FPGA Hardware Implementation of Computationally Efficient DOA Estimation of Coherent Signals

Ahmed A Hussain,

Department of Electrical Engineering

Prince Mohammad University, KSA

ahussain1@pmu.edu.sa

Nizar Tayem

Department of Electrical Engineering

Texas A & M Univ. – Commerce

Nizar.Tayem@tamuc.edu

Abdel-Hamid Soliman

School of Engineering

Staffordshire University, UK

A.Soliman@staffs.ac.uk

***Abstract***—**DOA estimation of highly correlated or coherent signals involves some preprocessing steps to de-correlate the signals before DOA estimates are computed. This increases the computational complexity of the estimation algorithms further, rendering hardware implementation a challenging task. In this paper, we present the hardware implementation of a novel and computationally efficient DOA estimation algorithm for coherent sources based on applying forward/backward averaging to the signal space matrix to deal with the incident coherent signals. The proposed algorithm is implemented on a Xilinx FPGA using LabVIEW FPGA modules. Simulations results as well as FPGA resource utilization and computation speed are presented to validate the efficacy of the proposed method and the efficiency of hardware implementation.**

***Keywords—coherent sources, forward/backward averaging, DOA estimation, FPGA, hardware implementation, computational complexity, LabVIEW***

# Introduction

DOA estimation of incident RF signals is an active area of research with applications in both civilian and military fields [1]-[6]. It is also one of the major enablers of the 5G and MIMO technologies currently being rolled out. There exists a large number of publications reporting a wide range of DOA estimation techniques including MUSIC, ESPRIT, and similar methods [7]-[10]. However, there is significant performance degradation seen in these methods in the presence of incident signals that are coherent or highly correlated [11]-[12].

In order to improve the performance of DOA estimation algorithms in dealing with coherent signals in multipath environments, some preprocessing techniques are usually applied to de-correlate the signal before DOA estimates are computed. This is required since the covariance matrix becomes singular and rank deficient, making accurate estimation of source DOAs difficult. These preprocessing techniques [13]-[25] help in improving the rank, but their performance and computational complexity vary. A clear drawback of these preprocessing techniques is an increase in the computational load of algorithms.

Spatial smoothing techniques have been widely used for treating highly correlated or coherent signals since first introduced in [13]-[14]. The methods proposed in [15]-[18] use forward/backward spatial smoothing to de-correlate the signals before further processing. Authors in [19] proposed a DOA algorithm based on a combination of the maximum likelihood (ML) estimator and the orthogonal matching pursuit (OMP) technique. While this algorithm is effective against coherent sources, its estimation time is high as it estimates the DOAs in each direction through an iterative one-dimensional search.

A detailed study of the existing literature [7]-[19] reveals that researchers have focused primarily on software simulations to validate the effectiveness and accuracy of DOA estimation algorithms. However, for real-world applications, hardware implementation and experimental validation are essential to establish the efficiency and accuracy of these algorithms.

Computational complexity of a DOA estimation algorithm is a critical consideration in the hardware implementation [20]-[25] of the algorithm, with direct impact on computation speed and resource consumption. An FPGA hardware implementation of a Bartlett DOA estimator is reported in [20], and implementations of MUSIC-based DOA algorithms are reported in [21]-[22]. The Bartlett DOA estimator in [20] is shown to be an efficient implementation in terms of computation time. FPGA real-time implementation based on QR, LU, and LDL/Cholesky decomposition methods have been reported in [23]-[25], respectively. These methods have been shown to be superior in performance to those of MUSIC and ESPRIT-based algorithms for hardware implementation in terms of estimation accuracy, computation speed, and resources consumption. However, one limitation of these methods is that they work well only for non-coherent sources.

In this paper, we propose an FPGA hardware implementation of a novel DOA estimation technique for coherent sources based on forward/backward averaging (FBA) [26]. The hardware implementation is realized on a Xilinx Virtex-5 SXT FPGA [@@1] using LabVIEW FPGA [@@2] modules. To the best of authors’ knowledge, this paper presents the first ever hardware implementation of a DOA estimation algorithm for coherent sources.

The proposed method reported in [26] relies on applying the FBA operation to the signal subspace matrix instead of the covariance matrix. The signal subspace matrix is obtained after decomposing the covariance matrix to extract the signal and noise subspaces, while the covariance matrix is decomposed using QR decomposition method instead of either eigenvalue decomposition (EVD) or singular value decomposition (SVD). The main advantage of applying FBA to the signal subspace matrix is that it decreases the computational complexity significantly when the number of antennas increases, especially in the case of MIMO systems. In the proposed method, constructing the data matrix based on signal space will reduce the computation cost (and memory requirements) significantly since it requires much smaller number of operations compared to the existing methods. This makes the proposed method more suitable for hardware implementation.

The hardware implementation of the proposed algorithm follows a pipelined architecture realized on the Xilinx Virtex-5 FPGA. LabVIEW FPGA simulations of the proposed method were performed to validate the efficacy of the method. A performance comparison is made with hardware implementations in [23]-[25].

This paper is organized as follows: Section II presents the system model and the proposed algorithm; section III presents the FGPA hardware implementation; section IV presents LabVIEW FPGA simulation results; and conclusions are drawn in section V.

# SYTEM MODEL AND PROPOSED METHOD

The system model shown in Fig. 1 considers *K* narrowband RF source signals in the far-field region of a uniform linear array (ULA) consisting of *M* omni-directional antennas. The distance between the adjacent antennas is  where  is the wavelength of the incident signals.



1. System model for DOA estimation employing a ULA

The antenna array is placed along the x-axis and contains *M* antenna elements. We consider *K* narrowband sources impinging on the antenna array with *θk* as an azimuth angle of the kth source.

The observed data from the antenna elements of the ULA at any time instant (*t*) can be expressed as:

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where  is the *i-*th incident source signal,  is the wavelength, (***d*** = λ/2) the spacing distance of ULA, and *nm*(*t*) is the noise at the m-th element.

The received data can be expressed in matrix form as:

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where  is the (*M* x *K*) array response matrix given as:

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where  for is the corresponding array response vector.

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where *S*(*t*) is the vector of received signals , and , where *N*(*t*) is the  additive white Gaussian noise (AWGN) vector.

The proposed algorithm computes the DOA in six steps which are described below.

Step 1: Compute Covariance Matrix

The *N* snapshots of the signal data received from the antenna array of the ULA are retrieved and used to compute the covariance matrix *Rx* according to the equation below:

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where is the column vector from the *ith* antenna element and ( )*H* is the conjugate transpose operation.

Step 2: Extract Signal Space

Using QR Decomposition:

The covariance matrix ***R****x* computed in Step 1 is factorized using QR decomposition to extract the signal space. Matrix decomposition using QR factorization applied on ***R****x* can be expressed as follows:

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where ***Q****s* is the (*M* x *K*) signal space matrix and ***Q****n* is the (*M* x (*K-M*)) noise space matrix, ***R****s* is the (*K* x *M*) upper triangular signal space matrix , and ***O*** is the lower triangular matrix that has all entities as zeros.

For further processing, we consider the signal space data contained in either ***Q****s* or ***R****s*. To estimate DOAs of *K* sources, we need to extract only the first *K* columns of ***Q****s*or ***R****sT*. For example, in the case of *K=*2, ***Q****s* is given by:

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**Using Cholesky Decomposition:**

In case of Cholesky factorization, matrix ***Rx*** is decomposed as follows:

(8)

where ***L*** is a unique lower triangular matrix with positive diagonal entries.

For two sources, only the first two columns of  need to be extracted to compute the DOA estimates. The submatrix of size *M* x 2 is obtained as:

 (9)

For the sake of convenience and consistency, we will call the matrix as ***Q****s*.

The signal space data matrix ***Q****s*(1*:M,*1:*K*) will be used to estimate the DOAs of pair of coherent sources (*s*1, and *s*2*=αs*1), where (0 < α ≤ 1). Note that when α = 1, the two sources are fully coherent.

Step 3: De-correlate Signals using Forward/Backward Averaging

In this step, the signal space is de-correlated by applying the forward/backward averaging method to the signal space data matrix in (7) for QR decomposition or in (9) for Cholesky decomposition.

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where ***J****M* is (*M* x *M*) and ***J****K* is (*K* x *K*) matrices with ones in the off diagonal elements and zeros in the rest of the elements; ( )\* is the conjugate operation. The dimension of is directly related to the number of sources *K*.

For further processing, we partition data matrix into two sub-matrices as follows:

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Step 4: Compute Direction Matrix Using Least Squares (LS) Solution

Since range of , there must exist a unique matrix ***T***, such that:

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where is the (*M x* 2) is the array response matrix, and  is a (2 *x* 2) diagonal matrix containing information about the DOAs of incident sources.

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since span the same signal space. This leads to both spaces being related by a nonsingular transform as follows:

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Equation (14) can be expressed as:

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The least square solution of (14) can be found as:

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Step 5: Compute Eigenvalues

The eigenvalues of  in (16) are computed which are then used to estimate the DOAs of incident sources. For a given matrix ***A***, the eigenvalues can be calculated as .

**Step 6: Compute DOA estimates**

In the final step, angle estimates are computed according to the following equation:

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where is the kth eigenvalue.

# Hardware Implementation

The hardware architecture for the implementation of the proposed method for DOA estimation of coherent sources is shown in Fig. 2. The implementation follows a 6-stage pipelined architecture for high throughput. These six stages correspond to the six steps of the proposed algorithm presented in Section II. The hardware platform used for implementation is the DSP-focused Xilinx Virtex-5 SXT FPGA with 40 MHz of onboard base clock and 512 MB of onboard RAM.

The FPGA was programmed in LabVIEW using built-in FPGA modules with high throughput mathematical operations. Fixed-point data type was selected and a data size of 16/8 was used (where the first number indicates word length in bits and the second number indicates integer length in bits).



1. Pipelined Architecture for FPGA Hardware Implementation

LabVIEW FPGA schematics (or block diagrams) were developed to implement the proposed DOA algorithm employing QR and Cholesky factorization for both a 4-element ULA and an 8-element ULA; and DOA of up to *K*=2 sources are estimated. These schematics files were compiled for FPGA simulation and performance evaluation of the DOA estimation algorithms. A successful compilation produces a report on the FPGA resources consumed and processing time required (in MHz).

Tables I and II show the count of arithmetic operations in the FPGA implementation of the proposed DOA estimation algorithm for the two cases of *M*=4 and *M*=8, respectively. FPGA codes for coherent sources with FBA operation applied to the covariance matrix for de-correlating the signals have also been compiled for comparison. The difference in the implementation between the non-coherent and coherent cases is the FBA operation which can be implemented with only a few addition operations. For *M*=4 case, the FBA operation requires only 8 addition operations for the proposed method but twice as many when FBA is applied to the covariance matrix (instead of the signal space data matrix in the proposed method), while for *M*=8 case, it requires 16 and 64 addition operations, respectively.

TABLE I. COUNT OF ARITHMETIC OPERATIONS FOR M=4

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Arithmetic**  **Operations** | **Non-coherent [@@3]** | | **Coherent**  **(conventional)** | | **Coherent**  **(proposed)** | |
| **QR** | **CHOL** | **QR** | **CHOL** | **QR** | **CHOL** |
| Addition | 20 | 0 | 36 | 16 | 28 | 8 |
| Subtraction | 0 | 5 | 0 | 5 | 0 | 5 |
| Multiplication | 0 | 11 | 0 | 11 | 0 | 11 |
| Complex-valued  Multiplication | 8 | 3 | 8 | 3 | 8 | 3 |
| Division | 16 | 2 | 16 | 2 | 16 | 2 |
| Square Root | 2 | 2 | 2 | 2 | 2 | 2 |
| **Total # of Operations** | 46 | 23 | 62 | 39 | 54 | 31 |

TABLE II. COUNT OF ARITHMETIC OPERATIONS FOR M=8

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Arithmetic**  **Operations** | **Non-coherent [@@3]** | | **Coherent**  **(conventional)** | | **Coherent**  **(proposed)** | |
| **QR** | **CHOL** | **QR** | **CHOL** | **QR** | **CHOL** |
| Addition | 44 | 0 | 108 | 64 | 60 | 16 |
| Subtraction | 16 | 13 | 16 | 13 | 16 | 13 |
| Multiplication | 32 | 27 | 32 | 27 | 32 | 27 |
| Complex-valued  Multiplication | 16 | 7 | 16 | 7 | 16 | 7 |
| Division | 2 | 2 | 2 | 2 | 2 | 2 |
| Square Root | 2 | 2 | 2 | 2 | 2 | 2 |
| **Total # of Operations** | 112 | 51 | 176 | 115 | 128 | 67 |

(to be completed soon)

# FPGA Simulation Results

(to be completed soon)

# Conclusions

FPGA hardware implementation of a computationally efficient DOA estimation method for coherent sources was proposed in this paper. The proposed algorithm reduces the computational complexity by applying forward/backward averaging technique to the signal space matrix instead of the covariance matrix in order to de-correlate the signals. The proposed method also does not require either EVD/SVD. The hardware implementation was done on a Xilinx FPGA and LabVIEW FPGA simulation results were presented to validate the efficacy of the proposed method and its hardware implementation. The reduced complexity and computation time and high accuracy of the DOA estimates make the proposed method suitable for practical applications in areas such as MIMO systems and 5G communications.

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