not take place among them (see Fig. 12). At low *Ras*, numerical methods are superior from two aspects. First of all, experimental results are more sensitive to measuring tools accuracy. In other words, because the *Nu* value is low, experimental errors get critical. The second is that most low *Ra*’s have been achieved by low temperature and not small fin spacing. Fortunately, numerical methods provide the opportunity to simulate natural convection at very low *Ras* with relatively high accuracy and also with a wide variety of temperature difference and fin spacing. In the present study, the results are valid in the range of .

|  |
| --- |
| **Table 1: Different definitions of *Ra* and the modification factor summary**  |
| Ref. | *Ra* | ModificationFactor | ApplicableRange |
| [22] |  |  |  |
| [23] |  |  |  |
| [9] |  | *---* |  |
| [10] |  |  |  |
| [24] |  |  |  |
| [7] |  |  |  |
| [26] |  |  |  |

In the present study, laminar natural convection over annular finned tube is studied numerically. Simulation is conducted in a wide range of geometrical parameters. Based on numerical flow visualisation, a new modification of *Rayleigh* number is proposed. This new modification is totally compatible with the idea of Elenbaas [18]. Later, the new modified *Ra* number is used to propose a simple but useful correlation to predict *Nusselt* number. This new correlation is much simpler in form and can be used easily in engineering applications. Finally, the proposed correlation is compared deeply in restricting cases, especially in very low *Ras* with previous correlations to show the weakness and strength of each correlation.

# Mathematical modelling

For a laminar steady flow over the circular fins, the governing equations and boundary conditions are as presented in the following section. All thermophysical properties with the exception of fluid density are assumed constant and are evaluated at the mean temperature between ambient and wall temperatures, i.e. film temperature.

|  |  |
| --- | --- |
|  | (12) |

## Main equations:

Continuity equation:

|  |  |
| --- | --- |
|  | (13) |

Momentum equation:

|  |  |
| --- | --- |
|  | (14) |
|  | (15) |
|  | (16) |

Energy equation in the fluid zone:

|  |  |
| --- | --- |
|  | (17) |

and energy equation in the solid zone:

|  |  |
| --- | --- |
|  | (18) |

Air is assumed as ideal gas for which:

|  |  |
| --- | --- |
|  | (19) |

## Boundary conditions:

A schematic view of an annular circular finned tube and computational domain is shown in Fig. 3. The computational domain is assumed as a cylinder with a diameter five times the fin diameter, *D*. In this figure, fin spacing and fin pitch are indicated by *S* and *c* respectively.

At the tube wall surface:

|  |  |
| --- | --- |
| ,  | (20) |

at the boundary far from the finned tube, pressure outlet boundary condition is assumed:

|  |  |
| --- | --- |
| ,  | (21) |

the left and right flat surfaces ( ) are assumed as symmetry surfaces:

|  |  |
| --- | --- |
|  | (22) |

and at the solid-fluid interface, the heat flux and temperature continuity are held:

|  |  |
| --- | --- |
|  | (23) |

The correctness of selected boundary conditions is discussed in Part 4.



Fig. 3. A schematic view of an annular circular finned tube and computational domain.

# Solution methodology

Hexagonal cells in the form of the structured grid were used to discretise the domain. A schematic grid distribution over the cross-section of the computational domain is shown in Fig. 4. Near the fin and the tube, grids are very fine and away from the tube, grids get course gradually. A grid study was carried out on the smallest and largest diameters to ensure the accuracy of the results. The results of the grid independence study are reported in Fig. 5, which shows for about number of elements, the numerical results get independent of the grid. The results of the grid independence study are reported in Fig. 5. The governing equations of this study were numerically solved by ANSYS CFX as a commercial CFD software [27]. The residual error settings of this solution were set to 10-4, at least while the total heat transfer from the fin was monitored during the simulation to ensure the convergence. Parallel computing in CFX was utilised to reduce the solving time and the high resolution scheme was used for discretising the advection terms. ANSYS CFX uses non-staggered grid layout to the decoupling mass flows and pressure field [27]. This method was basically proposed by Rhie and Chow [28] and modified later by Majumdar [29]. Since the grid number is relatively high, computing the solution is time-consuming. The processor which was used was Intel® Core™ i7-6800K Processor (15M Cache, up to 3.60 GHz). It took around three hours for each case to reach convergence.



*y*

*x*

*g*

Fig. 4. Grid arrangement over the cross-section of the computational domain.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *d*(mm) | *D*(mm) | *tf*(mm) | *S*(mm) | (K) |
| 25.4 | 57.15 | 0.4 | 25.24 | 40 |

Fig. 5. Variation of *Nu* with number of elements.

By the described method, nearly 70 different cases were simulated within the listed ranges of parameters in Table 2.

|  |
| --- |
| Table 2: Range of simulated parameters  |
| Dimension | *d*(mm) | *D*(mm) | *S*(mm) | *tf*(mm) | (K) |
| Max. | 120 | 140 | 25.24 | 0.4 | 80 |
| Min. | 18 | 57.15 | 3.22 | 0.4 | 40 |

According to cooling Newton’s law, the convective heat transfer coefficient can be calculated as (see Appendix 1 for a more detailed discussion):

|  |  |
| --- | --- |
|  | (24) |

and consequently, *Nu* as the main dimensionless parameter was calculated.

# More tests on boundary conditions

Two more tests were carried out to ascertain the accuracy of the mentioned boundary conditions, i.e*.* the domain independence test and the symmetry boundary condition test.

## Domain independence test

The domain should be large enough to not affect the heat transfer from the finned tube. Selecting a larger domain than the required value increases the computational time only, which is not favourable. However, the small domain also affects the accuracy of the results. Fig. 6 shows the variation of heat flux with respect to domain diameter. Based on this figure, five times the fin diameter was selected for the upper bound of the computational domain diameter.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *d*(mm) | *D*(mm) | *tf*(mm) | *S*(mm) | (K) |
| 25.4 | 57.15 | 0.4 | 4.68 | 40 |

Fig. 6. Variation of heat flux with domain diameter.

## Symmetry boundary condition

The symmetry boundary condition was also verified. In this regard, the transient simulation was conducted over a tube with 9 fins. Since the intension was only to examine the degree of flow field symmetry, a courser mesh was adopted in this test.

Temperature contours over the fin tube and a zoomed-in image of two middle fins are shown in Fig. 7. These figures prove the symmetry assumption is a reasonable boundary condition definition.

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |

Fig. 7. Flow temperature contours a) over the fin tube b) zoomed-in image of middle fins (4th, 5th and 6th fins from left).

# Flow pattern study

Flow pattern study is a powerful technique that is easier to achieve in numerical methods than in experimental methods. To study the behaviour of flow streamlines, temperature contour and streamlines on a plane in the middle of fin spacing are displayed in Fig. 8. In these images, the left sides are dedicated to temperature contours while the right sides are flow streamlines.

Because the temperature contours on the plane in the middle of fin thickness and in the middle of fin spacing were approximately similar, the temperature contour on the surfaces in the middle of fin thickness was not presented for the sake of brevity. The temperature contour shows a slender plume rising from the tube in a circular form. Its diameter is slightly greater than fin diameter and it is shaded in a short distance above the fin tip. The plume in its path upwards sucks the neighbour’s fresh air and gets colder. The right side of Fig. 8 shows streamlines. It is worthwhile to mention that the streamlines take the form of the ring between tube and fin and move in an annular shape. This circular pattern of flow can be observed even in the wider fin spacing (Fig. 8 c and Fig. 8 d).

|  |  |
| --- | --- |
| 1.jpg | 62.jpg |
| (a) | (b) |
| 55..jpg55..jpg55..jpg | 10.jpg |
| (c) | (d) |

|  |
| --- |
| Fig. 8. Temperature contour (left) and streamlines (right) on the middle fin space surface.*d*= 25.4 mm, *D*=57.15 mm, *tf*=0.4 mm |
|

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | (a) | (b) | (c) | (d) |
| *S* (mm) | 3.22 | 3.22 | 25.2 | 25.2 |
| () (K) | 20 | 40 | 20 | 40 |

 |

# Proposed new *Ra* modification

To recap on the historical background of natural convection from an extended surface, Elenbaas [18] was the first who proposed to modify the *Ra* number for a rectangular plate fin (see Fig. 1). The idea behind the modification was that the flow between extended surfaces is similar to the internal flow confined in a duct. Therefore, the flow travelling length shows the degree of boundary layer development. In this regard, he proposed the parameter, , as a modification of *Ra* in which *S* is the distance between two adjacent fins. According to Elenbaas’s philosophy, *H* in the  is the channel length which is passed by the fluid. However, this philosophy was lost in the work of Edwards and Chaddock [22]. They simplistically replaced *H* by *D* and after that, this modification was imitated by other researchers. Although their final proposed correlations may have adequate accuracy, this modification has lost its physical meaning. It is noteworthy that, although complicated correlations may result in good accuracy, it is expected that a simpler correlation reaches the same accuracy by using a proper modified *Ra* definition which is compatible with the nature of flow pattern.

As a virtual experiment, let us assume a simple case with two parallel discs; taking into account that *H* is the average length passed by the flow. Fig. 9a shows a simple view of parallel discs over which flow passes. Based on this figure, the average path length may be approximated as:

|  |  |
| --- | --- |
|  | (25) |

that *A* is a disc surface area. It is interesting that  is the chosen parameter by Tsubouchi and Masuda [7] to stand for *H* for parallel isothermal circular discs (Eq.(9)). The same method can be applied to an annular finned tube shown in Fig. 9b. For annular fins, according to Fig. 9b, fluid passes a circular path between two fins (the assumption of the circular path was studied previously in Fig. 8). In the same way, based on Fig. 9b, the average path length for annular fins may be approximated as:

|  |  |
| --- | --- |
|  | (26) |

ignoring the constant value,  , the proposed modified *Ra* is:

|  |  |
| --- | --- |
|  | (27) |

It is clear as  (that means parallel discs), Eq.(26) transforms to Eq.(25).

|  |  |
| --- | --- |
|  | Fluid path |
| (a) | (b) |

Fig. 9. Fluid path between a) two parallel disks, b) two parallel fins.

As the last step, to investigate the quality of the new proposed modified *Ra,* the calculated *Nu* numbers based on the current numerical procedure are plotted against two different *Rayleigh* number definitions, i.e*.* the traditional definition (Eq.(8)) and the proposed one (Eq.(27)). Fig. 10 illustrates the result of this comparison. As is clear in this figure, the proposed *Ra* brings more data points into a common correlation. A zoomed-in part of this graph is presented also and shows that using the new proposed *Ra* reduces the scattering in data points considerably.

 

Fig. 10. *Nu* number based on traditional *Ras* and proposed *Ra*.

# Proposed new *Nu* correlation

With this new definition of *Ra*, the following correlation is proposed to predict *Nu* (for more detail about , see Appendix 1):

|  |  |
| --- | --- |
| , for  | (28) |
|  | (29) |
|  | (30) |

The value of *R*2 for Eq.(28) is 0.996 and the standard error is 4.9%, which shows good agreement with the proposed correlation. The comparison of Eq.(28) with the numerical results is shown in a scatter plot in Fig. 11.

Fig. 11. The comparison of the present correlation with numerical results.

Eq.(28) is also verified against several experimental and numerical works. In this regard, it is important to note that:

|  |  |
| --- | --- |
|  | (31) |

where . Fig. 12 shows this comparison for *D*/*d*=1.94 as an example.

Fig. 12. Comparison of the present work with previous studies.

Fig. 12 shows very good agreement with the present work and other previous studies. Among them, the most agreement can be observed with the numerical work of Senapati et al. [26] and the experimental work of Jones and Nwizu [23].

The behaviours of different curves in Fig. 12, at two extremes of *Ras* (low and high *Ras*), are worth noting. At high *Ras*, results of Tsubouchi and Masuda [7] are slightly less than the others. Tsubouchi and Masuda worked on relatively thick fins and at the end, they separated the heat transfer from the fin rim and presented two separate equations for *Nu*, one for fin tip surface and another for tube- and fin-side surfaces. However, in the present study as well as in the work of Senapati et al. [26], the heat transfer from fin tip is also included. Unfortunately, the fin thickness in the work of Senapati et al. is not reported. However, in the present study, fin thickness is 0.4 mm. Therefore, the fin tip area is negligible in comparison with fin sides and the tube surface area and consequently, its heat transfer is not comparable. For this reason, the authors of this study guess that contribution of fin tip heat transfer is overpredicted in Tsubouchi and Masuda work. On the other hand, at low *Ras*, the present correlation and the work of Tsubouchi and Masuda are nearly the same. This is also true for other researchers except for Senapati et al., whose work is basically different from the others.

Based on Fig. 12, although all curves are adequately close to each other, their trends are not the same. For this reason, the present correlation at the limiting conditions is compared with those of other researchers. For this comparison, the work of Tsubouchi and Masuda is selected as an experimental work candidate and the work of Senapati et al. as a numerical work candidate. These works have wider working ranges than the others.

# Comparison of the present work with the others at limiting working conditions

It remains only to judge equations in limiting working conditions. Before starting this comparison, it must be reminded that:

|  |  |
| --- | --- |
|  | (32) |

The comparison may be divided into two categories. The first one is that tube diameter is approaching to zero. This case simulated parallel circular discs. The other case is that the tube diameter is different from zero. In each of these two main cases, two limiting cases are studied, i.e*.* small *Ras* and large *Ras.* Each of these cases is studied in the following section. The results of this discussion are also summarised in Table 3.

|  |
| --- |
| Table 3: Comparison between the proposed correlation and others in the limiting conditions  |
|  | $$d\rightarrow 0$$ | $$d\ne 0$$ |
| *Nu* | Small*Rayleigh* | Large*Rayleigh* | Small*Rayleigh* | Large*Rayleigh* |
| Tsubouchi and Masuda [7] (Eq.(10)) | $$0.053Ra\_{s}$$ | $$0.6Ra\_{s}^{0.25}$$ | $$0.053Ra\_{s}$$ | $$Ra\_{s}^{0.25}×\left(0.6-0.03\sqrt{1+152ξ^{2}}\right)$$ |
| Senapati et al. [26] (Eq.(11)) | $$undefined$$ | $$undefined$$ | $$less than zero$$ | $$0.05Ra\_{s}^{0.35}ξ^{-0.17}$$ |
| Present correlation (Eq.(28)) | $$0.092Ra\_{s}$$ | $$0.58Ra\_{s}^{0.25}$$ | $$\frac{Ra\_{s}}{1+ξ}$$ | $$\frac{0.58Ra\_{s}^{0.25}}{\left(1+ξ\right)^{0.25}}$$ |
| Expected value | $$Nu∝Ra\_{s}$$ | $$Nu∝Ra\_{s}^{0.25}$$ | $$Nu∝f\left(ξ\right)∙Ra\_{s}$$ | $$Nu∝f(ξ)∙Ra\_{s}^{0.25}$$ |

## Parallel circular discs ($d\rightarrow 0)$

This limiting condition is similar to a set of parallel discs which was the base of the work of Tsubouchi and Masuda [7]. For this special case, two limiting working conditions may be assumed, i.e*.* small *Rayleigh* number and large *Rayleigh* number:

**a) Small *Rayleigh* number (fully developed condition)**

* Tsubouchi and Masuda [7] (Eq.(10)):

$$\lim\_{Ra\_{s}\to 0}Nu\_{Ts}=\frac{Ra\_{s}}{6π}=0.053Ra\_{s}$$

* Senapati et al. [26] (Eq.(11)):

$$\lim\_{Ra\_{s}\to 0}Nu\_{se}=undefined$$

* Present correlation (Eq.(28)):

$$\lim\_{Ra\_{s}\to 0}Nu=0.092Ra\_{s}$$

In this case, both the experimental correlation of Tsubouchi and Masuda [7] (Eq.(10)) and the present correlation (Eq.(28)) show the behaviour of fully developed flow. In the fully developed condition, *Nu* will be proportional to *Ras* (see Eq.(3))*.* However, proportionality constant in Eq.(10) is more than Eq.(28). This might be due to heat transfer from the fin rim because, in such a small heat transfer, the heat transfer from the fin tip gets relatively important. On the other hand, Eq.(11) is undefined, which shows that this equation does not follow fully developed behaviour in small *Ras*.

**b) Large *Rayleigh* number (single vertical plate)**

In this case, because the thermal boundary layer is thin, all correlations are expected to resemble the natural convection over a single vertical plate. Therefore, under this condition, *Nu* will be proportional to $Ra\_{s}^{0.25}$.

* Tsubouchi and Masuda [7] (Eq.(10)):

$$\lim\_{Ra\_{s}\to \infty }Nu\_{Ts}=\frac{11.3}{6π}Ra\_{s}^{0.25}=0.6Ra\_{s}^{0.25}$$

* Senapati et al. [26] (Eq.(11)):

$$\lim\_{Ra\_{s}\to \infty }Nu\_{se}=undefined$$

* Present correlation (Eq.(28)):

$$\lim\_{Ra\_{s}\to \infty }Nu=0.58Ra\_{s}^{0.25}$$

In this case, Eq.(11) is again undefined while the experimental correlation of Tsubouchi and Masuda [7] (Eq.(10)) and the present correlation (Eq.(28)) are very close. The relative error between the present correlation and Eq.(10) () is presented against *Ras* in Fig. 13.

Fig. 13. The relative error between experimental and present correlation in large *Ras*.

## Annular finned tube ($d\ne 0)$

In the presence of an inner tube, again, two limiting working conditions, i.e*.* small *Rayleigh* number and large *Rayleigh* number, are considered:

**a) Small *Rayleigh* number (fully developed condition)**

* Tsubouchi and Masuda [7] (Eq.(10)):

$$\lim\_{Ra\_{s}\to 0}Nu\_{Ts}=\frac{Ra\_{s}}{6π}=0.053Ra\_{s}$$

* Senapati et al. [26] (Eq.(11)):

$$\lim\_{Ra\_{s}\to 0}Nu\_{se}=-3.827+2.548ξ^{-0.009}$$

* Present correlation (Eq.(28)):

$$\lim\_{Ra\_{s}\to 0}Nu=\frac{Ra\_{s}}{1+ξ}$$

In this comparison, Eq.(11) is not a function of *Ras* which is incorrect. On the other hand, *Nu* in both present equation and Eq.(10) is proportional to *Ras* which resembles the fully developed behaviour. However, the proportionality constant in contrast with the Tsubouchi and Masuda correlation is a function of $ξ$.

**b) Large Rayleigh number (single vertical plate)**

* Tsubouchi and Masuda [7] (Eq.(10)):

$$\lim\_{Ra\_{s}\to \infty }Nu\_{Ts}=\frac{Ra\_{s}^{0.25}}{12π}\left(23.7-1.1\sqrt{1+152ξ^{2}}\right)$$

* Senapati et al. [26] (Eq.(11)):

$$\lim\_{Ra\_{s}\to \infty }Nu\_{se}=0.05Ra\_{s}^{0.35}ξ^{-0.17} $$

* Present correlation (Eq.(28)):

$$\lim\_{Ra\_{s}\to \infty }Nu=\frac{0.58Ra\_{s}^{0.25}}{\left(1+ξ\right)^{0.25}}$$

Here, again, both Eq.(10) and Eq.(28) resemble natural convection over a single vertical plate (*Nu* is proportional to $Ra\_{s}^{0.25}$) but the proportionality constant is a function of $ξ$. However, the power of $Ra\_{s}$ in Eq.(11) is far from 0.25 (0.35 against 0.25).

The comparisons of the above three equations are shown in Fig. 14. Based on these graphs, the behaviour of the present correlation in small $ξ$ is more similar to Eq.(10), while in relatively large $ξ$, it approaches to Eq.(11).

Fig. 14. Variations of different *Nu* against *Ras* for various $ξ$ in larg *Ras*.

## Parallel circular discs over the entire range of *Ra*

It was claimed that the new proposed Rayleigh number includes the effect of central tube curvature. Therefore, it is expected that if one lets d=0, the new proposed equation, Eq.(28), will represent the parallel circular disc Nusselt number in the entire range of Ra. For this reason, Eq.(28) was compared with Eq.(9), which is a specific equation for parallel circular disc. It is worthwhile to note that in the case of *d*=0, both *Ra* and *Ras* are the same. The presented results in Fig. 15 show very good agreement. Therefore, Eq.(28) can present *Nu* for both annular fins and parallel circular discs.

Fig. 15. Comparison of Eq.(9) and Eq.(28) for *d*=0.

# Conclusion

In the present study, natural convection heat transfer from annular finned tubes was investigated. The following important conclusions are presented:

1. The visualisation of flow field shows that air passes a circular path around the tube. Therefore, the same expression for both parallel discs and annular fins may not be used for *Ra*.
2. A new expression for *Ra* was presented to consider the presence of the tube in a finned tube. This new expression is more compatible with flow pattern.
3. A new correlation was proposed for *Nusselt* number based on the modified *Rayleigh* number and compared deeply with previous correlations.
4. By this new correlation, the bottom limit of *Ras* is extended and *Nu* may be evaluated at the lower *Ras*.
5. The present correlation in spite of previous correlations resembles better the behaviour of flow in limiting conditions. In low *Ras*, it resembles the fully developed flow and in large *Ras*, it simulates natural convection over a single vertical plate.
6. It was shown that although previous correlations work well in moderate *Ras*, their accuracy is poor in limiting cases, i.e. low and high *Ras*.
7. It can represent *Nu* for both annular fins and parallel circular discs.

# Appendix 1 (non-isothermal fin)

All presented equations in the main text are based on the assumption of the isothermal fin where fin surface temperature is uniform and at the tube wall temperature, . However, it is known that due to fin finite thermal conductivity, the accuracy of this assumption is questionable. To compensate the non-isothermal fin temperature, it is recommended to use wall average temperature instead of wall tube temperature [11, 18, 30]. The average fin tube temperature may be introduced by the following combination:

|  |  |
| --- | --- |
|  | (33) |

where  and  are bare tube area and fin area respectively.

 in Eq.(33) is the fin efficiency:

|  |  |
| --- | --- |
|  | (34) |

and occurs when the fin surface transfers heat at bare tube temperature, .

|  |  |
| --- | --- |
|  | (35) |

It is clear that when fin also works at the bare tube temperature, , i.e*.* , the average temperature will be equal to the bare tube temperature. The fin efficiency *η* is calculated by [31]:

|  |  |
| --- | --- |
|  | (36) |
|  | (37) |
|  | (38) |

where *ks* and *tf* are fin thermal conductivity and fin thickness respectively. In this regard, for example, the proposed modified *Rayleigh* for non-isothermal fin is:

|  |  |
| --- | --- |
|  | (39) |

and also, the convective heat transfer coefficient can be calculated as:

|  |  |
| --- | --- |
|  | (40) |

In the present study, the above procedure was employed to account for this variable temperature.

# Nomenclature

|  |  |  |
| --- | --- | --- |
| *A= Af + Ab* | = | finned tube heat transfer area per unit length of the tube (m2) |
| *Ab* | = | bare tube heat transfer area per unit length of the tube (m2) |
| *Af* | = | fin surface area per unit length of the tube (m2) |
| *Cp* | = | air specific heat capacity (J/kgK) |
| *c* | = | fin pitch (m) |
| *D* | = | fin diameter (m) |
| *d* | = | tube outer diameter (m) |
| $$d\_{e}=(D+d)/2$$ | = | the mean diameter of fin and tube(m) |
| *g* | = | gravitational acceleration (m/s2) |
| *h* | = | average convective heat transfer coefficient (W/m2K) |
| *k* | = | fluid thermal conductivity (W m-1 K-1) |
| *ks* | = | fin thermal conductivity (W m-1 K-1) |
| *Nu* | = | *Nusselt* number |
| *P* | = | pressure (Pa) |
|  | = | heat transfer between finned tube and air (W) |
| *R*=287.1 | = | air gas constant (J/kgK) |
|  |  | proposed modified finned tube *Rayleigh* number,  |
|  | = | heat sink modified *Rayleigh* number, |
|  | = | horizontal tube *Rayleigh* number,  |
|  | = | finned tube modified *Rayleigh* number,  |
|  |  | finned tube *Rayleigh* number,  |
| *S* | = | fin spacing (m) |
|  | = | film temperature (K) |
|  | = | average fin surface temperature (K) |
| *Tw* | = | tube surface temperature (K) |
|  | = | ambient temperature (K) |
| *tf* | = | fin thickness (m) |
| *α* | = | air thermal diffusivity (m2/s) |
| *β* | = | air thermal expansion coefficient (1/K) |
| *η* | = | fin efficiency |
| *µ* | = | viscosity (Pa.s) |
| *ρ* | = | air density (kg/m3) |
| *ν* | = | air kinematic viscosity (m2/s) |
|  | = | tube to fin diameter ration |

# References

[1] Effendi N.S., Kim K.J., Orientation effects on natural convective performance of hybrid fin heat sinks, Appl. Therm. Eng., 123 (2017) 527-536.

[2] Micheli L., Reddy K., Mallick T.K., Thermal effectiveness and mass usage of horizontal micro-fins under natural convection, Appl. Therm. Eng., 97 (2016) 39-47.

[3] Tari I., Mehrtash M., Natural convection heat transfer from horizontal and slightly inclined plate-fin heat sinks, Appl. Therm. Eng., 61 (2013) 728-736.

[4] Charles R., Wang C.-C., A novel heat dissipation fin design applicable for natural convection augmentation, Int. Commun. Heat Mass Transf., 59 (2014) 24-29.

[5] Phiraphat S., Prommas R., Puangsombut W., Experimental study of natural convection in PV roof solar collector, Int. Commun. Heat Mass Transf., 89 (2017) 31-38.

[6] Lee D.-Y., Chae M.-S., Chung B.-J., Natural convective heat transfer of heated packed beds, Int. Commun. Heat Mass Transf., 88 (2017) 54-62.

[7] Tsubouchi T., Masuda H., Natural convection heat transfer from horizontal cylinders with circular fins, in: International Heat Transfer Conference 4, Begel House Inc., 1970.

[8] Sparrow E., Bahrami P., Experiments on natural convection heat transfer on the fins of a finned horizontal tube, Int. J. Heat Mass. Transf., 23 (1980) 1555-1560.

[9] Kayansayan N., Karabacak R., Natural convection heat transfer coefficients for a horizontal cylinder with vertically attached circular fins, Heat Recov. Syst. CHP, 12 (1992) 457-468.

[10] Hahne E., Zhu D., Natural convection heat transfer on finned tubes in air, Int. J. Heat Mass. Transf., 37 (1994) 59-63.

[11] Wang C.-S., Yovanovich M., Culham J., Modeling natural convection from horizontal isothermal annular heat sinks, J. Electron. Packaging, 121 (1999) 44-49.

[12] Güvenç A., Yüncü H., An experimental investigation on performance of fins on a horizontal base in free convection heat transfer, Heat Mass Transfer, 37 (2001) 409-416.

[13] Yildiz Ş., Yüncü H., An experimental investigation on performance of annular fins on a horizontal cylinder in free convection heat transfer, Heat Mass Transfer, 40 (2004) 239-251.

[14] Aziz A., Fang T., Thermal analysis of an annular fin with (a) simultaneously imposed base temperature and base heat flux and (b) fixed base and tip temperatures, Energ. Convers. Manage., 52 (2011) 2467-2478.

[15] Yaghoubi M., Mahdavi M., An investigation of natural convection heat transfer from a horizontal cooled finned tube, Exp. Heat Transfer, 26 (2013) 343-359.

[16] Terekhov V., Ekaid A., Yassin K., Laminar free convection heat transfer between vertical isothermal plates, J. Eng. Thermophys, 25 (2016) 509-519.

[17] Lee M., Kim H.J., Kim D.-K., Nusselt number correlation for natural convection from vertical cylinders with triangular fins, Appl. Therm. Eng., 93 (2016) 1238-1247.

[18] Elenbaas W., Heat dissipation of parallel plates by free convection, Physica, 9 (1942) 1-28.

[19] Graetz v.L., Ueber die wärmeleitungsfähigkeit von flüssigkeiten, Ann. Phys-Berlin, 254 (1882) 79-94.

[20] Bejan A., Convection heat transfer, John wiley & sons, 2013.

[21] Kreith F., Manglik R.M., Bohn M.S., Principles of heat transfer, Cengage learning, 2012.

[22] Edwards J., Chaddock J., An experimental investigation of the radiation and free convection heat transfer from a cylindrical disk extended surface, Trans. Am. Soc. Heat. Refrig. Air-Cond. Eng, 69 (1963) 313-322.

[23] Jones C.D., Nwizu E.I., Optimum spacing of circular fins on horizontal tubes for natural convection heat transfer, in: ASHRAE J., 1969, pp. 72.

[24] Knudsen J., Pan R., Natural convection heat transfer from transverse finned tubes, Chem. Eng. Progr, 59 (1963) 44-49.

[25] Rohsenow W.M., Hartnett J.P., Cho Y.I., Handbook of heat transfer, McGraw-Hill New York, 1998.

[26] Senapati J.R., Dash S.K., Roy S., Numerical investigation of natural convection heat transfer over annular finned horizontal cylinder, Int. J. Heat Mass. Transf., 96 (2016) 330-345.

[27] ANSYS-CFX, Release 18.0-User Manual, Canonsburg, PA, USA, (2018).

[28] Rhie C., Chow W.L., Numerical study of the turbulent flow past an airfoil with trailing edge separation, AIAA J., 21 (1983) 1525-1532.

[29] Majumdar S., Role of underrelaxation in momentum interpolation for calculation of flow with nonstaggered grids, Numer. Heat Transfer, 13 (1988) 125-132.

[30] Littlefield J., Cox J., Optimization of annular fins on a horizontal tube, Wärme-und Stoffübertragung, 7 (1974) 87-93.

[31] Nemati H., Samivand S., Simple correlation to evaluate efficiency of annular elliptical fin circumscribing circular tube, Arab. J. Sci. Eng., 39 (2014) 9181-9186.