Computationally Efficient Forward/backward Averaged DOA Estimation of Coherent Sources in Pairs

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***Abstract***—**DOA estimation of signal sources in multipath environments is a computationally complex problem. In this paper, authors present a computationally efficient DOA estimation algorithm effective against highly correlated or fully coherent sources in pairs. The proposed method is novel in that it applies forward/backward averaging to the signal subspace to de-correlate the signals unlike existing methods which apply spatial smoothing techniques to the correlation matrix. This significantly reduces the computational complexity and computation time of the algorithm and improves the estimation accuracy making the proposed method amenable to practical hardware implementation. Simulation results are presented to validate the efficacy of the proposed method and a performance comparison is made with Root-MUSIC method.**

***Keywords—coherent sources, forward/backward averaging, DOA estimation, computational complexity, computation time***

# Introduction

DOA estimation of incident RF signals is an area of increasing importance in both civilian and military applications [1]-[6]. It is also one of the major enablers of the 5G and MIMO technologies currently being rolled out. Existing literature reports a wide range of DOA estimation techniques including MUSIC, ESPRIT, and their variants [7]-[10]. However, the performance of these classical DOA estimation methods deteriorates significantly in the presence of incident signals that are highly correlated or coherent [11]-[12].

Coherent signals occur as an RF signal traverses in multipath environments. Improvement to DOA estimation of coherent signals has been an important research area in dealing with coherent signals problem in multipath environments. Preprocessing is usually needed for DOA estimation of coherent sources since the covariance matrix becomes singular and rank deficient and it is not possible to estimate the source DOA accurately [11]-[12]. Certain methods can be used at the preprocessing stage to improve the rank, but their performance and computational complexity vary [13]-[25]. These preprocessing techniques while being indispensable impose an additional burden on computational cost as well as resource requirements, especially for real-time hardware realizations of DOA estimation algorithms.

Spatial smoothing techniques have been widely used for treating highly correlated or coherent signals since they were first introduced in [13]-[14]. The methods proposed in [15]-[18] use forward/backward spatial smoothing as preprocessing techniques to de-correlate the signals before further processing. The method proposed in [19] combines Barlett method with Beamspace MUSIC [20] to reduce computational complexity. Authors in [21] proposed a DOA algorithm based on a combination of the maximum likelihood (ML) estimator and the orthogonal matching pursuit (OMP) technique. While this algorithm is effective against coherent sources, its estimation time is high as it estimates the DOAs in each direction through an iterative one-dimensional search. A multiple-Toeplitz matrix construction method for DOA estimation of coherent sources is proposed in [22] while a components separation algorithm (CSA) is proposed in [23] which can compute DOA estimates of mixed near-field and far-field source signals in multipath environments.

In this paper, we propose a novel DOA estimation technique for coherent sources based on forward/backward averaging (FBA) method for signal space in the preprocessing stage whereas existing method requires forward/backward averaging of the covariance matrix. The main shortcoming of applying FBA to the covariance matrix is that it increases the computational complexity significantly when the number of antennas increase especially in the case of MIMO systems. Another shortcoming is that since the covariance matrix also includes the noise along with the signal spaces it adversely affects the performance. In the proposed method, constructing the data matrix based on signal space will reduce the computation cost (and memory requirements) significantly since it requires much smaller number of operations compared to the existing methods. This will make the proposed method more suitable for hardware implementation. The proposed method relies on applying the preprocessing operation to the signal subspace matrix which is obtained after decomposing the covariance matrix to extract the signal and noise subspaces, while the covariance matrix is decomposed using QR decomposition method instead of either eigenvalue decomposition (EVD) or singular value decomposition (SVD). The contribution of the proposed method is in significantly reducing the computational complexity at two stages of the DOA estimation process – first, at the de-correlation stage, and second, at the signal extraction stage. Matlab simulations of the proposed method were performed to validate the efficacy of the method. A performance comparison is made with Root-MUSIC method [24] which shows comparable estimation accuracy at significantly lower computation cost.

This paper is organized as follows: Section II presents the system model and the proposed algorithm; section III presents computational complexity; section IV presents Matlab simulation results; and conclusions are presented in section V.

# SYTEM MODEL FOR DOA ESTIMATION

The system model shown in Fig. 1 considers *K* narrowband RF source signals in the far-field region of a uniform linear array (ULA) consisting of *M* omni-directional antennas. The distance between the adjacent antennas is  where  is the wavelength of the incident signals.



Fig. 1. System model for DOA estimation employing a ULA

The antenna array is placed along the x-axis and contains *M* antenna elements. We consider  narrowband sources impinging on the antenna array with *θk* as an azimuth angle of the kth source.

The observed data from the antenna elements of the ULA at any time instant (t) can be expressed as:

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where  is the *i-*th incident source signal,  is the wavelength, (***d*** = λ/2) the spacing distance of ULA, and *nm*(*t*) is the noise at the m-th element.

The received data can be expressed in matrix form as:

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where  is the (*M* x *K*) array response matrix given as:

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where  for is the corresponding array response vector.

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where ***S***(*t*) is the vector of received signals , and , where ***N***(*t*) is the  additive white Gaussian noise (AWGN) vector.

In the first stage of the proposed method, the *N* snapshots of the signal data received from the antenna array of the ULA are retrieved and used to compute the covariance matrix *Rx* according to the equation below:

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where is the column vector from the *ith* antenna element and ( )*H* is the conjugate transpose operation.

In the second stage, the covariance matrix ***R****x* computed in stage 1 will be factorized using QR decomposition to extract the signal space. Using QR factorization ***R****x* can be factorized into two matrices - a unitary matrix ***Q*** and upper triangular matrix ***R***. The DOA information can be extracted from the signal space of either ***Q*** matrix or ***R*** matrix. Least square (LS) approach of finding the direction matrix is applied next.

Matrix decomposition using QR factorization will be applied on ***R****x* so that it can be expressed as follows:

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where ***Q****s* is the (*M* x *K*) signal space matrix and ***Q****n* is the (*M* x (*K-M*)) noise space matrix, ***R****s* is the (*K* x *M*) upper triangular signal space matrix , and ***O*** is the lower triangular matrix that has all entities as zeros.

For further processing, we consider the signal space data contained in ***R****s*, whose transpose is given by ***R****sT*. To estimate DOAs of *K* sources, we need to extract only the first *K* columns of ***R****sT*. For example, with the case of *K=*2, ***R****sT* is given by:

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The data matrix ***R****sT*(1*:M,*1:2) will be used to estimate the DOAs of pair of coherent sources (*s*1, and *s*2*=αs*1), where (0 < α ≤ 1). Note that when α = 1, the two sources are fully coherent.

The third stage of the proposed method is to apply the forward/ backward averaging using the signal space in (7).

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where ***J****M* is (*M* x *M*) matrix with ones in the off diagonal elements and zeros in the rest of the elements. Similarly, ***J****K* is (*K* x *K*) matrix with ones in the off diagonal elements and zeros in the rest of the elements. The dimension of is directly related to the number of sources *K*, whereas in the existing methods the forward backward averaging of the covariance matrix is related to the number of antenna elements *M*. In most of the practical applications the number of antennas is much greater than the number of sources such that: *M >> K*. Thus, the proposed method will significantly reduce the computational complexity and improve the performance since it operates on the signal space alone. Note that the proposed method can be applied for any number of pairs of sources.

The fourth stage of the proposed method is to partition data matrix into two sub-matrices as follows:

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Since range of , there must exist a unique matrix ***T***, such that:

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where is the (*M x* 2) is the array response matrix, ,and  is a  diagonal matrix containing information about the DOAs of incident sources.

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since span the same signal space. This leads to both spaces being related by a nonsingular transform as follows:

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Equation (12) can be expressed as:

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The eigenvalues of the matrix are the diagonal elements of . Finding the eigenvalues of will lead to obtaining the DOAs for the incident sources. The least square solution of (12) can be found as:

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Computing the eigenvalues of  in (15) can be used to estimate the DOAs of incident sources using the following expression:

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where is the kth eigenvalue.

# Computational Complexity

Comparison of the proposed method with existing methods in terms of complex-valued multiplication and addition operations is presented in Table I. This comparison is based on the forward/backward averaging (FBA) method being applied to the signal space in the proposed method while it is applied to the covariance matrix in the existing methods [16]-[24]. M is the number of antenna elements; N, the number of snapshots; and K is the number of signal sources. The expressions for multiplication and addition operations for the proposed method are based on (8) while those for the existing methods are based on (16).

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where all matrices are of size *M* x *M*.

1. FBA Complexity Comparison

|  |  |  |
| --- | --- | --- |
|  | **Multiplications** | **Additions** |
| **Proposed Method** | M2K + MK2 | (M-1)MK |
| **Existing Methods** | 2M3 | M2 (2M-1) |

To get a clearer idea of how these complex-valued arithmetic operations scale with increasing antenna elements, refer to Table II below. The number of sources is fixed at *K*=4. The ratio of operations required for existing conventional methods against the proposed method is also calculated.

1. FBA Operations Comparison With Varying M

|  |  |  |
| --- | --- | --- |
| **K=4** | **Multiplication** | **Addition** |
| **M** | **Prop (P)** | **Conv. (C.)** | **Ratio C/P** | **Prop (P)** | **Conv. (C.)** | **Ratio C/P** |
| 4 | 128 | 128 | 1 | 48 | 112 | 2.3 |
| 8 | 384 | 1024 | 2.7 | 224 | 960 | 4.3 |
| 16 | 1280 | 8192 | 6.4 | 960 | 7936 | 8.3 |
| 24 | 2688 | 27648 | 10.3 | 2208 | 27072 | 12.3 |
| 32 | 4608 | 65536 | 14.2 | 3968 | 64512 | 16.3 |
| 64 | 17408 | 524288 | 30.1 | 16128 | 520192 | 32.3 |

It is clear from the above tables that the proposed method has much lower computational complexity compared with the existing methods. As the number of antenna elements increases, the number of arithmetic operations required increase significantly in the case of existing methods. For example, in case of MIMO systems with M=64 antenna elements, 30 times less multiplication and 32 times less addition operations are required with the proposed method. This is a significant improvement over existing methods, giving it a distinct advantage in the deployment of massive MIMO systems.

Another computational aspect of the proposed method is that it applies QR decomposition to decompose the covariance matrix into signal and noise subspaces instead of either EVD or SVD. QR decomposition has much lower complexity compared with either EVD or SVD [11]. QR requires O(2M3/3) operations whereas EVD/SVD require O(2M3) operations. A detailed computational complexity [25] considering the major signal processing steps in the proposed method is shown in Table III and that of Root-MUSIC (with forward/backward averaging) is shown in Table IV. For a numeric comparison with *N*=100, *M*=32, *K*=4, the number of operations for the proposed method are about 33% of the operations required for Root-MUSIC. It is clear from these tables that the computational complexity of the proposed method is much lower compared with Root-MUSIC and other EVD/SVD based methods in [16]-[24].

1. Computational Complexity of Proposed Method

|  |  |  |
| --- | --- | --- |
| **Signal Processing Step** | **Proposed Method** | **Operations** |
| Covariance Matrix | NM2 + M2 | 103424 |
| Signal Extraction (QR) | 2M3/3 | 21845 |
| Forward/Backward Avg. | M2K + MK2 + (M-1)MK | 8576 |
| Least Squares Solution | 3MK2+2K3 | 1664 |
| Compute Angles | K2/2 + K/2 | 10 |
| **TOTAL** (for N=100, M=32,K=4) | **135519** |

1. Computational Complexity of Root-MUSIC

|  |  |  |
| --- | --- | --- |
| **Signal Processing Step** | **Root-MUSIC** | **Operations** |
| Covariance Matrix | NM2 + M2 | 103424 |
| Forward/Backward Avg. | 2M3 + M2(2M-1) | 130048 |
| Signal Extraction (EVD) | 2M3 | 65536 |
| Eigenvectors Multiplication | M3 + M2K  | 36864 |
| Compute Polynomial Roots | 2M2(M-1) | 63488 |
| Compute Angles | 2(M-1) | 62 |
| **TOTAL** (for N=100, M=32,K=4) | **399422** |

The computation times in Matlab for the proposed method and Root-MUSIC method for computing DOA estimates for one pair of coherent sources is listed Table III. It is clear from the table that the proposed method has lower computation time and the difference becomes more pronounced as the number of antenna elements increase. The computation time is calculated for 100 iterations for each value of *M*.

1. Computation time comparison

|  |  |  |
| --- | --- | --- |
| **M** | **Proposed Method** | **Root-MUSIC** |
| 4 | 0.288299 | 0.320647 |
| 8 | 0.345337 | 0.357428 |
| 16 | 0.463409 | 0.6113 |
| 24 | 0.626055 | 1.043519 |
| 32 | 0.723993 | 1.608866 |
| 64 | 1.603823 | 7.810541 |
| 128 | 3.560806 | 41.90877 |

 The computation times listed in Table V have been plotted in Fig. 2. It is clear from the graph that the difference in the computation times increases significantly with the increase in antenna elements beyond *M* = 24.



Fig 2. Computation time of proposed method and Root-MUSIC

Now, having established the superiority of the proposed method in terms of complexity and computation time, the next section will analyze the estimation accuracy of the proposed method in terms of RMSE values calculated from Matlab simulations.

# Simulation Results

Matlab simulation results are presented below for different scenarios in Fig. 3 through to Fig.8. RMSE values for DOA estimates of the azimuth angle are plotted against number of antenna elements, varying SNR, and number of snapshots. Up to four sources are considered with different combinations of coherent and non-coherent signals. A comparison with Root-MUSIC is also shown in Fig. 7 and Fig. 8. All these plots show that the proposed method has low RMSE values indicating high estimation accuracy under different scenarios. Even at 0 dB SNR, the performance of the proposed method is very good as indicated in Fig. 5 and Fig. 6. Its comparison with Root-MUSIC is also favorable with Root-MUSIC doing only slightly better but at a much higher computation cost and resource requirements.

**Scenario 1:** *M*=8, SNR: [0:5:30] dB, source signals located at: [50o 110o]; *N* = 1024, *K* = 2 (one pair of coherent sources – [*s*1, *s*2*=αs*1]).



Fig 3. RMSE vs SNR: Pair of coherent sources

**Scenario 2:** *M*: [8, 16, 24, 32], SNR=20 dB, Source signals located at: [50o 80o 100o 130o]; *N* = 1024, *K* = 4 (two pairs of coherent sources - [*s*1, *s*2*=αs*1], [*s*3, *s*4*=βs*3]).



Fig 4. RMSE vs Antenna elements: Two pairs of coherent sources

**Scenario 3:** *M*=32, SNR: [0:5:30] dB, Source signals located at: [50o 70o 110o 120o]; *N* = 1024, *K* = 4 (two pairs of coherent sources - [*s*1, *s*2*=αs*1], [*s*3, *s*4*=βs*3]).



Fig 5. RMSE vs SNR: Two pairs of coherent sources

**Scenario 4:** *M*=32, SNR: 0:5:30 dB, Source signals located at: [50o 70o 110o 120o]; *N* = 1024, *K* = 4 (One pair of coherent sources and two non-coherent sources - [(*s*1, *s*2*=αs*1), *s*3, *s*4]).



Fig 6. RMSE vs SNR: One pair of coherent sources and two non-coherent sources

**Scenario 5:** *M*=8, SNR=20 dB, Source signals located at: [50o 110o ]; *N*: [100:100:1000], *K* = 2 (one pair of coherent sources – [*s*1, *s*2*=αs*1]).



Fig 7. RMSE vs Snapshots: One pair of coherent sources and comparison with Root-MUSIC

**Scenario 6:** *M*=8, SNR: [0:5:30] dB, Source signals located at: [50o 110o ]; *N*=1024, *K* = 2 (one pair of coherent sources – [*s*1, *s*2*=αs*1]).



Fig 8. RMSE vs SNR: One pair of coherent sources and comparison with Root-MUSIC

# Conclusions

A computationally efficient DOA estimation method for coherent sources was proposed in this paper. This method reduces the computational complexity by applying forward/backward averaging technique to the signal space matrix instead of the covariance matrix in order to de-correlate the signals. The proposed method also does not require either EVD/SVD. Simulation results were presented to validate the efficacy of the proposed method, which can compute DOA estimates of coherent sources in pairs. Performance of the proposed method has also been compared with Root-MUSIC method. The reduced complexity and computation time and high accuracy of the DOA estimates make the proposed method suitable for practical applications in areas such as MIMO systems and 5G communications.

##### References

1. V. Katkovnik,M.-S. Lee, and Y.-H. Kim, “Performance study of the minimax robust phased array for wireless communications,” IEEE Transactions on Communications, vol. 54, no. 4, pp. 608–613, 2006.
2. Y.H. Ko,Y. J.Kim,H. I.Yoo,W.Y.Yang, and Y.S.Cho, “2-DDoA estimation with cell searching for a mobile relay station with uniform circular array,” IEEE Transactions on Communications, vol. 58, no. 10, pp. 2805–2809, 2010.
3. Mukhopadhyay, M., B. K. Sarkar, and A. Chakrabarty, “Augmentation of anti-jam gps system using smart antenna with a simple doa estimation algorithm,” Progress In Electromagnetics Research, PIER 67, 231–249, 2007.
4. Z. Lei and T. J. Lim, “Estimation of directions of arrival of multipath signals in CDMA systems,” IEEE Transactions on Communications, vol. 48, no. 6, pp. 1022–1028, 2000.
5. X. Yuan, J. Wan, Z. Cheng, H. Liu, G. Li, A novel GNSS interference suppression and DOA estimation processing system, in: Proceedings 2011 International Conference on Transportation, Mechanical, and Electrical Engineering (TMEE), Changchun, 2011, pp. 1568–1571.
6. H. Zhang, Z. Gao, H. Fu, A high resolution random linear sonar array-based MUSIC method for underwater DOA estimation, in: Proceedings of the 32nd Chinese Control Conference, Xi’an, 2013, pp. 4592–4595.
7. R. Schmidt, “Multiple Emitter Location and Signal Parameter Estimation,” IEEE Trans. Antennas and Propagation, Vol. 34, No. 2, March, 1986, pp. 276–280.
8. Roy, R. and T. Kailath, “Esprit-estimation of signal parameters via rotational invariance techniques,” IEEE Transactions on Acoustics, Speech and Signal Processing, Vol. 32, No. 7, 984–995, July 1989.
9. H. Krim and M. Viberg, “Two decades of array signal processing research: the parametric approach,” IEEE Signal Processing Magazine, vol. 13, no. 4, pp. 67–94, 1996.
10. Z. M. Liu, Z. T. Huang, and Y. Y. Zhou, “An efficient maximum likelihood method for direction-of-arrival estimation via sparse Bayesian learning,” IEEE Transactions on Wireless Communications, vol. 11, no. 10, pp. 3607–3617, 2012.
11. G.H. Golub, C.F. Van Loan, Matrix Computations, third ed., John Hopkins University Press, Baltimore, 1996.
12. Balanis, C. A. and P. I. Ioannides, Introduction to Smart Antennas. San Rafael: Morgan & Claypool, 2007.
13. J.E. Evans, J.R. Johnson, D.F. Sun, “Application of advanced signal processing techniques to angle of arrival estimation in ATC navigation and surveillance systems,” Technical Report, MIT Lincoln Laboratory, June 1982.
14. T. Shan, M. Wax, and T. Kailath, “On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals,” IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. 33, No. 4, 1985, pp. 806–811.
15. S. U. Pillai and B. H. Kwon, “Forward/backward spatial smoothing techniques for coherent signal identification,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37,no. 1, pp. 8–15, 1989.
16. Y. H. Choi, “ESPRIT-based coherent source localization with forward and backward vectors,” IEEE Transactions on Signal Processing, vol. 58, no. 12, pp. 6416–6420, 2010.
17. G. Ni and C. He, "Direction of Arrival with Coherent Sources in Time Modulated Arrays," *2019 International Symposium on Antennas and Propagation (ISAP)*, 2019, pp. 1-3.
18. J. Pan, M. Sun, Y. Wang and X. Zhang, "An Enhanced Spatial Smoothing Technique With ESPRIT Algorithm for Direction of Arrival Estimation in Coherent Scenarios," in *IEEE Transactions on Signal Processing*, vol. 68, pp. 3635-3643, 2020, doi: 10.1109/TSP.2020.2994514.
19. S. N. Shahi, M. Emadi, and K. Sadeghi “High Resolution DOA Estimation In Fully Coherent Environments”, Progress In Electromagnetics Research C, Vol. 5, 135–148, 2008.
20. Zoltowski, M. D., G. M. Kautz, and S. D. Silverstein, “Beamspace root-MUSIC,” IEEE Transactions on Signal Processing, Vol. 41, No. 1, 344–364, Jan. 1993.
21. M. Rahmani, M. H. Bastani and S. Shahraeeni, "A computationally efficient direction-of-arrival estimation algorithm, robust against coherent sources and small sample size," 2013 21st Iranian Conference on Electrical Engineering (ICEE), 2013, pp. 1-4, doi: 10.1109/IranianCEE.2013.6599522.
22. W. Zhang, Y. Han, M. Jin and X. Qiao, "Multiple-Toeplitz Matrices Reconstruction Algorithm for DOA Estimation of Coherent Signals," in IEEE Access, vol. 7, pp. 49504-49512, 2019, doi: 10.1109/ACCESS.2019.2909783.
23. A. M. Molaei, B. Zakeri and S. M. Hosseini Andargoli, "Components Separation Algorithm for Localization and Classification of Mixed Near-Field and Far-Field Sources in Multipath Propagation," in IEEE Transactions on Signal Processing, vol. 68, pp. 404-419, 2020, doi: 10.1109/TSP.2019.2961226.
24. Shu Changgan and Liu Yumin, "An improved forward/backward spatial smoothing root-MUSIC algorithm based on signal decorrelation," 2014 IEEE Workshop on Advanced Research and Technology in Industry Applications (WARTIA), 2014, pp. 1252-1255, doi: 10.1109/WARTIA.2014.6976509.
25. Gentilho, E., Scalassara, P.R. and Abrão, T.,  “Direction-of-arrival estimation methods: a performance-complexity tradeoff perspective,” Journal of Signal Processing Systems 92 (2020): 239-256.