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Digitally Captured Signatures: a Method for the Normalization of Force through Calibration and the Use of the Zeta function.

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Abstract

Digitally captured signatures (DCS) are entering the scope of the Forensic Handwriting Examiner, and the meaningful comprehension and comparison of the captured information is essential in order to proceed to the examination of authenticity of DCS. In DCS solutions force is captured and presented as pressure levels, but the relation between exercised force and the assigned pressure levels is not provided by the manufacturers of these solutions. For this purpose, we constructed an experimental array that allows the correlation of exercised force and assigned pressure levels, in a combination of three different digitizers, six different styli and four different capturing software. This process let us calculate the correlation function that assigns pressure levels to force for each solution (called the Zeta Function). Through this process it was observed that different solutions follow different Zeta functions. To address this problem, a methodology for normalization of captured data between different solutions was created and demonstrated, using the calculated Zeta function and its inverse.

Keywords

Capturing hardware and software, digitally captured signature, biometric signature, calibration, zeta function, compatibility, normalization, hysteresis, forensic handwriting examination, electromagnetic resonance

Highlights

Correlation of force to assigned pressure levels for DCS
Calibration and normalization of digitizers

1. Introduction

Forensic handwriting and signature examination rely on the analysis, comparison and evaluation of a series of characteristics that describe both the static (pictorial) as well as the dynamic aspects of a given handwriting product. Amongst these features, a very important one that allows the study of the dynamic aspects of handwriting, is pressure. Identified as one of the three aspects of pen control (as point load, the others being pen position and pen hold) [1], pen pressure (point load or just pressure) can be defined as "the weight or pressure unconsciously applied to the pen during the act of writing" [2], and this definition in itself includes some ambiguity. In the ENFHEX Best Practice Manual (BPM) [3], as part of the "General Characteristics" of handwriting and signatures that have to be taken into account during a forensic examination of authenticity, "fluency/pressure" is mentioned indirectly as a description of "whether the writing appears to be skillfully or poorly produced, whether there is hesitation in the pen line, whether the writing line is smooth flowing and whether the writing line has variable pressure, or constant, hard pressure" and the association of quality, penmanship and "pressure" becomes obvious. Still, pressure (in any of the above references) remains an aspect of writing that is evaluated by the qualified Forensic Handwriting Examiner (FHE) qualitatively and not quantitatively through inspection of the written line.

In biometric signatures (also known as the Digitally Captured Signature - DCS or eSignatures) the software and hardware solutions employed to digitize and encrypt the hand/stylus movements during signature execution produce sets of interconnected data. The mainstream coordinates currently captured by these solutions worldwide are identified as the X, Y, T and F channels [4]. According to the ISO/IEC documentation [4], the X and Y channels capture the x and y spatial coordinates (i.e. the horizontal and vertical pen position on the surface of the digitizer) and the T channel captures the time data relative to the first data point. The Pen tip force channel (or F channel) is defined as "recording pen forces (pressure) data". The ISO documentation goes one step further recognizing Newtons (N) as the unit of measurement. This is not necessarily reflected in the output of the currently available software and hardware solutions (as will be discussed later) and the terminology used in the application of this technology can be misleading. The ease with which different measures are interchanged creates problems of perception as pressure and force are treated as equals, and even though the ISO documentation describes the F channel values as "Force" to be measured in N, most solutions refer to the F channel values as "Pressure".

This technology however allows the quantification of force, enabling further investigation into how this feature of the handwriting movement can be quantified and expressed in actual force units and how it should be interpreted by the FHE. For the purposes of this study the term force will be used for the captured Z axis coefficient and the measurements will be presented in N. The aim of this study is to define an applied force/pressure level assigning function (here forth called the Zeta function - ZF) for selected software/hardware solutions and construct a normalization method, which is necessary [5] for the valid comparison of DCS captured with different Software and Hardware solutions, mimicking expected casework conditions.

2. Materials and Methods

2.1. Equipment Used

In order to approach the issue of force recording in the case of DCS, an accurate mechanism of reproducible and measurable force load had to be constructed. This has been accomplished in the past [6, 7], but a simpler approach to apply measurable force was chosen. An XY-Plotter (the AxiDraw V.3 from Evil Mad Scientist Laboratories) was used as a stable holding mechanism for the stylus. The height of the XY-plotter, the height of the stylus fixture and the relative height of the scales (PCE-BS 3000 from PCE Instruments which is calibrated) to the XY-plotter allow for different percentage of the actual weight of the XY-plotter to be applied by the stylus tip. The Furthermore, the selected XY-plotter allows the positioning of additional weight in the stylus holding part hence providing the capability to change the force load applied by the stylus tip (see figures 1 and 2).

For each set of measurements, the stylus was placed in the pen mount perpendicular to the digitizer, at the center of the active area, and a measurement was recorded, noting the weight measurement. The collection per set was executed consecutively, adding weight from 0 to the maximum force threshold and back, adding or removing additional weight without changing the position of the plotter. The time interval between measurements was as long as it was required to collect the measurements. Initial tests were conducted to determine the minimum and maximum force thresholds per combination.The recorded values were subsequently transformed from weight (measured in grams) to force (measured in Newtons) through the formula $F = m * g$, where F is the force applied (in N), m is the mass (in Kg) and g is the acceleration of gravity on sea level (9.81 m/sec^2).

Two different Electromagnetic Resonance (EMR) technology digitizers were selected, coupled with not only their default styli but also with a third-party compatible stylus and a compatible inking pen. Two different software solutions as well as the digitizer manufacturer's SDK (Software Developer's Kit) were used for the capturing and acquisition of the data of the aforementioned combinations. Furthermore, an Apple iPad Pro (1st Gen) coupled with an Apple Pencil (1st Gen) which uses conductive technology [8] was used in combination with a dedicated DCS capturing application [9]. The different collection combinations are exhibited on Table 1.

The three software programs [10], [11], [12] with the same hardware solution (combinations 1, 2 and 3) allowed the evaluation of how force is captured from the software side with the same default stylus and digitizer. The choice of different hardware from the same manufacturer (combinations 1 and 5 for different digitizers with their default styli, combinations 1, 4, 7 and 8 for the STU 540 and 5 and 6 for STU 530 with different styli, but with the same digitizer and software, 4 and 6 with the same stylus over different digitizer) captured using the same software solution allowed the evaluation of how force is captured from the hardware side.
The selection of Wacom's Bamboo Spark inking pen was aimed at studies that compare signing behavior on paper and on glass (DCS) by using the available EMR technology

[13]. The inking pen is an EMR active stylus that also includes an inking cartridge thus allowing the pen to act as a traditional ballpoint pen (or felt tip, depending on the inking cartridge available), producing an ink trace on paper. The EMR circuit inside the inking pen allows the simultaneous capturing of the aforementioned four channels (X, Y, F and T) by the digitizer when the inking pen is moving over it and is within range. This technological advantage is used to study "Hybrid Signatures" which are created when executing a signature formation with an inking pen on a sheet of paper on top of the digitizer. Hybrid signatures are valuable for research purposes as they provide both a physical (paper) and a digital (DCS) representation of the same signature movement. This formation has been used before [13] with the Wacom STU 530 digitizer and the Wacom Bamboo Inking pen, using a sticky note attached on the glass surface of the digitizer, and for that reason the authors chose to include a sticky note attached on the surface of the digitizer during the measurement collection phase. The selection of the LAMY AL-star black EMR (which comes in two different versions which are equipped with different tips, a plastic one for the paper like surface variant and a rubber one for the glossy like surface variant– combinations 7 and 8) was aimed to the investigation of the forensic properties of a third party EMR pen, when used in combination with digitizers popular in the banking and government sector applications.

The experiment requires the collection of a set of measurements or different forces applied to each of the chosen digitizers within the minimum and maximum threshold for applied force. The datasets were then graphed and fitted using RStudio [14] and the resulting regressions will be compared and discussed.
In preliminary testing, polynomials of different powers (3rd, 4th, 5th and 6th) as well as a logarithmic fit were applied and compared. From the different models the 6th degree polynomial fit provided the best fit not only for the main body of the data set but also for the extremities (i.e. Iressure levels close to 0 or 1023) which are very important areas in the normalization process, hence the 6th degree polynomial fit was chosen for all calibrations.

3. Results

For the EMR technology combinations (see Table 1), a hysteresis effect was observed as expected as it occurs in ferromagnetic and ferroelectric materials [15]. Hysteresis expresses the dependence of the state of a system on its history – and for the recorded force measurements on EMR digitizers this means that the assigned pressure level per exercised force is dependent on the previous value and specifically if the value is ascending or descending. This can clearly be observed in the calibration curve on figure 3.
The recognition of the hysteresis effect in Wacom EMR digitizers should not be considered something new, as it is mentioned by Wacom in the relevant patent filings [16].
In order to calculate both phases of the hysteresis effect, two sets of measurements were collected between the minimum and maximum values, one set was collected with ascending weight load only and then one set was collected with descending weight load only.

For the seven EMR combinations the collected datasets were used to calculate the corresponding ascending load and descending load ZF and ZF^{-1}, whilst for the iPad Pro only one ZF and ZF^{-1} was calculated, presented in Table 2. The ZF^{-1} is calculated to allow computation of exercised force from assigned pressure level and is required for the normalization process presented in section 4.1.1.

3.1 Wacom STU 540 with default stylus and Wacom SDK

The Wacom STU 540 with default stylus, using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 4.9 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 3. As can be observed from figure 3, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated (see Table 2).

3.2 Wacom STU 540 with default stylus and Wacom Signature Scope

The Wacom STU 540 with default stylus, using Wacom Signature Scope (WSS) recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 3.8 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 4. As can be observed from figure 4, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated (see Table 2).

3.3 Wacom STU 540 with default stylus and Namirial FirmaCertaForensic

The Wacom STU 540 with default stylus, using Namirial FirmaCertaForensic (FCF) recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 3.8 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 5. As can be observed from figure 5, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.4 Wacom STU 540 with Bamboo Spark Inking Pen and Wacom SDK

The Wacom STU 540 with Bamboo Spark Inking Pen, using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 4.5 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 6. As can be observed from figure 6, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.5 Wacom STU 530 with default stylus and Wacom SDK

The Wacom STU 530 with default stylus, using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 3.5 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 7. As can be observed from figure 7, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.6 Wacom STU 530 with Bamboo Spark inking pen and Wacom SDK

The Wacom STU 530 with Wacom Bamboo Spark inking pen, using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 4.5 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 8. As can be observed from figure 8, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.7 Wacom STU 540 with LAMY AL-star black EMR (Glossy Surface) and Wacom SDK

The Wacom STU 540 with LAMY AL-star black EMR (Glossy Surface), using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 6 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 9. As can be observed from figure 9, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.8 Wacom STU 540 with LAMY AL-star black EMR (Paper Surface) and Wacom SDK

The Wacom STU 540 with LAMY AL-star black EMR (Paper Surface), using Wacom SDK recorded 1024 pressure levels (from 0 to 1023). During preliminary testing it was found that the device registers applied force between 0.14 and 6 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and the results are shown in figure 10. As can be observed from figure 10, both datasets show a pseudo-logarithmic response. The datasets were fitted and the polynomial functions with the best fit were calculated.

3.9 iPad Pro 9.7” (1st Gen) with Apple Pencil (1st Gen), captured with Forensic Signalyzer app.

The Apple iPad Pro, using Forensic Signalyzer app recorded 4096 pressure levels (from 0 to 4095). During preliminary testing it was found that the device registers applied force between 0.085 and 4.8 N. Measurements were collected ascending from minimum to maximum and then descending from maximum to minimum and are plotted on figure 11. As can be observed from figure 11, both datasets show a linear response and are identical. The datasets were fitted and the polynomial functions with the best fit were calculated.

The ascending load Zeta function is $Z_u(x) = -23.47 + 851.3x$. The descending load Zeta function is $Z_d(x) = -23.47 + 851.3x$. For uniformity with the previous, the fitting function used was a 6th degree polynomial, which produced the linear equation. It is obvious that the iPad Pro does not exhibit a hysteresis effect (as expected as it is not EMR technology), since the difference of the two is zero. Therefore, there is only one Zeta function for the iPad Pro and we proceed with the calculation of only one inverse Zeta function, which is $Z^{-1}(x) = 2.757 * 10^{-2} + 1.175 * 10^{-3}x$.

4. Discussion

4.1. Comparison of different technologies (combinations 1 to 8 and 9)

As demonstrated in figures 1b and 9b, different technologies can exhibit radically different models of pressure level assignment to force (e.g. linear vs. polynomial). It is evident that direct comparison of unnormalized assigned pressure levels for DCS captured with solutions using different capturing technology should only be undertaken and interpreted with the necessary care in a forensic context as it can be misleading (e.g. see figures 12a and 12b). The difference between solutions of the same EMR technology is less dramatic but also not negligible between any of the tested combinations. As demonstrated in Table 4, the same pressure level may refer to significantly different force and hence their unnormalized comparison and evaluation may be problematic.

4.1.1 Normalization Process

The goal is to normalize the force data of a source solution so that they are comparable to the force data of a target solution. When a signature formation is executed and captured by the source solution, the exercised force on the source digitizer is unknown – only the assigned pressure levels are known and are assigned by use of the Zeta function. From the ZF, its inverse (ZF^{-1}) can be calculated, i.e. the function that given the value of the assigned pressure level will calculate the initially exercised force, hence the initially exercised force values can be calculated in N. Then, using these reconstructed force values with the target solution's ZF we can calculate the pressure levels that would have been assigned by the target solution if the signature was captured by it.

This process has to happen for both ascending and descending loads to take into account the hysteresis effect – if it exists, and a criterion needs to be set to distinguish which type of load (and hence ZF) is to be used. This criterion is the difference of force between two consecutive points (ΔF). If ΔF>0 then the force is increasing and the Ascending Load ZF should be used, if the ΔF<0 then the force is decreasing and the Descending Load ZF should be used. Finally different minimum and maximum force thresholds should be taken into account.

To demonstrate the importance of normalization we captured two signatures and we applied the process to normalize the data of the source solution to the target solution. The source solution was chosen to be the Apple iPad Pro with the Apple Pencil captured with Forensic Signalyzer app, and the target solution was chosen to be the Wacom STU 530 with the default stylus captured with the Wacom SDK. The reference DCS, captured with the target solution, is visualized with R Studio in figure 12a, with the pressure levels represented as shades of gray. The unnormalized DCS, captured with the source solution, is visualized with R Studio in figure 12b, with the pressure levels represented as shades of gray.

The normalization process is exhibited on Table 3 (for a small but indicative part of the data of the unnormalized signature), and the process is demonstrated in a flowchart (figure 13). The resulting normalized DCS is visualized in figure 12c.

4.2. Comparison of same hardware with different software

Combinations 1, 2 and 3 calibrate the Wacom STU 540 with the default stylus, captured with three different software solutions, i.e. the Wacom SDK, the Wacom Signature Scope and Namirial's Firma Certa Forensic. As shown on figure 14, the calibration curves of the 3 combinations are similar, so the conclusion reached is that these three software attribute the same pressure level to the same applied force for the same device. This result cannot be generalized for software other than the aforementioned three, as the literature indicates that this is not always the case [17, 18, 19].

4.3. Comparison of different hardware with same software

4.3.1 Comparison of different digitizer with corresponding default stylus, same software

To explore the way pressure levels are allocated by different hardware solutions under the same software, we compared the calibration curves of Wacom's STU 530 and Wacom's STU 540 with their default styli, accordingly, captured with the Wacom SDK (combinations 1 and 5, Table 1). As shown in figure 15, the calibration curves are clearly different.

The experiment shows that pressure levels are allocated in a similar (pseudologarithmic) manner but at different levels, not allowing quantitative comparison of data captured with these two digitizers (without normalization).

4.3.2 Comparison of same digitizer, different stylus, same software

4.3.2.1 Wacom STU 530, Wacom SDK, default stylus vs. Wacom Bamboo Spark inking pen

To explore the way pressure levels are allocated when using different styli but the same digitizer under the same software, we compared the calibration curves of Wacom's STU 530 with the default stylus and the Bamboo Spark inking pen captured with the Wacom SDK (combinations 5 and 6). As shown in figure 16, the calibration curves are different.

Pressure levels are allocated in a similar (pseudologarithmic) but different model (see Table 4), not allowing quantitative comparison of data captured with these two styli (without normalization). These results can be explained by the fact that the correlation of exercised force to assigned pressure level is related to the pressure sensor inside the stylus and not the digitizer itself for the EMR technology solutions.
This result may explain the statistical difference observed by Heckeroth et al. [13] during the comparison of signatures collected on Wacom STU 530 digitizers both with the default stylus and the Bamboo Spark inking pen, exploring the characteristics of signature execution between DCS on glass and traditional signature on paper surfaces.

4.3.2.2 Comparison between Wacom STU 540, Wacom SDK, default stylus vs. Wacom Bamboo Spark inking pen vs. LAMY Al-Star black EMR (paper surface and glass surface variants)

To further explore the way pressure levels are allocated when using different styli but the same digitizer captured by the same software, we compared the calibration curves of Wacom's STU 540 with the default stylus, the Bamboo Spark Inking Pen Wacom Bamboo Spark Inking Pen the two variants of the LAMY Al-Star black EMR (paper surface and glass surface), captured with the Wacom SDK (combinations 1, 4, 7 and 8). As exhibited in figure 17, the calibration curves for the two variants of the LAMY Al-Star black EMR (combinations 7 and 8) are similar, so the conclusion can be drawn that both LAMY Al-Star black EMR styli attribute applied force to pressure levels in the same way, regardless of the variant (which has to do with the material at the tip of the stylus).

Proceeding with the comparison of the calibration curves of combinations 1, 4 and 7 (which also represents 8), we observe strong differences as exhibited in figure 18. Pressure levels are allocated in a similar (pseudologarithmic) but different model, not allowing quantitative comparison of data captured with these two styli (without normalization). These results can again be explained by the fact that the correlation of exercised force to assigned pressure level is related to the pressure sensor inside the stylus and not the digitizer itself.

4.4. Limitations

The calibration procedure collected measurements from the center of the active area of each digitizer. Variations due to the position on the digitizer are therefore not expected for EMR technology, since the force sensor is positioned within the stylus. This seems to be confirmed by prior research [7]. Furthermore, the direction of the stylus during collection of measurements was perpendicular to the active area of the digitizer, not exploring the possibility of variation of the registered force by the pressure sensor in an angle. This is not expected to change the accuracy of the force recording by the pressure sensor as this is positioned along the axis of the stylus [9, 16], although this was not confirmed. The measurements have been collected with the stylus being placed statically over the digitizer's active area, whilst the execution of a DCS would include a moving stylus. This is a limitation but again, due to the mechanics involved, the actual force exercised parallel to the axis of the stylus (which is what the pressure sensor registers force) should be recorded without issues. The reliability of the measurements was confirmed when comparing preliminary measurements, the data presented in the experiment and during repetition of some of the measurements, but repeatability was not exclusively tested. However, the results from the two variants of the LAMY Al-star EMR styli (which have a different tip but apparently the same pressure sensor) suggest repeatability. Lastly, the aging of the pressure sensor [5] was not examined in this study as new styli were used for the calibrations. The aging factor is important as demonstrated in the literature [5] and will be examined in further research.

5. Conclusion

Different hardware solutions (both of the same and of different sensor technology) will probably attribute different pressure levels for the same applied force. This result does not allow the direct comparison of force data and from different sources without further analysis [18], without necessarily precluding qualitative comparison of DCS captured with different solutions of the same type (i.e. both following linear models or both following pseudo-logarithmic models). It is expected that on a practitioner level, FHEs will encounter cases where either the known material is from mixed sources or the questioned material is from a different source from the known material, or both. To allow the meaningful and scientifically valid comparison of such data, the methodology described allows the normalization for the force data – either to a selected target solution or to purely as numerical force values. There are limitations on the range of comparable values when the two solutions have different force thresholds (as exhibited in the example in 4.1.1), still this method allows the comparison of the two different DCS within the interval of overlapping force values. The different methods (e.g. polynomial vs. linear) of assigning pressure levels to force are derived by the hardware used and its properties. In contrast to this, in traditional FHE methodology to examine handwriting produced on paper, there is no approach to quantitatively evaluate writing pressure. Therefore the examination of quantitative force values in DCS is new to the forensic field. This leads to the question which system of force to pressure level correlation is more suitable for DCS comparisons (polynomial or linear) if any.

The construction of a calibration methodology through the calculation and use of the ZF, together with the use of styli with high force sensitivity range (e.g. the LAMY AL-star black EMR) allows researchers to gauge the actual force range of signature execution, construct population studies and enrich the FHE community's contributions and suggestions as to the necessary hardware and software requirements towards the industry [19]. Still, the exploration of the force to pressure level correlation is not complete. Further research needs to be carried out regarding the various limitations already mentioned.

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Combination Number	Digitizer	Stylus	Software Used for Capturing
1	Wacom STU 540	Default (540)	Wacom SDK
2	Wacom STU 540	Default (540)	Wacom Signature Scope
3	Wacom STU 540	Default (540)	Namirial FirmaCertaForensic
4	Wacom STU 540	Wacom Bamboo Spark Inking Pen	Wacom SDK
5	Wacom STU 530	Default (530)	Wacom SDK
6	Wacom STU 530	Wacom Bamboo Spark Inking Pen	Wacom SDK
7	Wacom STU 540	LAMY AL-star black EMR (Glossy Surface)	Wacom SDK
8	Wacom STU 540	LAMY AL-star black EMR (Paper Surface)	Wacom SDK
9	Apple iPad Pro 9.7” (1^{st} Gen)	Apple Pencil (1^{st} Gen)	Forensic Signalyzer

Table 1 Combination of capturing conditions.

Combination Number	Solution Description	Ascending Load Zeta Function	Ascending Load Inverse Zeta Function	Descending Load Zeta Function	Descending Load Inverse Zeta Function
1	Wacom STU 540 with default stylus and Wacom SDK	$Z_u(x) = -131.5267 +980.8542x - 282.43 46x^2 -17.9142x^3 + 28.158x^4 - 5.7823x^5 +0.3858x^6$	$Z_u^{-1}(x) = 0.5604 - 8.463 * 10^{-3}x + 7.519 * 10^{-5} x^2 -2.678 * 10^{-7}x^3 +4.832 * 10^{-10}x^4 -4.266 * 10^{-13}x^5 +1.482 * 10^{-16}x^6$	$Z_d(x) = -28 2.6441 + 2047.5107x - 15$ $x^2 +664.6895x^3 -1 59.6381x^4 +20.2146x^5 -1.0508x^6$	$Z_d^{-1}(x) = 5.729 - 9.391 * 10^{-2}x + 5.798 * 10^{-4} x^2 -1.701 * 10^{-6}x^3 +2.586 * 10^{-10}x^4 -1.965 * 10^{-12}x^5 +5.92 * 10^{-16}x^6$
	Adjusted R-squared	0.9992	0.9991	0.9989	0.9955
2	Wacom STU 540 with default stylus and Wacom Signature Scope	$Z_u(x) = - 285.902 + 1868.527x - 1364.392x^2 + 613.673x^3 -169.38x^4 + 26.018x^5 - 1.673x^6$	$Z_u^{-1}(x) = 0.1989 - 2.47 * 10^{-3}x + 3.316 * 10^{-5} x^2 - 1.303 * 10^{-7}x^3 + 2.48 * 10^{-10}x^4 - 2.268 * 10^{-13}x^5 + 8.122 * 10^{-17} x^6$	$Z_d(x) = -278.947 + 2968.9$	$Z_d^{-1}(x) = 0.7408 - 1.847 * 10^{-2}x + 1.59 * 10^{-4}x^2 - 5.757 * 10^{-5}x^3 + 1.019 * 10^{-9}x^4 - 8.675 * 10^{-13}x^5 +2.852 * 10^{-16} x^6$
	Adjusted R-squared	0.9969	0.9987	0.9999	0.9948
3	Wacom STU 540 with default stylus and Namirial FIrmaCertaForensic	$Z_u(x) = -128.4184 + 1426.6966 02x^2 +315.1193x^3 -6 6.8796x^4 + 7.8297x^5 -0.3933x^6$	$Z_u^{-1}(x) = 1.824 - 3.583 * 1$	$Z_d(x) = -2 00.386 + 2149.792x - 175 4x^2 +785.736x^3 -1 94.391x^4 + 24.851x^5 -1 .28x^6$	$Z_d^{-1}(x) = 26.50 - 3.889 * 10^{-1}x + 2.152 * 10^{-3} x^2 -5.822 * 10^{-6}x^3 +8.266 * 10^{-9}x^4 -5.908 * 10^{-12}x^5 +1.678 * 10^{-15}x^6$,
	Adjusted R-squared	0.9996	0.9855	0.998	0.9109
4	Wacom STU 540 with Bamboo Inking Pen and Wacom SDK	$Z_u(x) = -151.6523 + 1264.8094 93x^2 +188.6995x^3 -1 6.2854x^4 - 2.3297x^5 -0.3846x^6$	$Z_u^{-1}(x) = 2.655 - 4.477 * 10^{-2}x + 2.841 * 10^{-4} x^2 -8.305 * 10^{-7}x^3 +1.252 * 10^{-9}x^4 - 9 .391 * 10^{-13}x^5 +2.79 * 10^{-16}x^6$	is $Z_d(x) = -1 52.0626 + 1660.6545x - 1$ $x^2 +570.4155x^3 -153.10 55x^4 + 22.07x^5 -1.3165 x^6$	$Z_d^{-1}(x) = 1.314 - 2.968 * 10^{-2}x + 2.295 * 10^{-3} x^2 -7.66 * 10^{-7}x^3 +1.273 * 10^{-9}x^4 -1.029 * 10^{-12}x^5 +3.247 * 10^{-16}x^6$
	Adjusted R-squared	0.9982	0.9976	0.9997	0.9977
5	Wacom STU 530 with default stylus and Wacom SDK	$Z_u(x) = -273.122 + 2351.661x +$ $x^2 +1150.935x^3 -3 41.306x^4 +53.126x^5 -3.387x^6$	$Z_u^{-1}(x) = 14.74 - 1.713 * 10^{-1}x + 8.007 * 10^{-4} x^2 -1.898 * 10^{-6}x^3 +2.434 * 10^{-9}x^4 -1.608 * 10^{-12}x^5 +4.318 * 10^{-16}x^6$	$Z_d(x) = -179.926 + 3971.2$	$Z_d^{-1}(x) = 59.94 - 7.151 * 1$
	Adjusted R-squared	0.9995	0.9987	0.9979	0.9935
6	Wacom STU 530 with Bamboo Inking Pen and Wacom SDK	$Z_u(x) = -65.8149 + 877.1544x + 75x^2 +31.6239x^3 + 6.15 14x^4 - 1.7193x^5 +0.1053x^6$	$Z_u^{-1}(x) = 4.036 - 5.824 * 10^{-2}x + 3.35 * 10^{-4}x^2 -9.142 * 10^{-7}x^3 +1.306 * 10^{-9}x^4 -9.398 * 10^{-13}x^5 +2.711 * 10^{-16}x^6$	$Z_d(x) = -1 30.3994 + 1422.9675x - 7$ $x^2 +203.8218x^3 -11.67 x^4 - 3.6521x^5 + 0.4728 x^6$	$Z_d^{-1}(x) = 833.31 - 7.848 x + 2.971 * 10^{-2}x^2 -5.825 * 10^{-5}x^3 +6.269 * 10^{-8}x^4 -3.525 * 10^{-11}x^5 +8.112 * 10^{-15}x^6$
	Adjusted R-squared	0.9998	0.9951	0.9988	0.9948
7	Wacom STU 540 with LAMY Al-star EMR (Glossy Surface) and Wacom SDK	$Z_u(x) = -18.5877 + 1696.0044x$ $94x^2 +653.7729x^3 -1 61.1162x^4 +20.1305x^5 -0.9985x^6$	$Z_u^{-1}(x) = 6.335 - 8.087 * 10^{-1}x + 3.86 * 10^{-3}x^2 -9.133 * 10^{-6}x^3 +1.154 * 10^{-8}x^4 -7.477 * 10^{-12}x^5 +1.96 * 10^{-16}x^6$	$Z_d(x) = 383.3673 + 1249.2$	$Z_d^{-1}(x) = 1.918 * 10^5 - 1 .353 * 10^3x + 3.964 * x^2 -6.176 * 10^{-3}x^3 +5.397 * 10^{-6}x^4 -2.509 * 10^{-9}x^5 +4.846 * 10^{-13}x^6$
	Adjusted R-squared	0.9977	0.9992	0.9883	0.9955

8	Wacom STU 540 with LAMY Al-star EMR (Paper Surface) and Wacom SDK	$Z_u(x) = -26.551 + 1835.345x - x^2 + 927.681x^3 - 261.062x^4 + 36.934x^5 - 2.058x^6$	$Z_u^{-1}(x) = 16.76 - 2.316 * 10^{-1}x + 1.192 * 10^{-3} x^2 - 3.038 * 10^{-6}x^3 + 4.142 * 10^{-9}x^4 - 2.898 * 10^{-12}x^5 + 8.217 * 10^{-16}x^6$	$Z_d(x) = 380.0815 + 1251.7$	$Z_d^{-1}(x) = 2.836 * 10^4 - 2.151 * 10^2x + 6.769 * 10^{-1}x^2 - 1.131 * 10^{-3} x^3 + 1.058 * 10^{-6}x^4 - 5.259 * 10^{-10}x^5 + 1.085 * 10^{-13}x^6$
	Adjusted R-squared	0.9955	0.9983	0.9869	0.9943
9	iPad Pro with Apple Pencil (1st Gen) and Forensic Signalyzer	$Z(x) = -23.47 + 851.3x$	$Z^{-1}(x) = 2.757 * 10^{-2} + 1.$	$Z(x) = -23.47 + 851.3x$	$Z^{-1}(x) = 2.757 * 10^{-2} + 1.$
	Adjusted R-squared	1	1	1	1

Table 2 Calculated Zeta Functions and their inverse, for ascending and descending loads for all tested solutions. For combination 9, the functions for ascending and descending load are the same.

F Data from iPad Pro	F (N)	F with STU 530 Thresholds	STU 530 Ascending	STU 530 Descending	ΔF	F Data Normalized to STU 530
2405	2.85	2.85	911	956	+	911
2439	2.89	2.89	914	959	+	914
2472	2.93	2.93	918	961	+	918
2464	2.92	2.92	917	961	-	961
2464	2.92	2.92	917	961	0	961
2497	2.96	2.96	920	964	-	920

Table 3 Example of pressure level values captured with the Apple iPad Pro with Pencil, and their transformation stages until they are normalized to Wacom's STU 530 and default stylus values.

	Corresponding force in Newtons		
Pressure Levels	STU 540 with Default Stylus	STU 540 with Bamboo Spark Inking Pen	STU 530 with Default Stylus
300	0.54	0.43	0.28
600	1.07	0.93	0.79
800	1.77	1.68	1.62
1000	4.28	3.38	3.94

Table 4 Example of different pressure levels values assigned for the same force on different digitizers.

FIG. 1 The experimental setup.;

FIG. 2 The experimental setup. ;

FIG. 3 Wacom STU 540 calibration curve with the default stylus and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 4 Wacom STU 540 calibration curve with the default stylus and Wacom Signature Scope, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 5 Wacom STU 540 calibration curve with the default stylus and Namirial FirmaCertaForensic, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 6 Wacom STU 540 calibration curve with the Wacom Bamboo Spark Inking Pen and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 7 Wacom STU 530 calibration curve with the Default Stylus, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 8 Wacom 530 calibration curve with the Wacom Bamboo Inking pen and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 9 Wacom STU 540 calibration curve with the LAMY AL-star black EMR (Glossy Surface) and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 10 Wacom STU 540 calibration curve with the LAMY AL-star black EMR (Paper Surface) and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 11 iPad Pro 9.7" (1st Gen) with Apple Pencil (1st Gen), captured with Forensic Signalyzer app, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements. ;

FIG. 12a The reference DCS, captured with the target solution. pressure levels are represented as shades of gray, with black for 0 and white for the maximum pressure level value. ;

FIG. 12b The unnormalized DCS, captured with the source solution. Pressure levels are represented as shades of gray, with black for 0 and white for the maximum pressure level value. ;

FIG. 12c The normalized DCS, captured with the source solution but normalized to appear as if it was captured with the target solution. Pressure levels are represented as shades of gray, with black for 0 and white for the maximum pressure level value. ;

FIG. 13 The workflow of the steps followed for the normalization. ;

FIG. 14 Comparison of the calibration curves of the Wacom STU 540 with the default stylus, captured with Wacom SDK, Wacom Signature Scope and Namirial FirmaCertaForensic. ; FIG. 15 Comparison of the calibration curves of the Wacom STU 540 with its default stylus and Wacom STU 530 with its default stylus, both captured with Wacom SDK. ;

FIG. 16 Comparison of the calibration curves of the Wacom STU 530 with its default stylus and Wacom Bamboo Spark inking pen, both captured with Wacom SDK. ;

FIG. 17 Comparison of the calibration curves of the LAMY Al-Star black EMR in its two variants (Paper and Glossy surface tips) captured on a Wacom STU 540 with Wacom SDK. ;

FIG. 18 Comparison of the calibration curves of the default stylus, the Wacom Bamboo Spark Inking Pen and the LAMY Al-Star black EMR captured on a Wacom STU 540 with Wacom SDK.

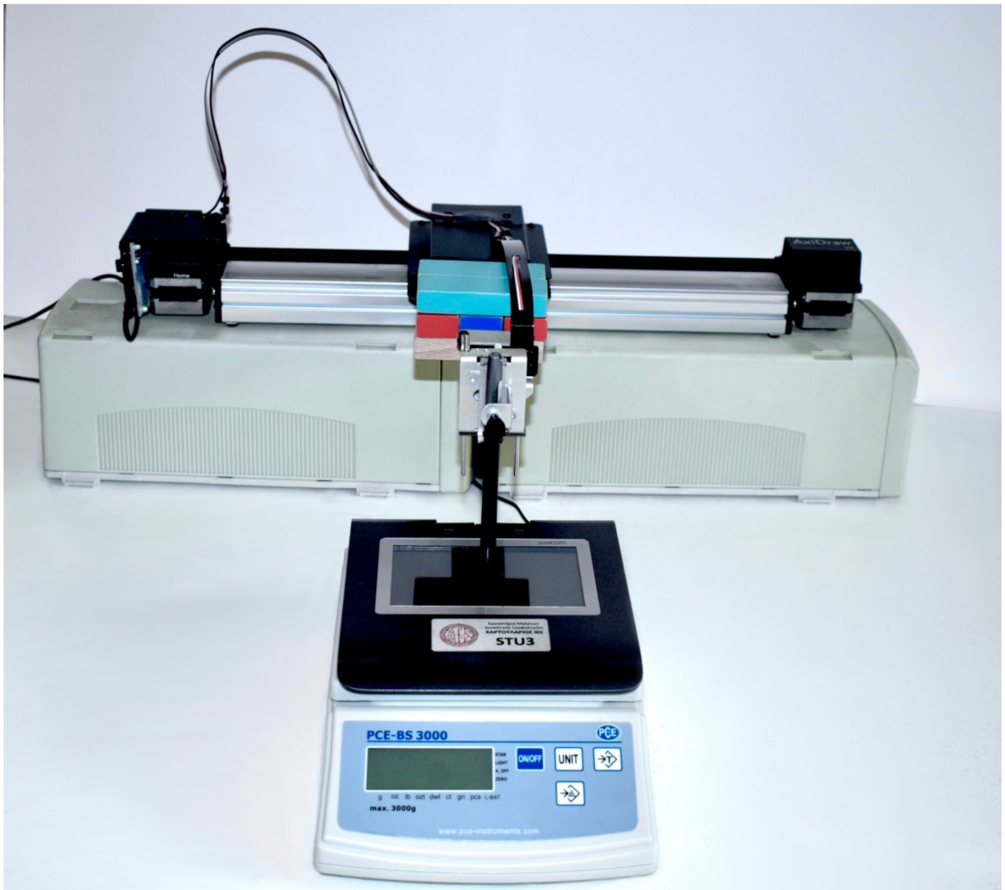

Figure 1: The experimental setup.

172x154mm (300 x 300 DPI)

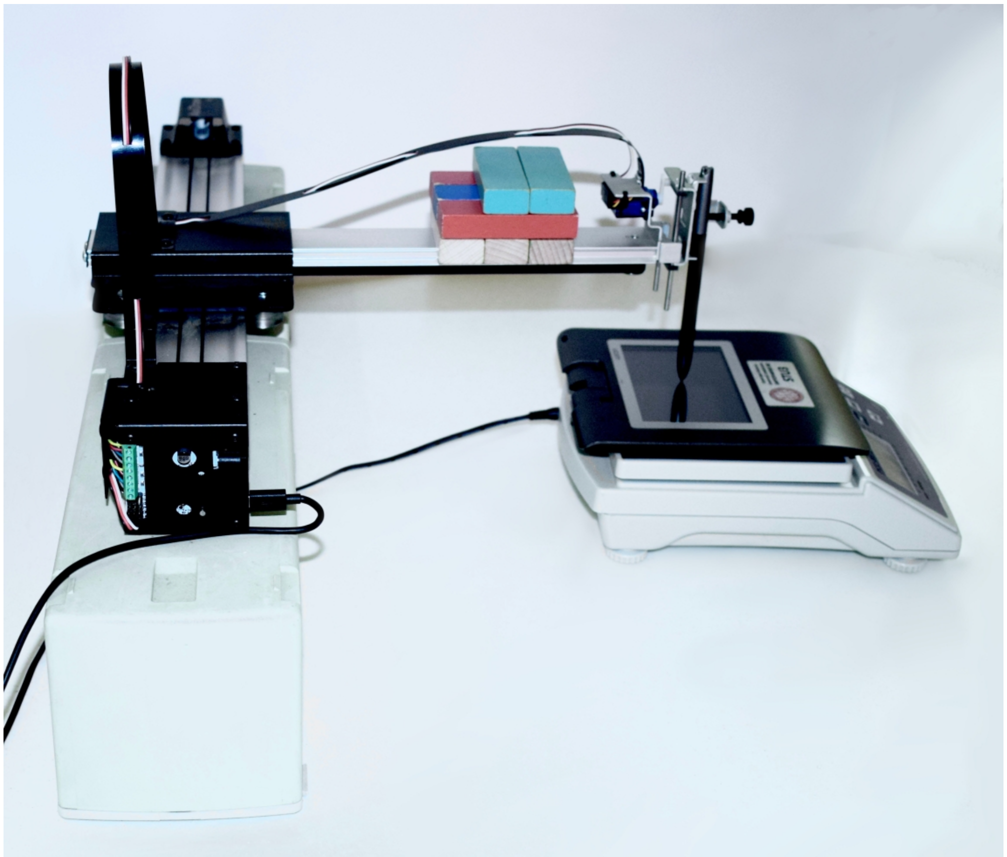

Figure 2: The experimental setup.

336x289mm (300 x 300 DPI)

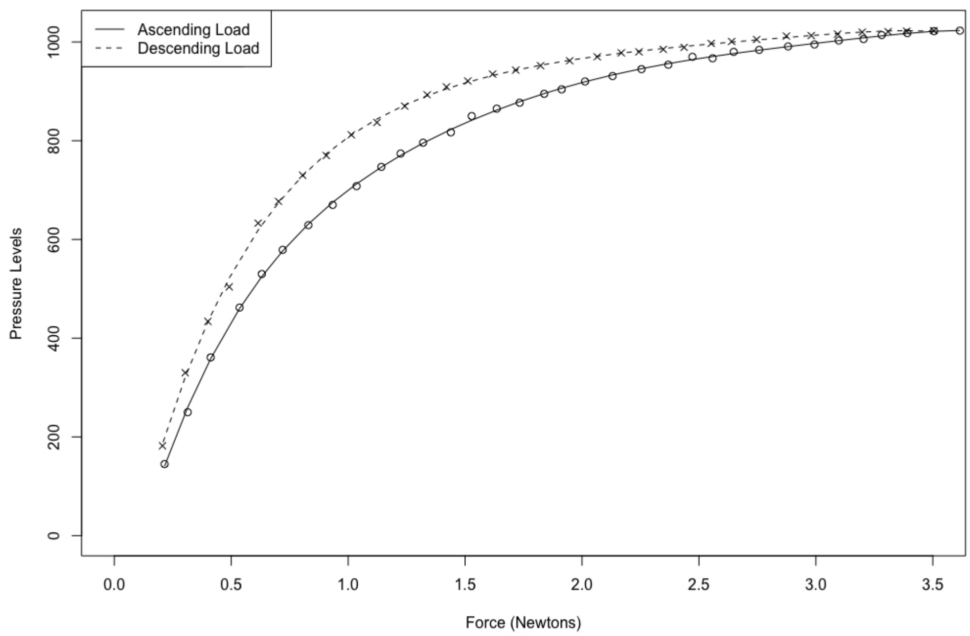

Figure 3: Wacom 540 calibration curve with the default stylus and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x62mm (300 x 300 DPI)

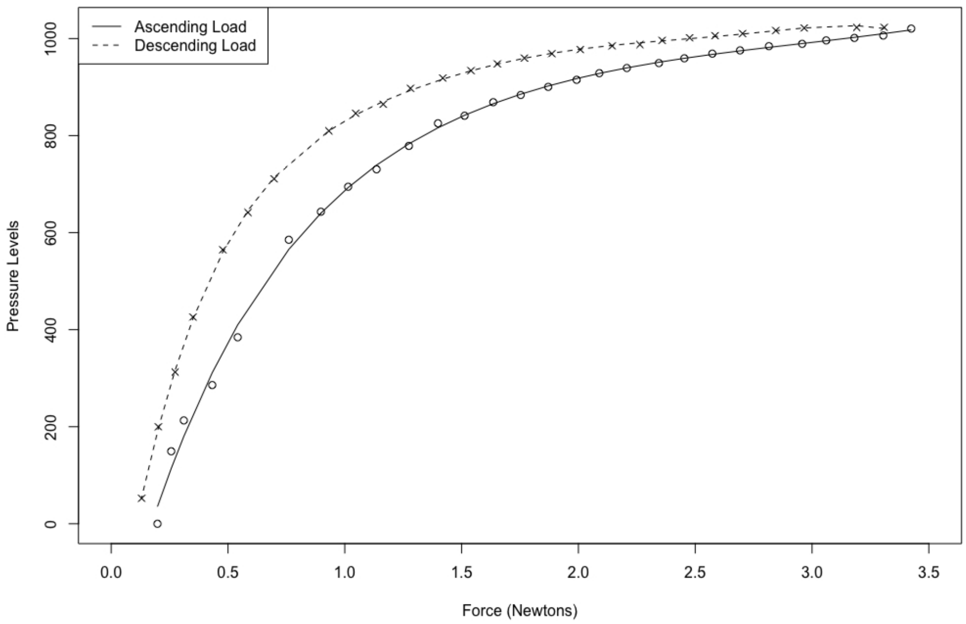

Figure 4: Wacom 540 calibration curve with the Default stylus and Wacom Signature Scope, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

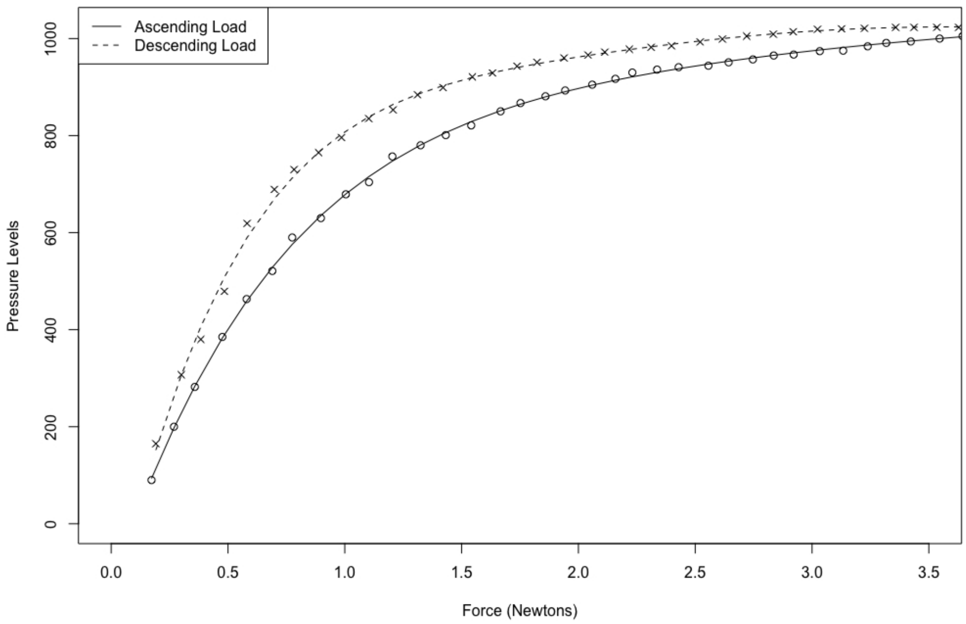

Figure 5: Wacom 540 calibration curve with the Default stylus and Namirial FirmaCertaForensic, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

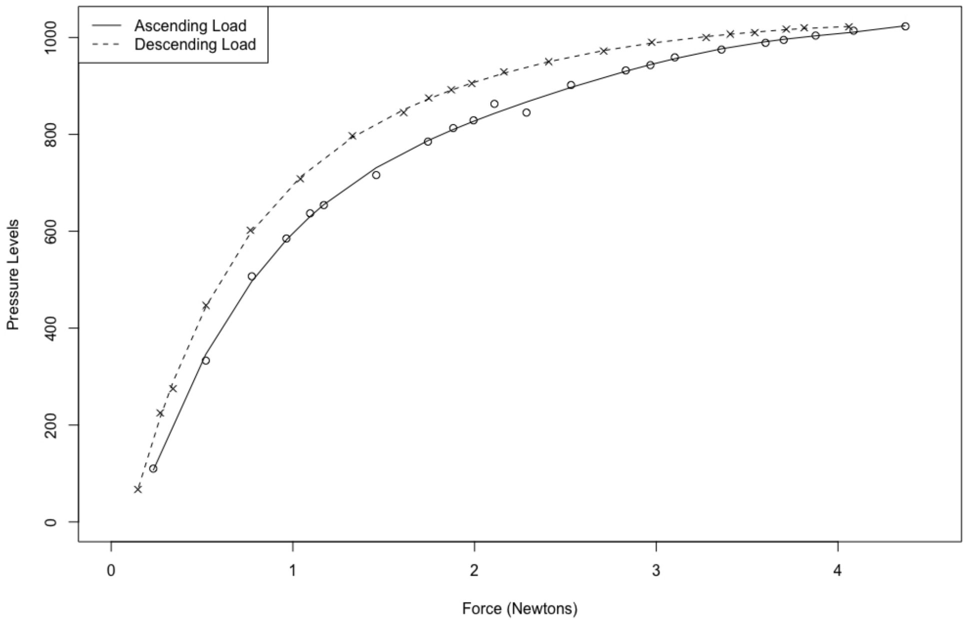

Figure 6: Wacom 540 calibration curve with the Wacom Bamboo Spark Inking Pen and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

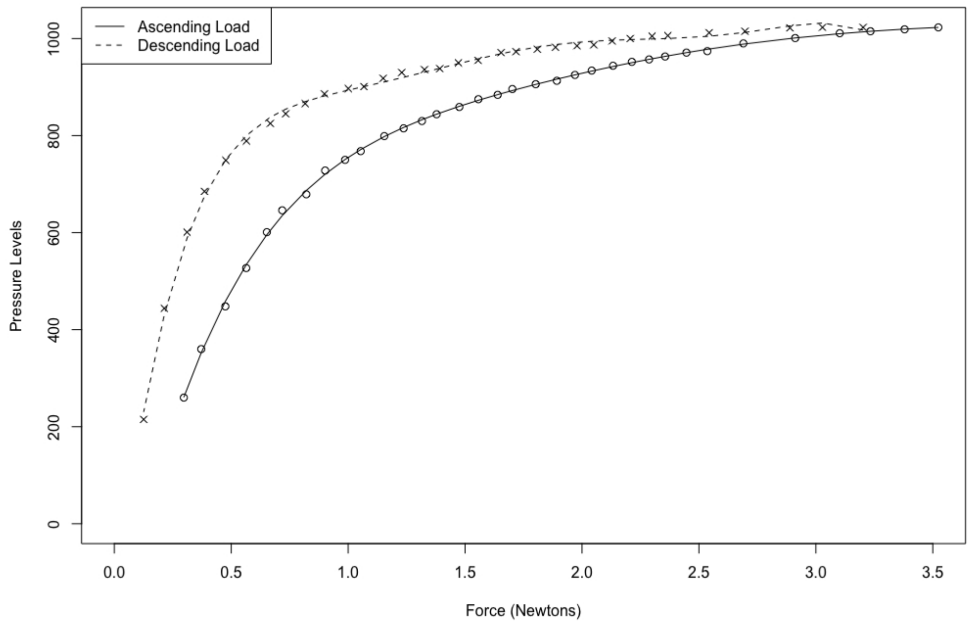

Figure 7: Wacom 530 calibration curve with the Default Stylus, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

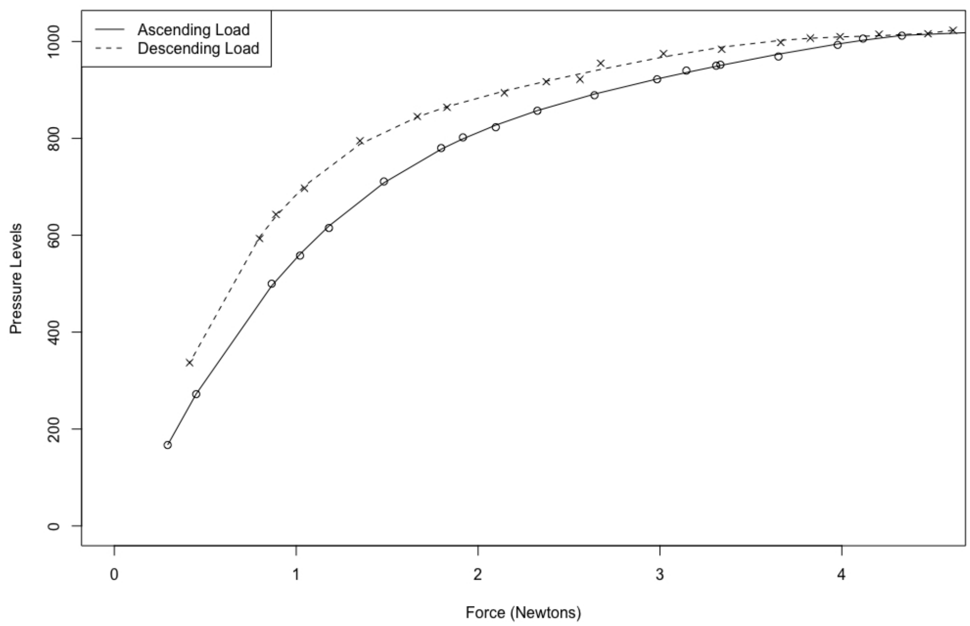

Figure 8: Wacom 530 calibration curve with the Wacom Bamboo Inking pen and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

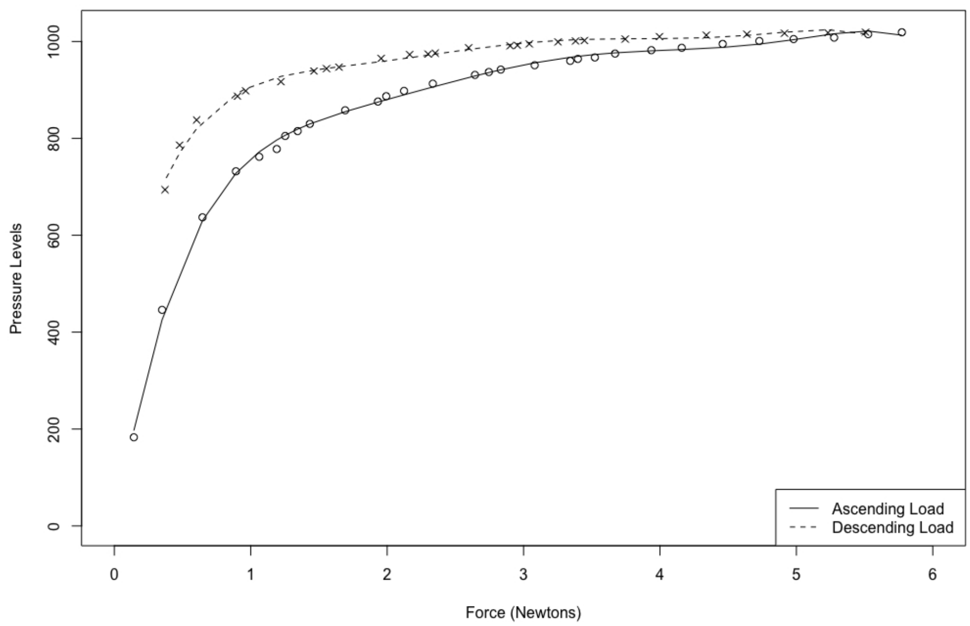

Figure 9: Wacom 540 calibration curve with the LAMY AL-star black EMR (Glossy Surface) and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

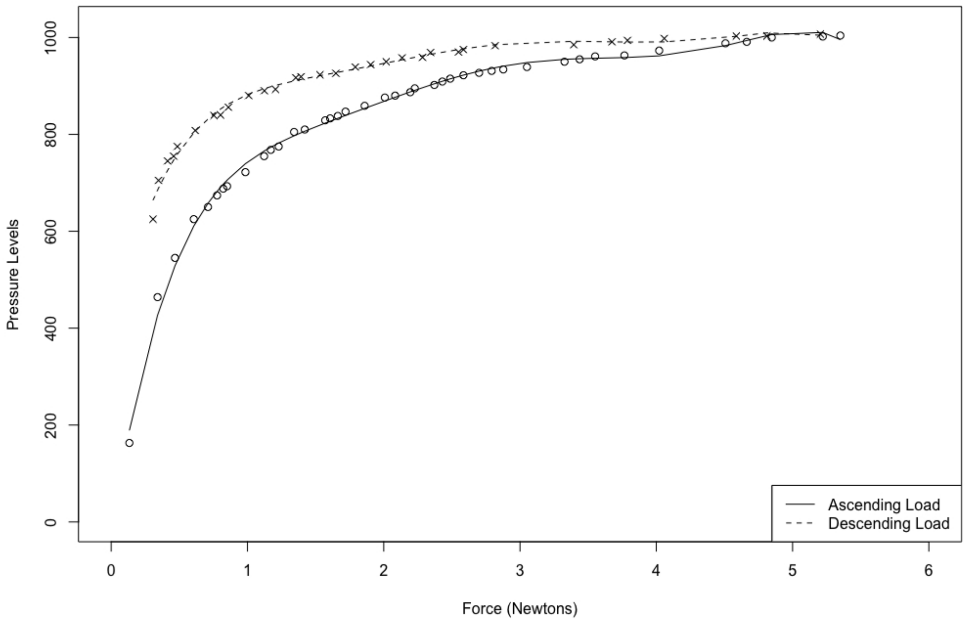

Figure 10: Wacom 540 calibration curve with the LAMY AL-star black EMR (Paper Surface) and Wacom SDK, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

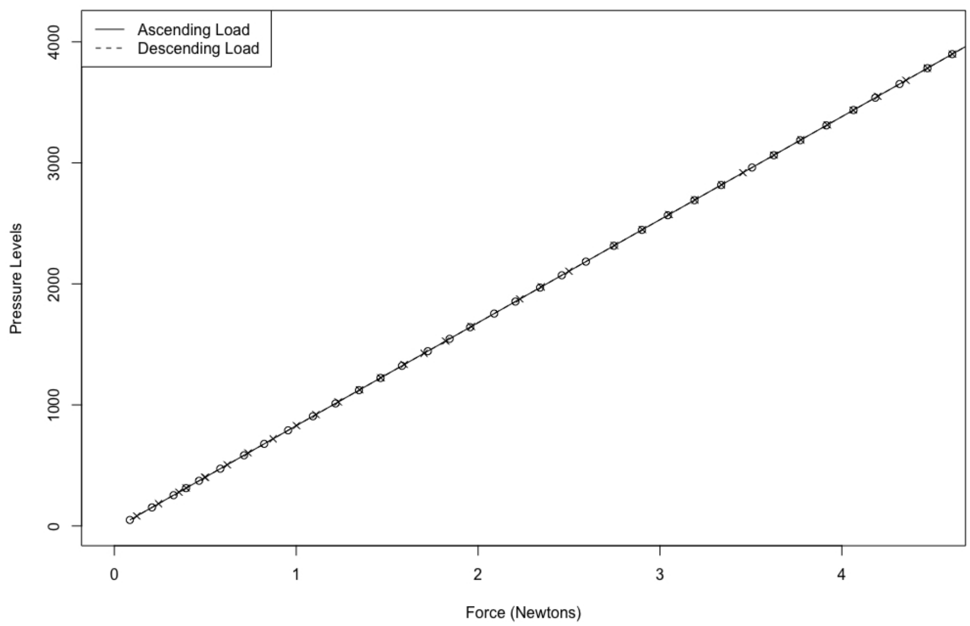

Figure 11: iPad Pro 9.7" (1st Gen) with Apple Pencil (1st Gen), captured with Forensic Signalyzer app, o and x for the ascending and descending load measurements respectively. The fitted zeta functions are plotted with solid line for ascending and dotted line for descending load measurements.

86x61mm (300 x 300 DPI)

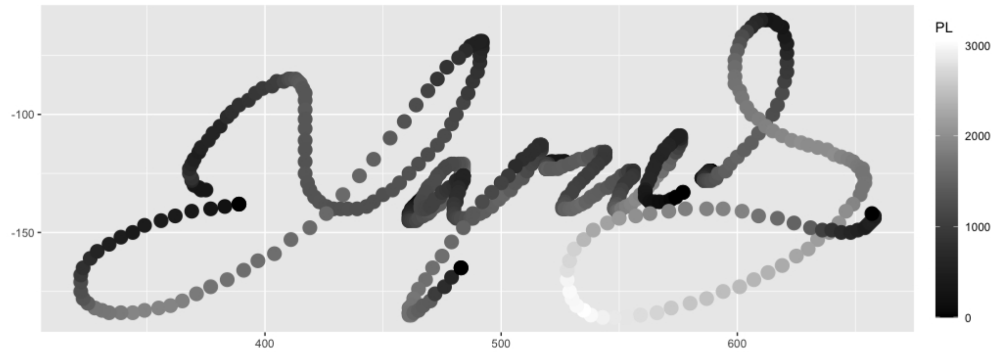

Figure 12a: The reference DCS, captured with the target solution. PL are represented as shades of gray, with black for 0 and white for the maximum PL value.

86x61mm (300 x 300 DPI)

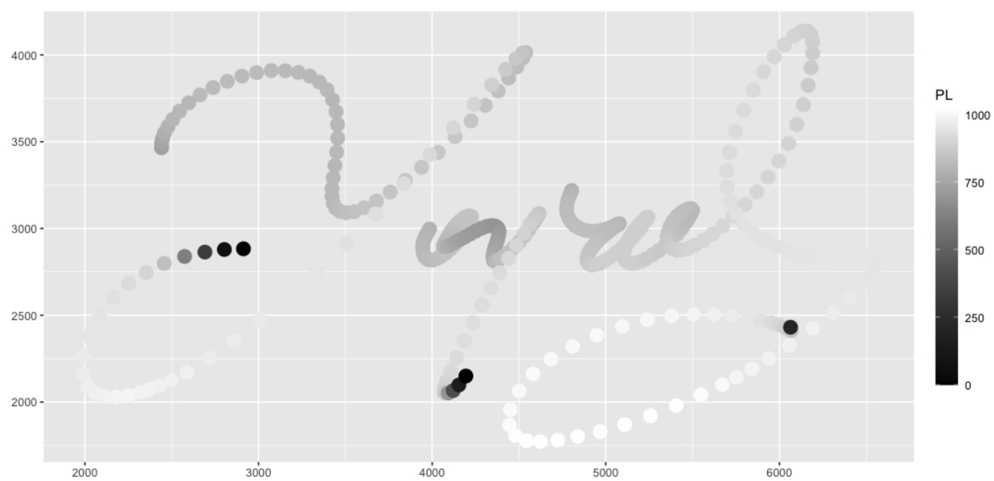

Figure 12b: The unnormalized DCS, captured with the source solution. PL are represented as shades of gray, with black for 0 and white for the maximum PL value.

86x61mm (300 x 300 DPI)

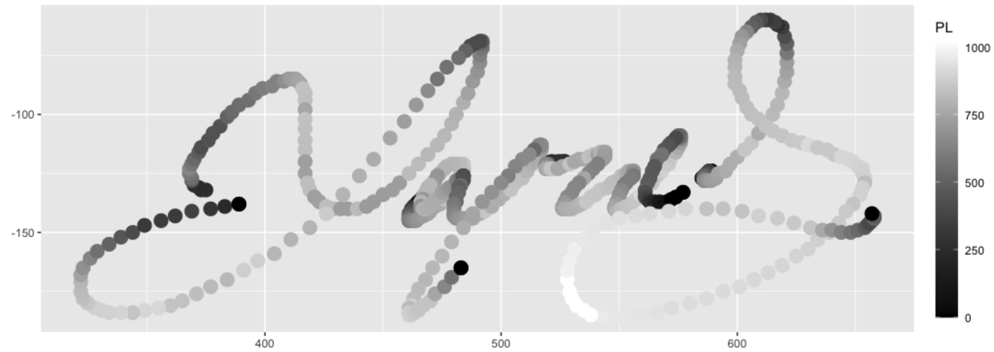

Figure 12c: The normalized DCS, captured with the source solution but normalized to appear as if it was captured with the target solution. PL are represented as shades of gray, with black for 0 and white for the maximum PL value.

86x61mm (300 x 300 DPI)

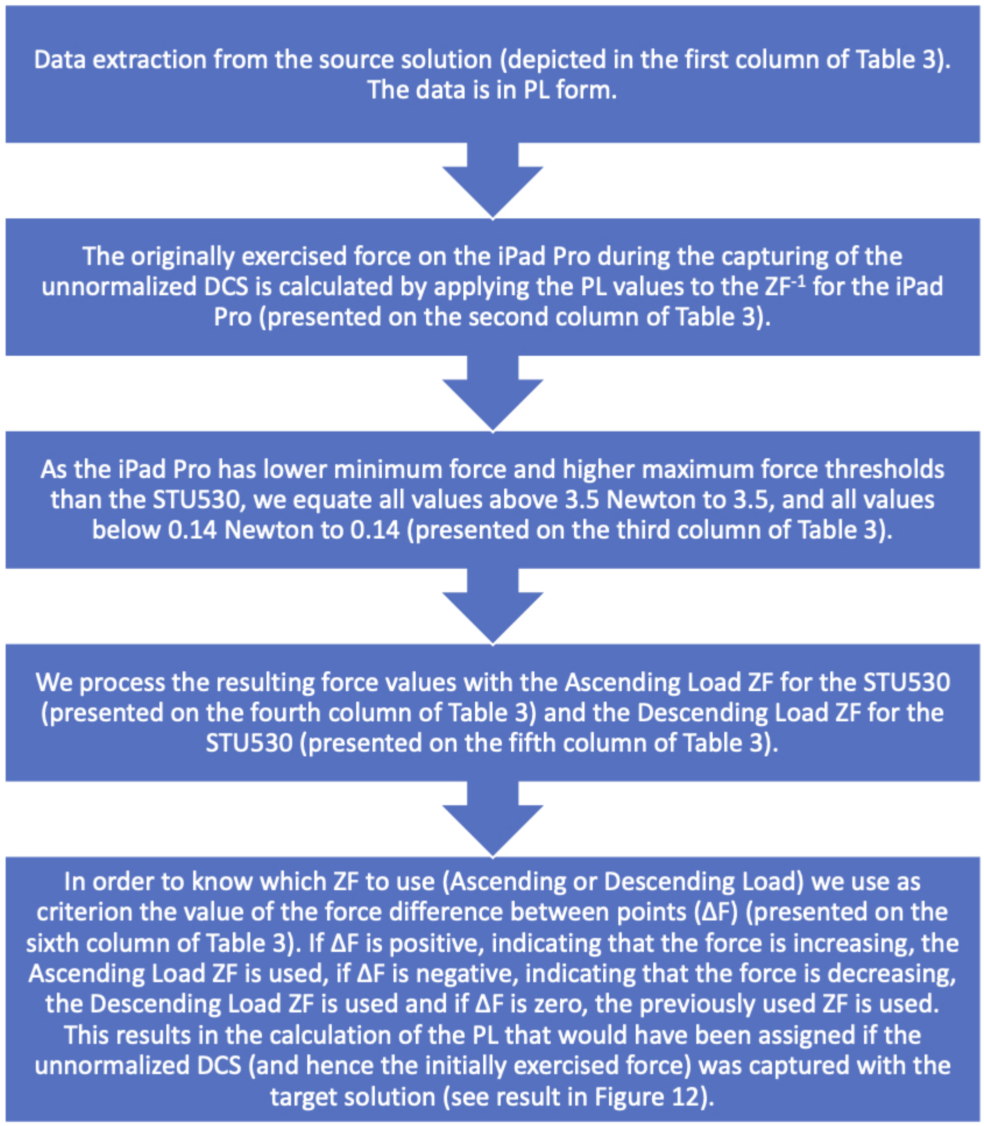

Figure 13: The workflow of the steps followed for the normalization.

72x83mm (300 x 300 DPI)

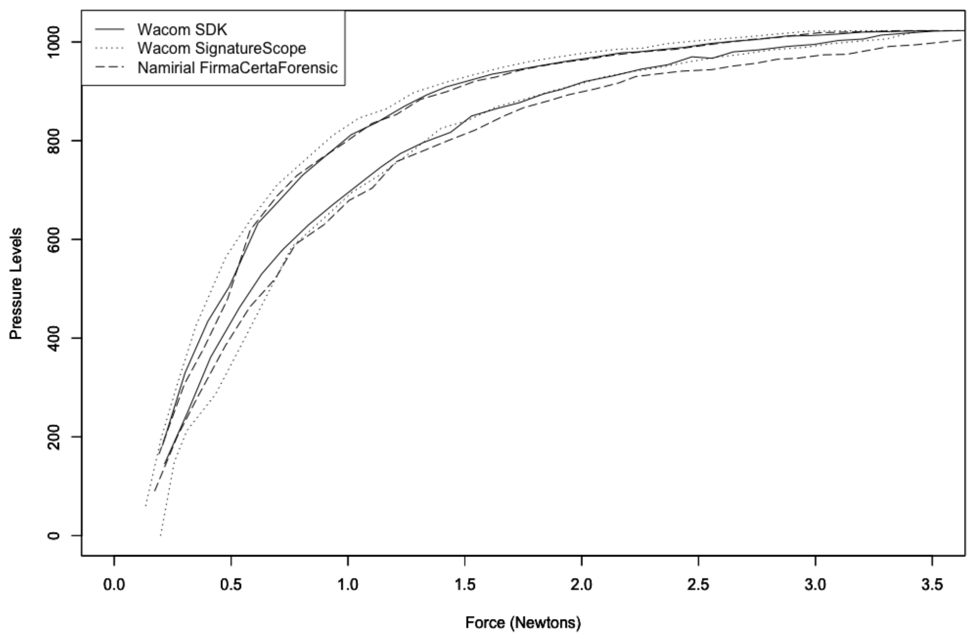

Figure 14: Comparison of the calibration curves of the Wacom STU540 with the Default stylus, captured with Wacom SDK, Wacom Signature Scope and Namirial FirmaCertaForensic.

86x62mm (300 x 300 DPI)

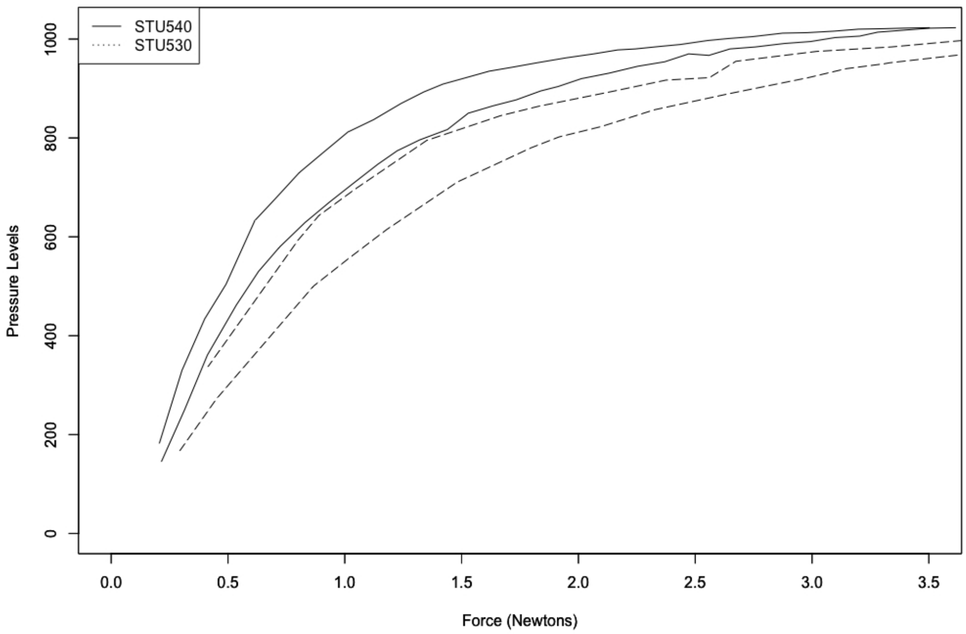

Figure 15: Comparison of the calibration curves of the Wacom STU540 with its Default stylus and Wacom STU530 with its Default stylus, both captured with Wacom SDK.

86x62mm (300 x 300 DPI)

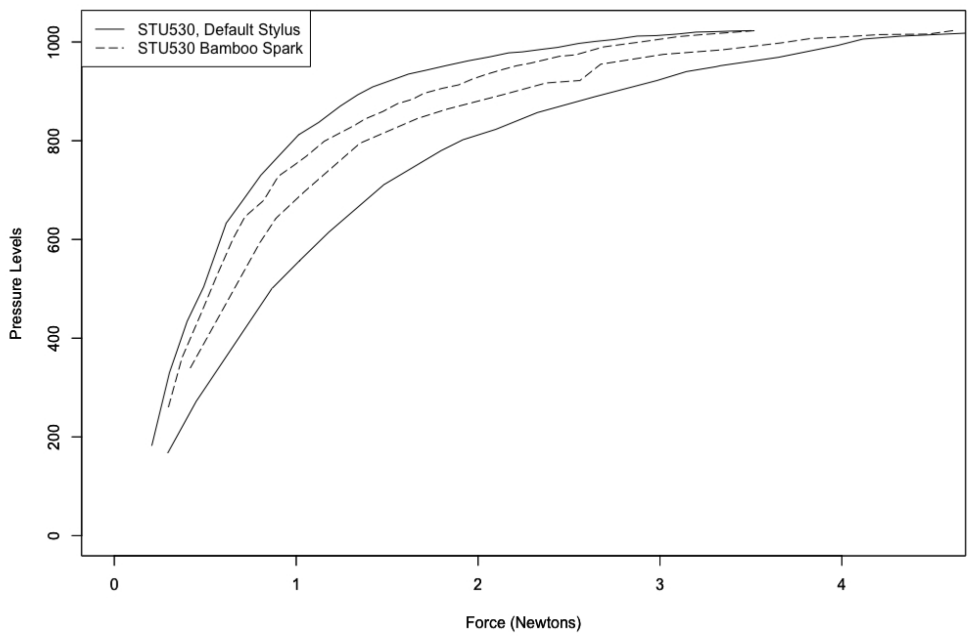

Figure 16: Comparison of the calibration curves of the Wacom STU530 with its Default stylus and Wacom Bamboo Spark inking pen, both captured with Wacom SDK.

86x62mm (300 x 300 DPI)

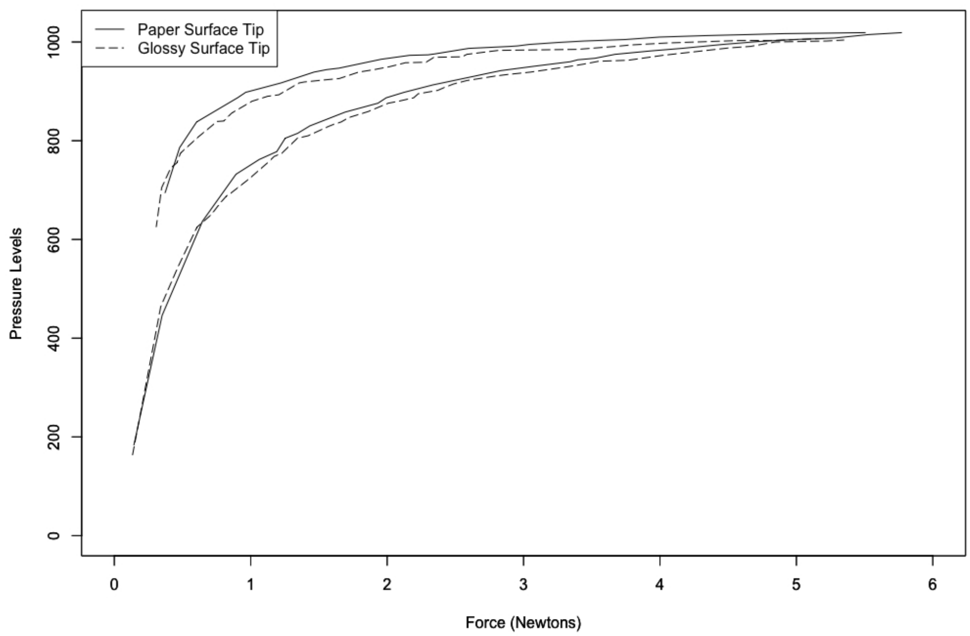

Figure 17: Comparison of the calibration curves of the LAMY Al-Star black EMR in its two variants (Paper and Glossy surface tips) captured on a Wacom STU540 with Wacom SDK.

86x62mm (300 x 300 DPI)

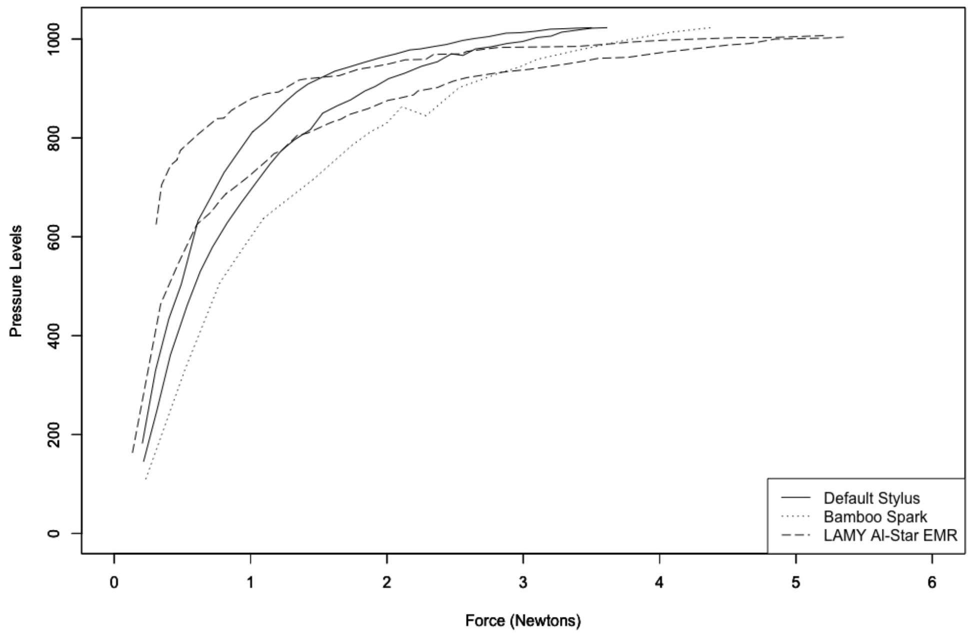

Figure 18: Comparison of the calibration curves of the Default stylus, the Wacom Bamboo Spark Inking Pen and the LAMY Al-Star black EMR captured on a Wacom STU540 with Wacom SDK.

86x62mm (300 x 300 DPI)