Propagator Rooting Method Direction of Arrival Estimation Based on Real Data

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*Abstract*—In this paper, we present a novel and computationally efficient DOA estimation method that works equally well for both non-coherent and coherent sources. This method is based on applying the propagator method as a linear operator to the covariance matrix of the received data taken from a single snapshot of signals impinging on a uniform linear array. A Toeplitz Hermitian data matrix is constructed and transformed to a real-valued data matrix which significantly reduces computational complexity. The propagator method obviates the need to use either eigenvalue decomposition or singular value decomposition in calculating the DOA. Finally, the Root-MUSIC method is employed in conjunction with proposed method to estimate the angles of arrivals from the received signal. Simulation results demonstrate the efficacy of the proposed method.

Keywords— complexity, coherent sources, propagator method Root-MUSIC, real-valued data, DOA estimation

# Introduction

The direction-of-arrival (DOA) estimation is widely used in wireless communications, radar systems, and other related fields [1-2]. Over the last several decades, traditional subspace approaches like the MUSIC [3], ESPRIT [4] algorithms have received a lot of attention. These approaches rely on the signal/noise subspace produced from the sample covariance matrix's eigenvalue decomposition (EVD) or the data matrix's singular value decomposition (SVD). Unfortunately, the subspace decomposition procedure is highly computationally intensive, time-consuming, and costly. Furthermore, in the case of highly correlated and coherent source signals that facilitate the rank loss of the covariance matrix, these subspace-based approaches fail in real-time applications.

Using alternative methods to extract signal and noise subspaces [5-6] have significantly reduced complexity but suffered from accuracy degradation under coherent narrowband signals. Instead of EVD/SVD, a propagator method (PM) based on a linear operator was first proposed in [7-8]. A modified orthogonal PM method was proposed in [9] that used spline interpolation to rebuild the noise free diagonal elements of the data covariance matrix. While the use of interpolation reduces the impact of noise, it nevertheless increases the computational complexity. Moreover, the method in [9] works well only for uncorrelated signals. A fast DOA estimator using PM for an L-shaped array was proposed in [10], and one for coherent sources using PM without EVD/SVD was presented in [11]. An extension of the traditional PM called generalization propagator method (GPM) was presented in [12]. For a more effective utilization of the received data, it considers different block structures of array manifold to get a new spectral function without EVD. The methods in [10-12] are effective in reducing the computational complexity. In [13], authors present an efficient decomposition method based on QR requiring only a single snapshot of the received data for constructing a Toeplitz data matrix. This method works well for DOA estimation of both coherent and non-coherent sources.

In this paper, we present a novel DOA estimation method that significantly reduces computational complexity, compared with existing methods, for estimating both coherent and non-coherent source signals even at a low signal-to-noise ratio (SNR) with high estimation accuracy. The main features of the proposed method are: 1) It uses a single snapshot from a uniform linear array (ULA) to generate the Toeplitz data matrix. 2) It transforms complex data into Toeplitz Hermitian Symmetric form. Then, constructs straightforward unitary transformation to transform the complex Toeplitz Hermitian Symmetric form into real-valued data. It reduces computational complexity by working on real-valued data. 3) Uses the Propagator Method (PM) based on the linear operators without any eigen decomposition such as EVD/SVD on the covariance matrix of the received data. A coordinate mapping technique is utilized to convert the derivative of the standard PM cost function into a new parameter domain, and a real coefficient function whose order is only related to the number of array elements constructed. 4) Employs the root-MUSIC algorithm [14] to estimate the angles of arrivals from the received signal. The above mentioned features make the proposed method highly effective for DOA estimation of both non-coherent and coherent sources at a significantly reduced complexity.

The rest of this paper is organized as follows. The system model and the proposed method is explained in Section II. Computational complexity of the proposed method and its comparison with Root-MUSIC is presented in Section III. Section IV presents the simulation results and performance evaluation of the proposed method. Conclusions are drawn in Section V.

# SYSTEM MODEL AND PROPOSED METHOD

Consider a uniform linear array (ULA) consisting of 2*N +* 1 receiving antennas with interspacing *d* equal to half the wavelength of incident signals as shown in Fig. 1.



1. Uniform Linear Array

Suppose that there are *K* narrow band source signals assumed to be arriving at the array with angles of incidence (, with *K <* 2*N* + 1. We assume that all incident sources have the same carrier frequency. For a given snapshot t, the output signal from the *n-th* element on the ULA is given by:

|  |  |  |
| --- | --- | --- |
|  |  | 1 |

where is the signal from the *i-*th incident source, and *nn(t)* is the additive white Gaussian noise (AWGN) at the *n-*th element, and *xn(t)* is the received signal at the time *t* at the n-th element where

Considering the element at the center of the array as reference p­oint, the 2*N*+1 output vector from 2*N*+1 antenna elements placed on the linear axis can be written as:

|  |  |  |
| --- | --- | --- |
|  |  | 2 |

where dimension of *X(t)* are 2*N +*1*,* and

|  |  |  |
| --- | --- | --- |
|  |  |  |

is the (2*N +* 1) *× K* is array response matrix, where

|  |  |  |
| --- | --- | --- |
|  |  |  |

is the corresponding (2*N* +1) *×* 1array transfer vector, with

|  |  |  |
| --- | --- | --- |
|  |  |  |

***S****(t)* is the signal vector of *K* received signals

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| --- | --- | --- |
|  |  |  |

and,

|  |  |  |
| --- | --- | --- |
|  |  |  |

where is the (2*N +* 1)noise vector. The superscripts *T* and \* denote the transpose and conjugate operations, respectively.

In the proposed method, first we construct the data matrix ***Y*** with the dimension (2*N+*1) × (2*N+*1)using the observation data vector ***X****(t)* as follows:

Referring to (2), we let. Then the data matrix ***Y*** can be written as:



The ***Y*** data matrix in (9) is in a Toeplitz structure form in the noiseless data case, which means all elements along a diagonal and off-diagonal have the same value. The ***Y*** matrix satisfies the following properties:

and 

For noiseless data, it is clear to observe that:

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| --- | --- | --- |
|  |  |  |

However, the real-time data is noisy, and each antenna element will add its own random noise. Because of this, the off-diagonal elements are not conjugate of each other. This can be shown mathematically as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

If this condition is not satisfied, one is not able to convert the complex data ***Y*** into real data. To resolve this issue, we propose a new data matrix that includes both the original data and its Hermitian. The proposed data matrix is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The matrix ***Q*** is in the form of symmetric Hermitian Toeplitz structure whether the data is noiseless or noisy. Also, the structure of Toeplitz Hermitian data matrix in (13) allows the estimation of coherent and noncoherent sources since it always has the full rank that equals the number of DOA of incident sources. The ***Q*** matrix can be expanded as follows:



Observing the Toeplitz Hermitian matrix ***Q***, one can see that diagonal elements are same, while off-diagonal elements are conjugate of each other. For example:

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| --- | --- |
|  |  |

and,

|  |  |
| --- | --- |
|  |  |

A unique unitary transformation is needed to convert the complex data in (14) to real data. We propose a unitary matrix ***U***with dimensions (2*N*+1)× (2*N*+1)*,* which transforms ***Q***into a real matrix:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where ***I***is an Identity matrix and ***J*** is an exchange matrix with ones along the secondary diagonal and zeros elsewhere. Note that ***J*** *=* ***J****\* =* ***J****-1,* where \* stands for conjugate transpose. Therefore, the unitary matrix is expressed as follows:

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| --- | --- | --- |
|  |  |  |

Some of the existing methods have two structures of the unitary matrix, which depend on the parity of the number of antennas. In the proposed method, the structure of the unitary matrix in (18) works for both even or odd number of antennas.Thus, the real-valued matrix is obtained as:

|  |  |  |
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Matrix operations such as EVD, which applies on cross correlation matrix and SVD which applies on data matrix requires high computational load and cost. The proposed method employs the propagator method, which is a linear operator and does not require either SVD or EVD operations. This will reduce significantly the hardware complexity and signal processing.

In the proposed method, the propagator method is used in conjunction with Root-MUSIC method to estimate the DOAs of multiple incident sources whether the sources are coherent or non-coherent.

The propagator method is based on partitioning the proposed real data matrix into two submatrices as follows.

(20)

where the dimensions of and are and respectively.

A unique propagator matrix with dimension, which links the two submatrices and is defined as:

(21)

Using the propagator method in (21), a new matrix can be expressed as:

(22)

The combination of (21) and (22) can be presented as:

= (23)

Equation (23) verifies that the real data matrix in (20) is orthogonal to the columns of in (22). The value of the propagator method can be obtained by minimizing the cost function :

- (24)

where represents the Frobenius norm.

The optimal solution can be obtained by minimizing the cost function in (24).

(25)

The following spectrum function can be formed to estimate the DOAs of incident sources:

(26)

We observe from (26) that the DOA of incoming signals can be obtained by one dimensional spectrum peak search over the range of . To reduce the computational complexity further, the estimation in (26) can be modified to calculate the DOAs based on polynomial roots. The modified estimator can be described as follows:

Assuming , we can rewrite as

(27)

Then, the denominator in (26) can be expressed in the polynomial form as:

(28)

Similar to Root-MUSIC, the polynomial function in (28) can be used to calculate the roots that are directly related to the incident sources locations as follows:

(29)

The proposed method is unlike the existing Root-MUSIC in the following aspects: It is based on real data, can estimate the coherent and non-coherent sources, and uses only a single snapshot. These advantages make the proposed method a good candidate for high-speed communication and can be applied for stationary and non-stationary targets.

# Computational Complexity

Table I shows the computational complexity of major processing steps of the proposed method and Root-MUSIC. We observe from the table that Root-MUSIC requires much higher computational complexity compared to the proposed method, when the number of antennas increase and the requirement in the number of snapshots in the Root-MUSIC. Consider the scenario where the number of snapshots is (*M*=100), number of antenna elements (*N*=64), number of sources (*K*=3). For this case, the Root-MUSIC requires approximately three times more operations compared with the proposed method . Furthermore, DOA estimation of coherent sources using classical Root-MUSIC method requires spatial smoothing which adds to its computational load, while the proposed method works equally well for both non-coherent and coherent sources (without spatial smoothing). The proposed method also does not require the construction of a covariance matrix.

1. Computational Complexity of Proposed Method and Root-MUSIC

|  |  |  |
| --- | --- | --- |
| **Operations of major signal processing steps** | **Proposed Method using Root-MUSIC** | **Root-MUSIC** |
| Covariance Matrix | *-* |  |
| Real Data Matrix | *N* | *-* |
| Real Covariance Matrix | *-* | 2 |
| Real EVD decomposition | *-* |  |
| Propagator Estimation |  | *-* |
| Compute Polynomial Roots |  |  |
| Compute Angle |  |  |

# Simulation Results

To verify the efficacy of the proposed method, Matlab simulations were carried out for computing DOA estimates of the azimuth angle under different scenarios, and simulation results are presented below in Fig. 2 through to Fig.9 in the form of RMSE vs SNR plots, RMSE vs *N* (antenna elements) plots, and histograms. A comparison of the proposed method is made with the classical Root-MUSIC.

For RMSE vs SNR plots, consider the following simulation parameters: *N* = 25 antenna elements; three sources (*K* = 3) located at 80o (S1=DOA1), 110o (S2=DOA2), and 130o (S3=DOA3) lying in the far-field region of a ULA; SNR = [5:5:30] dB; DOA estimates computed for 100 Monte-Carlo trials for each SNR value; single snapshot for proposed method, and 25 snapshots for Root-MUSIC; DOA estimates computed for both non-coherent and coherent sources; for coherent case, source signals S1 and S2 are taken as coherent (S2 = S1).

Fig. 2 shows the RMSE vs SNR plot for the case of three non-coherent sources. It can be observed that overall the proposed method has better estimation accuracy with increasing SNR values while Root-MUSIC performs slightly better at low SNR.



1. RMSE vs SNR: Three non-coherent sources’ DOAs estimated using proposed method (PM) and Root-MUSIC (RM)

Fig. 3 shows the RMSE vs SNR plot for the case of two coherent sources and one non-coherent source. For computing the DOA estimates, forward/backward spatial smoothing [15] had to be applied to the Root-MUSIC method to obtain accurate DOA estimates whereas the proposed method works well without any spatial smoothing technique. It is clear from the plot that the proposed method outperforms the Root-MUSIC method. Fig. 4 shows the effect of coherent signals on DOA estimation for the Root-MUSIC method when no spatial smoothing is applied. For the two coherent signals (S1 and S2), RMSE values do not improve with increasing SNR values. This demonstrates the effectiveness of the proposed method in estimating coherent sources without additional complexity.



1. RMSE vs SNR: One non-coherent and two coherent sources’ DOAs estimated using proposed method (PM), and Root-MUSIC (RM) with spatial smoothing.



1. RMSE vs SNR: One non-coherent and two coherent sources’ DOAs estimated using proposed method (PM), and Root-MUSIC (RM) without spatial smoothing.

For RMSE vs Antenna Elements (*N*) plots, consider the following simulation parameters: *N* = [9, 13, 17, 21, 25, 29], SNR=25 dB; three sources (*K* = 3) located at 80o (S1=DOA1), 110o (S2=DOA2), and 130o (S3=DOA3) lying in the far-field region of a ULA; DOA estimates computed for 100 Monte-Carlo trials for each value of *N*; single snapshot for proposed method, and 25 snapshots for Root-MUSIC; DOA estimates computed for both non-coherent and coherent sources; for coherent case, source signals S1 and S2 are taken as coherent (S2 = S1).

Fig. 5 shows the RMSE vs *N* plot for the case of three non-coherent sources while Fig. 6 shows the RMSE vs *N* plot for the case of two coherent sources and one non-coherent source. Both plots show superior performance and estimation accuracy of the proposed method compared with Root-MUSIC, for both non-coherent and coherent sources.



1. RMSE vs *N*: Three non-coherent sources’ DOAs estimated using proposed method (PM) and Root-MUSIC (RM).



1. RMSE vs *N*: One non-coherent and two coherent sources’ DOAs estimated using proposed method (PM), and Root-MUSIC (RM) with spatial smoothing.

For histogram plots, consider the following simulation parameters: *N =* 13, SNR=20 dB; three sources (K = 3) located at 80o (S1=DOA1), 110o (S2=DOA2), and 130o (S3=DOA3) lying in the far-field region of a ULA; DOA estimates computed for 100 Monte-Carlo trials; single snapshot for proposed method, and 25 snapshots for Root-MUSIC; DOA estimates computed for both non-coherent and coherent sources; for coherent case, source signals S1 and S2 are taken as coherent (S2 = S1).

Fig. 7 shows the histogram plot for the proposed method and Fig. 8 shows the histogram plot for Root-MUSIC without spatial smoothing, for the case of two coherent sources and one non-coherent source. It is observed that highly accurate estimation is achieved for all trials for all sources for the proposed method, while the DOA estimates for the two coherent sources with Root-MUSIC method are not accurate overall with only about 20 trials producing accurate estimates.



1. Histogram for proposed method: DOA estimation of one non-coherent and two coherent sources.



1. Histogram for Root-MUSIC method: DOA estimation of one non-coherent and two coherent sources without spatial smoothing.

However, with spatial smoothing the performance of Root-MUSIC improves significantly, as shown in Fig. 9. It is also worth reiterating here that the proposed method does not require spatial smoothing for estimation of coherent sources.



1. Histogram for Root-MUSIC method: DOA estimation of one non-coherent and two coherent sources with spatial smoothing.

All the above plots show that the proposed method has higher estimation accuracy under different scenarios compared with Root-MUSIC.

# Conclusions

The proposed method does not require high computational complexity to estimate the DOA of multiple sources since it is based on single snapshot. The proposed methods work well for both coherent and non-coherent sources. The novelty of the proposed method lies in its use of real-valued data constructed from a single snapshot of the received signal at an ULA. Further, the use of the propagator method as a linear operator reduces the computational complexity further and does not require either EVD or SVD. This significant reduction in the complexity of the proposed method makes it suitable for real-time applications especially when the source is non-stationary.

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