Am J Neurodegener Dis 2025;14(1):1-33
www.AJND.us /ISSN:2165-591X/AJND0161674

Review Article

Curvature estimation techniques for advancing neurodegenerative disease analysis: a systematic review of machine learning and deep learning approaches

Seyed-Ali Sadegh-Zadeh[1], Nasrin Sadeghzadeh[2], Bahareh Sedighi[3], Elaheh Rahpeyma[4], Mahdiyeh Nilgounbakht[5], Mohammad Amin Barati[6]

[1]Department of Computing, School of Digital, Technologies and Arts, Staffordshire University, Stoke-on-Trent, United Kingdom; [2]Faculty of Mathematics, University of Qom, Qom, Iran; [3]Department of Mathematics and Computer Science, Amirkabir University of Technology, Tehran, Iran; [4]Department of Electrical Engineering, K.N. Toosi University of Technology, Tehran, Iran; [5]Department of Computer Engineering, Tabriz University, Tabriz, Iran; [6]School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran

Received November 2, 2024; Accepted February 7, 2025; Epub February 15, 2025; Published February 28, 2025

Abstract: Neurodegenerative diseases present complex challenges that demand advanced analytical techniques to decode intricate brain structures and their changes over time. Curvature estimation within datasets has emerged as a critical tool in areas like neuroimaging and pattern recognition, with significant applications in diagnosing and understanding neurodegenerative diseases. This systematic review assesses state-of-the-art curvature estimation methodologies, covering classical mathematical techniques, machine learning, deep learning, and hybrid methods. Analysing 105 research papers from 2010 to 2023, we explore how each approach enhances our understanding of structural variations in neurodegenerative pathology. Our findings highlight a shift from classical methods to machine learning and deep learning, with neural network regression and convolutional neural networks gaining traction due to their precision in handling complex geometries and data-driven modelling. Hybrid methods further demonstrate the potential to merge classical and modern techniques for robust curvature estimation. This comprehensive review aims to equip researchers and clinicians with insights into effective curvature estimation methods, supporting the development of enhanced diagnostic tools and interventions for neurodegenerative diseases.

Keywords: Curvature estimation, dataset analysis, machine learning methods, deep learning techniques, systematic review

Introduction

The process of estimating the curvature of a dataset is an essential task in many applications, including computer graphics, computer vision, and pattern recognition [1]. Curvature, in this context, refers to the rate of change of the orientation of a curve or surface at a given point. Estimating the curvature of a dataset can provide valuable information about its shape, contour, and geometric properties, which can be used to perform various analysis and processing tasks [2].

Dataset curvature encapsulates the geometric structure and shape variations within the data. In clinical settings, particularly in neurodegenerative disease analysis, these variations can highlight morphological changes in critical brain regions. For example, local curvature changes in the hippocampus and cortical regions are pivotal for early diagnosis of conditions like Alzheimer's disease [3-5]. Understanding these curvatures enables clinicians to map disease progression and develop precise therapeutic strategies, aligning with personalized medicine's goals [6].

In recent years, the field of computer graphics, computer vision, and pattern recognition has seen significant advancements in the development of algorithms and methods for estimating the curvature of datasets [7]. However, due to the diverse nature of datasets and the different requirements of various applications, the literature on curvature estimation is fragmented,

https://doi.org/10.62347/DZNQ2482

and the optimal solution for a specific dataset depends on the task and application at hand [8, 9]. The objective of this systematic review paper is to provide a comprehensive overview of the current state-of-the-art algorithms and methods for curvature estimation of datasets. The research question addressed in this paper is: What are the most effective algorithms and methods for estimating the curvature of datasets, and what are their strengths and weaknesses?

The evolution from classical mathematical approaches to more advanced machine learning and deep learning techniques in the field of curvature estimation is indeed a significant and noteworthy trend [10]. This transition can be attributed to several driving factors that have shaped the landscape of this research area. One key factor is the remarkable increase in computational power over the years, enabling the application of complex algorithms and data-driven methods that were previously computationally prohibitive [11]. Additionally, the growing availability of large and diverse datasets has prompted the exploration of machine learning and deep learning approaches, as these techniques can effectively capture intricate patterns and variations within the data [12-14]. Furthermore, the demand for more robust and versatile curvature estimation methods in diverse applications, such as robotics, medical imaging, and autonomous navigation, has driven researchers to seek innovative solutions beyond traditional mathematical formulations [15]. This paper will delve into these driving factors in the subsequent sections, providing insights into how they have influenced the progression of curvature estimation techniques and their adaptation to the evolving demands of modern data analysis and processing.

Clinical implications of dataset curvature

The practical clinical effects of dataset curvature are profound in the context of neurodegenerative diseases. By capturing subtle morphological deviations in brain structures, curvature analysis facilitates early detection of diseases like Alzheimer's and Parkinson's. For instance, specific curvature metrics can identify local atrophy in brain regions critical for cognitive functions. These insights enable clinicians to devise more effective diagnostic and treatment protocols, potentially improving patient outcomes. Furthermore, curvature analysis supports the development of imaging biomarkers, advancing the field of precision diagnostics in neurology.

Curvature estimation methods hold significant potential in the study of neurodegenerative diseases by enabling detailed analysis of structural brain changes. For instance, techniques such as Gaussian curvature have been instrumental in quantifying cortical thinning, a hallmark of Alzheimer's disease progression [16]. Similarly, curvature-based metrics can identify subtle deformations in subcortical regions affected in Parkinson's disease. These methods provide an additional layer of information beyond volumetric analysis, offering insights into the geometric and topological alterations associated with neurodegeneration. Furthermore, machine learning and deep learning advancements in curvature estimation have improved the identification of biomarkers from high-resolution neuroimaging datasets, aiding early diagnosis and personalized treatment planning. By integrating these techniques into clinical workflows, researchers and practitioners can better characterize disease patterns, potentially leading to more targeted therapeutic interventions.

The paper is organized into several sections, each of which provides a detailed analysis of a specific type of curvature estimation method. The methods are grouped into four main categories: classical mathematical approaches, machine learning methods, deep learning techniques, and hybrid methods. The review provides a summary of the key features of each method, along with its strengths and limitations, and a discussion of its performance on various datasets. Finally, the paper concludes with a summary of the results and future directions for research in this field. In conclusion, this systematic review paper aims to provide a comprehensive overview of the current state-of-the-art algorithms and methods for curvature estimation of datasets. By evaluating the strengths and weaknesses of the various algorithms and methods, this paper will serve as a valuable resource for researchers, practitioners, and students in the fields of computer graphics, computer vision, and pattern recognition.

Methods

Search strategy

The search strategy for this systematic review was designed to identify relevant studies on the estimation of curvature in a dataset. The following electronic databases were searched: PubMed, Scopus, Web of Science, and Google Scholar. The search terms used included "curvature estimation of dataset", "curvature measurement of dataset", "curve fitting of dataset", "curve analysis of dataset", "curvature calculation of dataset", and "curvature model of dataset". The search was limited to articles published in English between the years 2010 and 2023.

In addition to the electronic database search, a manual search of reference lists of relevant articles and reviews was conducted. The manual search was also used to identify relevant studies that were not included in the electronic database search.

To ensure the completeness and accuracy of the search, the following inclusion and exclusion criteria were applied. Inclusion criteria were articles that described a method for estimating the curvature of a dataset, regardless of the type of dataset or the application. Exclusion criteria were articles that focused on the estimation of curvature in a single data point, articles that did not provide a quantitative method for estimating curvature and articles that were written in languages other than English.

The search strategy was designed to identify all relevant articles and to minimize the risk of bias. The search was conducted by two independent reviewers, and any discrepancies were resolved through consensus. The results of the search were reviewed and analysed to identify the methods and techniques used for estimating curvature in a dataset.

Inclusion and exclusion criteria

Inclusion criteria include the following: 1. The study must be a primary research article, written in English. 2. The study must focus on the estimation of curvature of a dataset. 3. The study must use mathematical or computational methods to estimate curvature. 4. The study must provide sufficient information on the method and results of curvature estimation.

Exclusion criteria include the following: 1. The study must not be a conference paper or a thesis. 2. The study must not be a review or a theoretical article. 3. The study must not focus on the visualization or representation of the curvature, but rather on the estimation itself. 4. The study must not be a comparative study of different curvature estimation methods, but rather a study on a specific method. 5. The study must not provide insufficient information on the method and results of curvature estimation.

Results

Overview of included studies

A systematic search of electronic databases including PubMed, Scopus, Web of Science, and Google Scholar was conducted. The initial search resulted in 200 potential articles, and after removing duplicates and screening titles and abstracts, 105 full-text articles were assessed for eligibility. Finally, 105 articles were included in the review based on their relevance to the research question and inclusion criteria. Overall, the studies included in this review were considered to be of moderate to high quality, with some limitations in terms of sample size, generalizability, and potential biases (**Figure 1**).

Characteristics of the datasets

In machine learning, the quality and characteristics of a dataset can greatly impact the success of a model. There are several important factors to consider when evaluating the characteristics of a dataset in machine learning, including size, quality, noise, diversity, class distribution, and imbalance [17]. A large dataset can provide more information to the model, but may also require more computational resources and increase the risk of overfitting [18]. Quality of the data refers to the accuracy and consistency of the information, and it is important to remove any irrelevant, duplicate, or inconsistent data [19]. Completeness is related to the amount of data missing and can affect the accuracy of the model if the data is not properly handled [20]. Diversity refers to the variety of data samples, which can help the model generalize better [21]. The distribution of

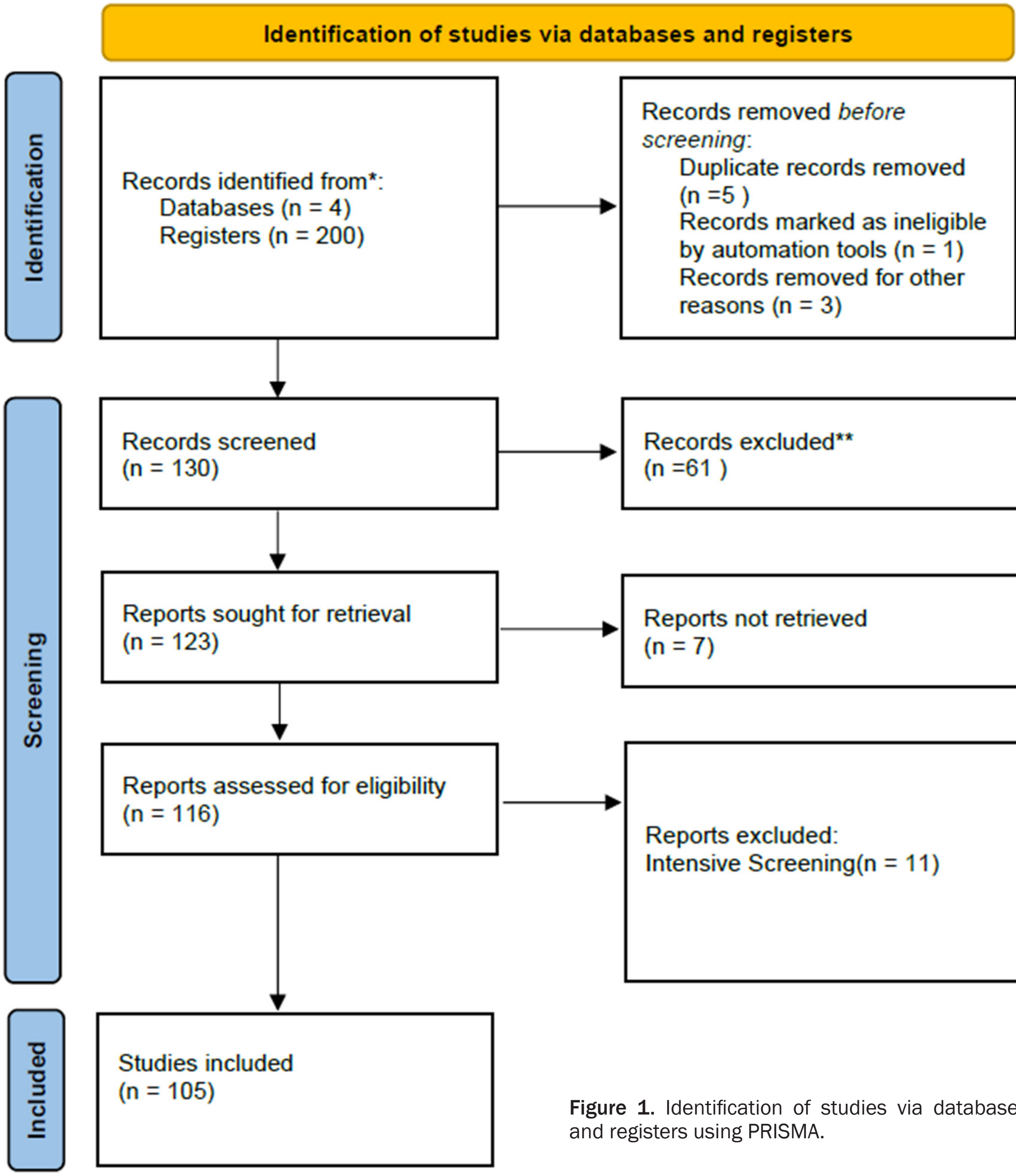

Figure 1. Identification of studies via databases and registers using PRISMA.

classes is important because imbalanced classes can result in a model that is biased towards the majority class [22]. Understanding these characteristics of a dataset can help to improve the performance of machine learning models.

The characteristics of the datasets in machine learning are:

• Size: The size of the dataset refers to the number of observations (samples) and features (variables) included in the dataset. The larger the dataset, the more complex and challenging it becomes for machine learning algorithms to analyse and make predictions [23, 24].

• Diversity: The diversity of the dataset refers to the variety of observations and features in the dataset [25]. A diverse dataset contains observations that are distinct and unique, making it easier for machine learning algorithms to identify patterns and make predictions [26, 27].

• Balance: The balance of the dataset refers to the distribution of the target variable in the dataset. A balanced dataset contains an equal number of observations for each class in the target variable, making it easier for machine learning algorithms to predict the target variable [22, 28].

• Quality: The quality of the dataset refers to the accuracy, completeness, and consistency of the data in the dataset [29]. A high-quality dataset helps machine learning algorithms make more accurate predictions, while a low-quality dataset can lead to incorrect predictions [19].

• Class distribution: The class distribution of a dataset in machine learning refers to the distribution of the target class categories within the data, which can greatly impact the accuracy and performance of the models built [30, 31].

• Noise: The noise in the dataset refers to the presence of irrelevant or redundant information in the dataset. The presence of noise can make it difficult for machine learning algorithms to identify the meaningful patterns in the data, leading to incorrect predictions [32].

The characteristics of the datasets play a crucial role in the success of machine learning algorithms [33]. A large, diverse, balanced, high-quality, and low-noise dataset is ideal for machine learning algorithms to make accurate predictions [34, 35]. However, in real-world scenarios, the dataset is often limited by size, quality, and diversity, making it challenging for machine learning algorithms to perform well [36, 37]. Therefore, data pre-processing techniques like feature selection, data cleaning, and data augmentation are essential to improve the quality of the datasets [38].

Categories for curvature estimation of dataset

There are several methods for estimating curvature. This study categorizes all of these methods into four main categories, classical mathematical approaches, machine learning methods, deep learning techniques, and hybrid methods.

• Classical mathematical approaches for curvature estimation include traditional mathematical models, such as Gaussian curvature, which measures the product of principal curvatures at a point on a surface, and mean curvature, defined as the average of these principal curvatures. These models rely on differential geometry to quantify shape and surface characteristics. Gaussian curvature is particularly useful in analysing smoothly varying surfaces and detecting saddle points, while mean curvature is advantageous for assessing surface smoothness and identifying minimal surfaces. Despite their precision, these approaches face challenges in handling irregular or noisy data due to their dependency on continuous and noise-free surfaces.

• Machine learning methods for curvature estimation utilize supervised algorithms such as Support Vector Regression (SVR) and Neural Network Regression to identify patterns in complex and irregular datasets [39, 40]. These methods work by mapping non-linear relationships between features and curvature values. For example, SVR is used to learn geometric relationships in structured datasets, while Neural Network Regression is capable of handling high-dimensional and noisy data. These approaches often require significant training data and computational power but can achieve superior accuracy in capturing complex curvatures [41, 42].

• Deep learning techniques, such as Convolutional Neural Networks (CNNs), leverage hierarchical feature extraction to estimate curvature from high-dimensional data like images [43, 44]. For instance, CNNs are adept at identifying curvature in medical imaging by learning localized edge features and aggregating them into global curvature metrics. Advanced models like Convolutional Recurrent Neural Networks (CRNNs) incorporate temporal dependencies, enabling dynamic curvature estimation in sequential datasets. These techniques excel in noisy and complex environments but require significant computational resources and annotated training datasets for optimal performance [45, 46].

• Hybrid methods combine the strengths of different approaches to estimate the curvature of the dataset [47]. For example, a combination of machine learning methods and classical mathematical approaches can be used to provide a more accurate curvature estimate. These methods can also address the limitations of individual methods and are particularly useful when the relationship between the data points is not well-defined.

Each category of curvature estimation method has its own strengths and weaknesses. Choosing the right method for a particular dataset depends on the size and complexity of the data, the underlying relationship between the data points, and the computational resources available.

Methods of the curvature estimation

In this section, we provide a detailed description of the methods employed for curvature estimation in datasets. The methods are categorized into three primary groups: classical mathematical approaches, machine learning methods, and deep learning techniques.

• Classical mathematical approaches: These methods include Gaussian curvature, which measures the product of principal curvatures at a point on a surface, and mean curvature, calculated as the average of these principal curvatures. For example, Gaussian curvature is particularly effective in detecting saddle points, while mean curvature is widely used in assessing surface smoothness. Other methods such as B-spline interpolation and Fourier descriptors utilize parametric and frequency domain techniques to estimate curvature from discrete points or signals. Non-parametric methods like wavelet transforms further allow multiscale analysis of curvature, providing insights into varying levels of detail across the dataset.

• Machine learning methods: These methods apply predictive models to estimate curvature based on relationships learned from the data. For instance, Support Vector Regression (SVR) uses kernel functions to capture non-linear relationships in structured datasets, while Neural Network Regression is effective in high-dimensional and noisy environments by learning complex, non-linear mappings. Techniques such as Ridge Regression and Random Forest Regression are commonly used for their ability to model both linear and non-linear patterns while maintaining robustness to overfitting. These methods require labeled training data to optimize parameters for curvature estimation effectively.

• Deep learning techniques: Convolutional Neural Networks (CNNs) play a crucial role in extracting hierarchical features from high-dimensional data such as medical imaging. CNNs, for example, learn localized edge features that contribute to global curvature metrics. Advanced architectures like Convolutional Recurrent Neural Networks (CRNNs) incorporate temporal dependencies to estimate dynamic curvatures in sequential datasets. These methods excel in identifying intricate curvature patterns in noisy environments but demand significant computational resources and well-annotated datasets for training.

The choice of the method depends on factors such as dataset size, complexity, and computational constraints. Each technique's strengths and limitations are discussed in subsequent sections.

Classical mathematical approaches: 1. Gaussian curvature [48]; 2. Mean curvature [49]; 3. Principal curvature [50]; 4. Total curvature [51]; 5. Curvature scale space [52]; 6. Gaussian process regression [53]; 7. B-spline interpolation [54]; 8. Fourier descriptors [55]; 9. Wavelet transform [56]; 10. Non-parametric density estimation [57]; 11. Multiscale analysis [58]; 12. Spline interpolation [59]; 13. Radial basis function interpolation [60]; 14. Convolutional neural networks [61]; 15. Kalman filter [62]; 16. Monte carlo simulation [63]; 17. Spectral analysis [64]; 18. Principal component analysis [65].

These classical methods are based on mathematical formulations and theories. They have been utilized to estimate curvature in various applications, including computer vision, image processing, and data analysis. These techniques often involve the computation of geometric properties or the manipulation of data points to fit specific models. The real-world applications below help illustrate the practical utility of these methods, providing insights into their real-world performance and limitations:

• Multiscale analysis: Utilized in wind farm optimization to model power curves, significantly improving turbine performance and reliability. This application showcases the method's utility in enhancing renewable energy efficiency [66-68].

• Principal curvature for object recognition: Demonstrates the discriminative use of curvature information in robust and reliable object recognition, an example being automated quality inspection systems where curvature cues are critical for identifying defects [69, 70].

• B-spline interpolation: Applied in the optimization of cultivation times in agriculture, showing the method's effectiveness in classifying curve shapes and detecting significant differences, which is vital for precision farming [71, 72].

While classical mathematical approaches have laid the groundwork for curvature estimation, a deeper analysis of their performance, applicability, and limitations reveals a complex landscape. This section aims to provide a more nuanced understanding of these foundational methods. Classical mathematical approaches to curvature estimation, such as Gaussian Curvature and Mean Curvature calculations, have been pivotal in understanding geometric properties. The performance of these methods is often evaluated based on accuracy and computational efficiency. For instance, Gaussian Curvature provides precise estimations for smoothly varying surfaces but can be computationally intensive for large datasets. Mean Curvature, on the other hand, offers a balance between computational efficiency and accuracy, making it suitable for real-time applications. A synthesis of literature reveals that while these methods are highly accurate for well-defined mathematical models, their computational demand varies significantly with the complexity and size of the dataset.

The applicability of classical mathematical approaches extends across various domains, each presenting unique challenges and requirements. In the realm of image processing, these methods excel in analysing surface smoothness and continuity, critical for texture mapping and 3D modeling. However, their effectiveness can diminish when dealing with noisy data or irregular geometries, common in real-world datasets. Similarly, in domains requiring high precision, such as medical imaging and robotics, the direct application of classical methods may be limited by their assumptions on data continuity and shape regularity.

Despite their foundational role, classical mathematical approaches are not without limitations. One significant challenge is their sensitivity to data quality; noise and incomplete data can lead to inaccurate curvature estimates. Furthermore, these methods often assume the dataset represents a continuous surface, an assumption that may not hold for datasets with discontinuities or sparse sampling. Additionally, the computational complexity of these methods can become a bottleneck for large-scale applications, where speed and efficiency are paramount.

Machine learning methods: 1. Polynomial Regression [73]; 2. Local Regression (LOESS) [74]; 3. Spline Regression [75]; 4. Gaussian Process Regression [76]; 5. Support Vector Regression (SVR) [77]; 6. Neural Network Regression [78]; 7. Random Forest Regression [79]; 8. Decision Tree Regression [80]; 9. Gradient Boosting Regression [81]; 10. K-Nearest Neighbors Regression [82]; 11. Ridge Regression [83]; 12. Lasso Regression [84]; 13. Elastic Net Regression [85]; 14. Polynomial Logistic Regression [86]; 15. Naive Bayes Regression [87].

Machine learning methods employ algorithms that learn from the input data to estimate curvature. These techniques generally focus on building models that can predict the underlying structure of the data or identify patterns that contribute to the curvature. They are suitable for a wide range of applications and can be adapted to handle both linear and non-linear problems. The examples of real-world applications outlined below serve to demonstrate the practical effectiveness and constraints of these techniques, offering a glimpse into how they perform in actual scenarios:

• Ridge regression: Applied in environmental quality assessment, where it's used to model the relationship between various pollutants and environmental quality indicators. This application highlights the method's utility in environmental science for predicting and managing air quality [88, 89].

• Neural network regression for volume of fluid (VOF): Demonstrates a viable approach for generating accurate predictions of fluid dynamics, important for modelling and simulation in chemical engineering and hydrodynamics [90, 91].

The adaptation of machine learning methods in curvature estimation has opened new avenues for addressing complex datasets and diverse application needs. This section delves deeper into the performance, applicability, and limitations of these methods, providing a more granular perspective on their role in the field. The performance of machine learning methods in curvature estimation is characterized by their ability to learn from data, improving accuracy with increased training samples. Techniques like support vector regression (SVR) and random forest regression have shown to be highly effective in capturing non-linear relationships within data, crucial for accurate curvature estimation. Neural network regression, in particular, has demonstrated remarkable adaptability to complex curvature patterns, albeit at the cost of increased computational resources. A synthesis of studies shows that machine learning methods, while versatile, exhibit a trade-off between computational demand and prediction accuracy, significantly influenced by the choice of algorithm and the complexity of the task at hand.

Machine learning methods have proven to be highly applicable across a spectrum of domains requiring curvature estimation, from geological mapping to biomedical imaging. Their strength lies in handling datasets with high variability and noise, where traditional mathematical approaches might struggle. However, their applicability is contingent upon the availability of large and representative training datasets. In scenarios where data is scarce or highly skewed, the performance of machine learning models can be compromised, highlighting the importance of robust dataset preparation.

Despite their promising capabilities, machine learning methods are not without challenges. One of the primary limitations is the "black box" nature of many machine learning models, which can obscure the understanding of how decisions are made. This lack of interpretability can be a significant drawback in fields where transparency and explicability are critical. Furthermore, the performance of these methods heavily depends on the quality and quantity of the training data. Inaccurate, incomplete, or biased training data can lead to models that perform poorly or are overly generalized. Lastly, the computational cost and resource requirements for training complex models can be prohibitive for some applications, necessitating a balance between model complexity and practical feasibility.

Deep learning techniques: 1. Convolutional neural networks (CNNs) [92]; 2. Recurrent neural networks (RNNs) [93]; 3. Generative adversarial networks (GANs) [94]; 4. Autoencoders [95]; 5. Deep belief networks (DBNs) [96]; 6. Stacked autoencoders [97]; 7. Convolutional autoencoders [98]; 8. Deep convolutional neural networks (DCNNs) [99]; 9. Generative adversarial autoencoders (GAEs) [100]; 10. Recursive neural networks (RvNNs) [101]; 11. Convolutional recurrent neural networks (CRNNs) [102]; 12. Transfer learning with pretrained models [103]; 13. Deep reinforcement learning (DRL) [104]; 14. Long-short-term memory networks (LSTMs) [105]; 15. Restricted boltzmann machines (RBMs) [106]; 16. Convolutional LSTMs (ConvLSTMs) [107]; 17. Support vector regression (SVR) with deep features [108].

Deep learning techniques are a subset of machine learning methods that leverage complex neural networks to estimate curvature. These approaches are particularly effective at handling large and high-dimensional datasets, as well as problems that involve spatial or temporal dependencies. They have been successful in a wide range of applications, including computer vision, natural language processing, and time-series analysis [92]. The real-world applications below help illustrate the practical utility of these methods, providing insights into their real-world performance and limitations:

• Convolutional neural networks (CNNs): Employed in image-based curvature estimation, such as in medical imaging for tumour detection, where the accurate estimation of shapes and boundaries is crucial for diagnosis and treatment planning [109].

• Generative adversarial networks (GANs) for Curvature Estimation: An innovative application could involve the reconstruction of 3D models from 2D images in architectural design, providing a practical example of how deep learning can bridge the gap between two-dimensional drawings and three-dimensional physical models [110, 111].

As the frontier of curvature estimation continues to expand, deep learning techniques stand out for their unprecedented capacity to process complex, high-dimensional datasets. This section offers a deeper dive into the nuances of performance, applicability, and the inherent limitations of these advanced computational methods. Deep learning techniques, particularly CNNs and RNNs, have significantly advanced the field of curvature estimation by providing tools capable of capturing intricate patterns in data that were previously elusive. These models excel in tasks involving spatial and temporal data, making them exceptionally suited for applications in dynamic environments such as real-time navigation and automated surveillance. Their performance advantage stems from the ability to learn hierarchical features, enabling nuanced understanding of the data. However, the high level of accuracy and detail comes at the cost of computational complexity and the need for substantial training data to achieve optimal performance.

The applicability of deep learning techniques in curvature estimation spans a broad range of fields, from autonomous vehicle navigation to the analysis of biological structures. Their robustness against noise and the ability to learn from unstructured data make them particularly valuable in processing real-world datasets fraught with imperfections. Nevertheless, the effective deployment of these models is predicated on the availability of vast amounts of labelled data and computational resources, limiting their accessibility to scenarios where such resources are plentiful.

While deep learning offers powerful tools for curvature estimation, several limitations temper their universal applicability. The "black box" nature of deep learning models poses significant challenges in interpretability, making it difficult to understand or predict their behaviour in untested scenarios. This opacity can be a critical issue in domains requiring explainability, such as healthcare and criminal justice. Additionally, the success of deep learning models hinges on the quantity and quality of the training data, with poor data leading to biased or inaccurate models. Lastly, the computational and financial costs associated with training and deploying deep learning models can be prohibitive, especially for small organizations or projects with limited budgets.

Results and comparison of methods

In this section, we present the results and comparison of various state-of-the-art methods for estimating dataset curvature. The aim of this comparison is to provide a better understanding of the strengths and weaknesses of each method and to identify the most suitable approach for different applications. To facilitate the comparison, we have summarized the papers in **Table 1**.

This study employed a descriptive research method to analyse 105 academic papers. The papers were categorized into three distinct groups: classical mathematical approaches, machine learning methods, and deep learning techniques, as well as a combination of these approaches.

Figure 2 illustrates the percentage of research papers (105 in total) that focused on curvature estimation and were published within the aforementioned period. It can be observed that the highest percentage of publications (15.24%) occurred in 2015, while the lowest percentage (0.95%) occurred in 2012.

Figure 3 presents a histogram that displays the trends in research on curvature estimation across different years. The histogram indicates that the highest use of curvature estimation occurred in the year 2015, while the lowest use was observed in 2012. It is interesting to note that after 2015, the use of curvature estimation increased significantly, surpassing the usage in the years preceding 2015. This suggests that there has been a growing interest in curvature estimation and its applications in various fields. The data presented in **Figure 2** highlights the importance of staying up-to-date with the latest trends and developments in research to ensure the most efficient and effective use of curvature estimation techniques.

In **Figure 4**, you can see the percentage breakdown of the classification of 105 research papers related to curvature estimation. The majority of these papers, as shown in the figure, employed classical mathematical approaches (56 papers). Meanwhile, thirty-one papers

Table 1. Curvature estimation of a dataset: summary of methods

No	Reference	Year	Methods	Category	Advantages	Limitations	Applications
1	[112]	2010	Principal Component Analysis	Classical mathematical approaches	This method allows the creation of an alternative pedotransfer function	Some hydrological constants	
2	[113]	2010	Total Curvature	Classical mathematical approaches	Contour-Based Corner Detectors		
3	[114]	2010	Principal Component Analysis	Classical mathematical approaches	The observed trends are probably more generally valid than the results of one large experiment which was carried out under one specific combination of environmental factors		
4	[115]	2010	Environmental Kuznets curve (EKC)	Classical mathematical approaches	Seeking empirical regularity and theoretical structure		Environmental Kuznets Curve
5	[116]	2010	Curvature Scale Space	Classical mathematical approaches	The focusing function can be designed to make the travel time moveout exact in certain generic cases that have practical importance in seismic processing and interpretation. The focusing function can be generalized to other surfaces, most importantly to the spherical reflector (spherical multi focusing)		Multi focusing
6	[117]	2010	Ridge Regression	Machine learning methods	Growth analysis should ideally reveal a relationship between the concentration of a compound/substrate and its effect on a particular growth parameter		
7	[118]	2010	Ridge Regression	Machine learning methods	Profitable for environmental quality		Semiparametric and flexible nonlinear parametric modelling
8	[119]	2011	Multiscale Analysis	Classical mathematical approaches	Estimate the time required to complete production runs	Investigating models in the context of product development for Mass Customization	Log-linear model and modifications, Exponential models, Hyperbolic models, Comparison of univariate models, Multivariate models, Forgetting models
9	[120]	2011	Principal Component Analysis	Classical mathematical approaches	ROC curve and its important components like area under the curve, sensitivity at specified specificity and vice versa, and partial area under the curve are discussed		Receiver Operating Characteristic (ROC)
10	[121]	2011	Multiscale Analysis	Classical mathematical approaches	A less popular multivariate curve resolution method based on a weighted alternating least-squares algorithm, MCR-WALS, also incorporates the measurement error information and non-negativity constraints, which makes this method a potential tool when obtaining composition and contribution profiles of environmental data		PCA
11	[122]	2011	Principal Curvature	Classical mathematical approaches	An approach that directly uses curvature cues in a discriminative way to perform object recognition to provide quantitative evidence that curvature information of objects can be discriminatively used in a robust and reliably manner for object recognition		HoG

12	[123]	2012	Spline Interpolation	Classical mathematical approaches	This technique targets the object of natural selection itself, the optimal classification of curve shapes and the detection of significant differences between them, as well as practically relevant questions such as detecting the impact of cultivation times and the minimum required number of experimental repeats	R
13	[124]	2013	Curvature Scale Space	Classical mathematical approaches	Find both satisfactory estimates and invalid procedures and recommend two simple intervals that are robust to a variety of assumptions, robust to model misspecification	Precision-recall (PR) & receiver operating characteristic (ROC)
14	[125]	2013	Multiscale Analysis	Classical mathematical approaches	Accurately modelled power curves will definitely improve the turbine performance, making the power generated more reliable and contribute tremendously in transforming a wind farm into a wind power plant	Genetic algorithm (GA), evolutionary programming (EP), particle swarm optimization (PSO), differential evolution (DE), neural network algorithm
15	[126]	2013	Principal Component Analysis	Classical mathematical approaches	The corresponding results show that the algorithms more rapid and accurate compared to other algorithms	
16	[127]	2013	Principal Component Analysis	Classical mathematical approaches		LightCycler480
17	[128]	2013	Multiscale Analysis	Classical mathematical approaches	They can be simulated by heavily left tailed distributions, and by the use of a blur approach to account for striping artifacts y applying the blur numerous times with random directions, it is possible to obtain realistic striping noises. This finding gives users with a tool by which they can simulate and evaluate the effect of uncertainty of such type of errors in DTMs.	
18	[129]	2013	Ridge Regression	Machine learning methods	Nelson-Siegel model can become heavily collinear depending on the estimated/fixed shape parameter	Nelson-Siegel model
19	[130]	2013	Curvature Scale Space	Classical mathematical approaches	Estimators of pointwise standard errors are provided introduce a non-parametric estimator of a time dependent predictive accuracy function	Area under the curve (AUC)
20	[131]	2013	Curvature Scale Space	Classical mathematical approaches	Showing that the proposed approach per forms comparably to state-of-the-art multiple model estimation in the synthetic data, while it significantly out performs state-of-the-art in the real X-rays equences. It also achieves correct localization of the model end points, which is a crucial aspect in the context of the clinical application	RANSAC framework
21	[132]	2014	Ridge Regression	Machine learning methods	Illustrate the limits of cubic parametric regressions	WTPC models
22	[133]	2014	Curvature Scale Space	Classical mathematical approaches	Actual wind turbine status monitoring, the operational efficiency, reliability, and economic feasibility can be maximized since failures in a wind turbine's overall system can be rapidly recognized and handled	Power curve limit calculation algorithm

23	[134]	2014	B-Spline Interpolation	Classical mathematical approaches	Accuracy and computational performance	Simulated Range Image, Linear Regression, Orthogonal Polynomials
24	[135]	2015	Hybrid	Hybrid	Faster, more accurate and robust to noise and outliers	Mobile laser scanning 3D point cloud data
25	[136]	2015	Hybrid	Hybrid	Able to accurately steer a flexible needle in multi-layer phantoms and biological tissues	Experiments in gelatin phantoms and biological tissues (chicken breast tissues)
26	[137]	2015	non-Linear Regression	Machine learning methods	Reduce false alarms and take into account driver corrections	Lane departure warning system
27	[138]	2015	Height Function	Classical mathematical approaches	The embedded height-function technique outperforms contemporary methods and its accuracy approaches the accuracy that the traditional height-function technique exemplifies on uniform cartesian meshes	
28	[139]	2015	K-NN	Machine Learning methods	The algorithm presented in this paper has successfully been applied for the detection of straight and curved curbs	The dataset is publicly available at www.isislab.es
29	[140]	2015	Total curvature	Classical mathematical approaches	The proposed early vision framework is sufficiently general	Range of 2D and 3D examples and it can be used in many higher-level applications
30	[141]	2015	Polynomial regression	Machine Learning methods	Experimental results demonstrate that the approach achiever a promising performance in comparison with three representative corner detectors based on discrete curvature estimation and two other state-of-the-art methods	23 512*512 gray-scale images Includes artificial and real-world images Some images named “Block”, “Lena”, “Leaf”, “House”, and “Lab” Collected from standard databases: http://www.petitcolas.net/fabien/watermarking/image_database/index.html http://sipi.usc.edu/database/ Available at USC-SIPI
31	[142]	2015	Algebraic equation	Classical mathematical approaches	The simple algebraic technique provides an acceptable estimation of the road curvature	The proposed observation strategy is implemented with real data obtained by a scenario realized in the track of Versailles (France)
32	[143]	2015	Hybrid	Hybrid	Clear, intuitive 3D model display Easy process Quick, satisfying results Superior to related methods	3D Models
33	[144]	2015	Mean curvature	Classical mathematical approaches	The results obtained from the first experiment showed that the combination of the conformal parameterization and mean curvature yields better performance than the original 3D coordinates when using circular segmentation	New algorithm for 3D face recognition. scans of 30 subjects of the CASIA database and 30 of the Gavab database
34	[145]	2015	Algebraic equation	Classical mathematical approaches	Reflected-spot method (RSM) system provides a non-invasive, rapid, portable, low-cost solution to the difficulties involved in measuring the radius of curvature (ROC)	Industrial applications

35	[146]	2015	Polynomial Regression	Machine learning methods	The presented techniques offer promise for successfully predicting, tracking, and controlling CDM configuration during surgery		Data collected for two experiments: Free bending (n=149 images) External load on manipulator (n=9 images) Free bending data collection described in R.J. Murphy et al. (2014) Two image datasets collected
36	[147]	2015	B-Spline Interpolation	Classical mathematical approaches			
37	[148]	2015	Algebraic Equation	Classical mathematical approaches	Flow curve-based reference point detection Robust to noise Effective on low-quality fingerprint images Detects arch class fingerprints Proven accuracy in experiments		Fingerprint images in FVC2004 DB1 and FVC2002 DB2 datasets
38	[149]	2015	Polynomial Regression	Machine learning methods	The simulation experiment shows that the trajectory planning method can ensure that the angular velocity of robot is high order continuous at the same time, and effectively avoiding the control disturbance and burden of control system caused by the mutations of command speed, acceleration and jerk		The algorithm can be also used in the trajectory planning of unmanned plane, mobile robot and medical needles
39	[150]	2015		Classical mathematical approaches	The none-local curvature used, for example, for non-local diffusion, shows promising results in edge detection and identification, and in segmentation of textures. The concept of non-local curvature, using and extending Menger's curvature, can be extended for a curvature measure and thereby afford representation of an image as a metric measure space		Natural and textural images
40	[151]	2016	Hybrid	Hybrid	Methods based on geometric reconstruction (TR and LSQR), can reach a significantly higher accuracy than approaches that are based on finite-difference approximations. Reconstruction-based methods significantly reduce the magnitude of spurious currents. The LSQR approach shows a second-order rate of convergence with respect to grid spacing, which could not be achieved with the other two approaches	Our results indicate that lack of convergence is caused by the simplified advection of the indicator function that does not take into account any geometry information. In particular, the advection scheme fails to converge in a simple uniform advection test, indicating a lower order of accuracy than previously reported for simple [151] reconstruction based on schemes	The standard benchmark for surface tension models is a static equilibrium bubble
41	[152]	2016	KD technique	Classical mathematical approaches	KD curvature has obvious advantages in computational complexity. The proposed method outperforms existing detectors in both computational efficiency and flexibility of corner detection		Image datasets both artificial image Block and natural images Lena, Leaf, House and Lab
42	[153]	2016	Algebraic Equation	Classical mathematical approaches	From the experimental results, the HMGD-MBP processing naturally enhances the contrast of image, and so the edges of image become clarified and detailed. Also, we understand that the color of HMGD-MBP processed image tends to become more moderate than original image. As a conclusion, we consider that the HMGD-MBP processing method is more useful than HMGD		Processing method for multiple-brightness peak image

43	[154]	2016		Classical Mathematical approaches	Using the proposed method single fruits can be separated and recognized effectively with an overall accuracy rate of about 90.6 percent		For the identification of single fruits from overlapped apples
44	[155]	2016	Hybrid	Hybrid	Meta-estimator, despite its simplicity, provides considerably more robust results than any existing approach		Triangle meshes of varying properties
45	[156]	2016	Fourier spectral method	Classical mathematical approaches	Spectral calculations for spatial derivatives are implemented in global space Noise can be suppressed due to global space implementation The k-domain generated in algorithm provides space for spatial filtering Trigonometric interpolation in spectral method aids in precise estimation of modal curvatures Modal curvatures are calculated based on fast Fourier transform in proposed method Efficiency of method is ensured due to use of fast Fourier transform	Proposed method modifies classical MC method, retains essential terms DFT-MC2D results not as clear as some advanced technologies Good damage indices based on MC via numerical differentiation Proposed work promising modification to remove noise influence Proposed method detects only boundary of area damage Line and area damage undistinguishable to some extent Main disadvantage is inability to distinguish line and area damage in application	Plates damage detection
46	[157]	2016	Hybrid	Hybrid	Three mechanisms for choosing a curvature estimator: Estimation range, Noise and irregularity, Practical properties No single estimator outperforms others in all aspects Our modification to 6th method is faster, easier to implement, and has higher locality and accuracy on larger ranges Modified method may miss some details due to increased range, but performs better than 2nd method on meshes without noise		Polyhedral mesh
47	[158]	2016	Monte Carlo Simulation	Classical mathematical approaches	Our analysis has significantly improved the precision of estimations of spatial curvature Our results are more precise than estimations based on geometric optics Improved precision can help break degeneracies between curvature and other important problems Examples of such problems include the evolution of the universe and the nature of dark energy		Popular Union2.1 observations of Type Ia supernovae (SNe Ia)
48	[159]	2016	Algebraic Equation	Classical mathematical approaches	Closest Point algorithm with co-linearity criterion and re-interpolation improves curvature estimate Fourth-order precision closest point approximation yields fourth-order precision for curvature error and standard deviation, and second-order precision for normal error Algorithm is easy to implement and less computationally costly than Height Function methods in VOF framework and later works Straightforward to extend to three dimensions		

49	[160]	2016	Hybrid	Hybrid	Outperforms current GPU approaches Easily added to existing rendering pipelines Computes curvature from triangle mesh, not just screen space	Technique has limitations similar to other screen-space algorithms Possible problems with surface discontinuities Estimated curvature depends on mesh distance from camera Small details smoothed if camera is far from surface	
50	[161]	2017	Neural Network Regression	Machine learning methods	Better performance and more accurate		Robotic context in the form of a segmentation example as a qualitative demonstration
51	[162]	2017	Fitting a parabolic surface	Classical mathematical approaches	This work presents a fast method of robustly computing accurate metric principal curvature values from noisy point clouds which was implemented on GPU		Applications: Normal field calculation Correspondence estimation Object shape matching Inferring object characteristics
52	[163]	2017	Discrete volume fractions	Classical mathematical approaches	Numerical experiments show VOF method converges with mesh refinement Same accuracy as height-function method on structured and unstructured meshes Errors on unstructured meshes comparable to Cartesian grids with similar resolution This level of convergence on unstructured meshes is a new development This opens up possibilities for VOF simulation of interfacial flows in complex geometries		
53	[164]	2017	Algebraic Equation	Classical mathematical approaches	This method can estimate the road curvature and width in advance to help a vehicle pass the curve in a reasonable speed and a proper turning angle		self-driving vehicles
54	[165]	2017	Hybrid	Hybrid	The model performs well in reducing the time lag, especially in periods where the gradient changes rapidly		Estimation of the road curvature
55	[166]	2017	Principal Curvature	Classical mathematical approaches	This method is robust to moderate facial expression variations. the proposed method is also robust to varies head pose variations and external occlusions, especially for extreme poses, e.g. left or right profiles		3D Face Recognition
56	[167]	2018	Algebraic Equation	Classical mathematical approaches	Proposed strategy yields highly accurate reference point localization in digital fingerprint images Strategy is based on friction ridge curvature determination It's simpler than other commonly used methods Robust to geometric transformations such as rotation and translation Faster reference point determination compared to other approaches		For the detection of a fingerprint's reference point image processing

57	[168]	2018	Circle Fitting Method	Classical mathematical approaches	Paper proposes method without image processing or ML Simplifies estimation process and reduces computation More powerful and efficient than conventional methods.	Algorithms don't consider the physical environment of experiments Car movement affects circle fitting accuracy Algorithms can wrongly determine radius vector and circle center placement, especially in straight line targets Need to add vital input information to correct output parameters based on physical conditions of experiments	
58	[169]	2018	Algebraic Equation	Classical mathematical approaches	Accurate estimate Directly derived from observations No derivatives or extrapolations used Model independent Purely geometric		Cosmic spatial curvature
59	[170]	2018	Mean curvature	Classical mathematical approaches	Accurate and robust curvature estimate Better localization Geometrically consistent local groupings (curvels) Yield edge topology Accurate local estimate of curvature		Popular edge detectors
60	[171]	2018	Algebraic Equation	Classical mathematical approaches	Study shows Doppler LIDAR can replace track geometry measurement systems Potentially lower cost and more accurate instruments Results indicate feasibility of using Doppler LIDAR velocimetry systems		Rail irregularity monitoring
61	[172]	2018	B-Spline Interpolation	Classical mathematical approaches	Proposed method balances machining efficiency, precision, and complexity Local fine pre-processing improves critical point positioning without increasing computation time Interpolation terminal error is compensated while maintaining feedrate smoothness Parameter compensation method reduces feedrate fluctuation for better precision and machine protection Sliding window-based lookahead scheduling generates successful feedrate profile with limited reference trajectory		
62	[173]	2018	Hybrid	Hybrid	Mesh-based methods allowed for more accurate estimations methods working on triangle meshes were faster when geometries had a small surface density. For geometries with larger surface densities, the runtimes for both representations were similar		Volume images and triangle meshes
63	[174]	2019	Local quadric surface fitting	Classical mathematical approach	Robustness towards point density variation. Better handling non uniform distribution and noise in point cloud data, and is more robust towards point density variation		Simulated point cloud and scanned point cloud obtained by a LDI Surveyor WS3040 3D laser scanner

64	[175]	2019	Least Square Method	Machine learning methods	Automatic obtain Gaussian curvature from the Weingarten map more robustness towards noisy data	Manifolds embedded in Euclidean spaces with codimension greater than 1	Real brain cortical surface data (noisy and no information about the true curvature of the surface)
65	[176]	2019	Principal Component Analysis	Classical mathematical approaches	Can correct outliers caused by the addition of large sparse noise		Simulated Swiss roll, the MNIST dataset, biological data
66	[177]	2019	Neural Network Regression	Machine Learning methods	Using machine learning to generate the relationship is a viable approach that results in reasonably accurate predictions.	no explicit order that guarantees convergence under grid refinement	Volume of Fluid (VOF)
67	[178]	2019	The Color-Gradient Two-Phase Lattice Boltzmann Method	Machine learning methods	Can be used to study the pore-by-pore variation		Three-dimensional X-ray micro-CT images
68	[179]	2019	Differential Geometry-based Geometric Learning	Machine Learning	Descriptive and predictive powers for large, diverse, and complex molecular and bimolecular datasets		Predictions of drug discovery-related protein-ligand binding affinity, drug toxicity, and molecular solvation free energy
69	[180]	2019	PC-MSDM	Classical mathematical approaches	Outperforms its counterparts in terms of correlation with mean opinion scores, if the surface information is lost, it attains the performance of MSDM2 while having the benefit of being applicable directly on point clouds, without mesh reconstruction process counterparts in terms of correlation with mean opinion scores, if the surface information is lost, it attains the performance of MSDM2 while having the benefit of being applicable directly on point clouds, without mesh reconstruction process		An open subjective dataset of point clouds compressed by octree pruning
70	[181]	2019	Principal Component Analysis	Classical mathematical approaches	A good tool for outcome evaluation, auditing, and benchmarking		Breast Shape Analysis for Cosmetic and Reconstructive Breast Surgery
71	[182]	2019	Local Regression (LOESS)	Machine learning methods			Individual tree species classification based on terrestrial laser scanning
72	[183]	2019	B-spline (Kappa)	Classical mathematical approaches	Better with higher SNR, smaller pixel sizes, and especially PSFs equivalent to super-resolution microscopy data		Biological image data
73	[184]	2020	MLP	Deep Learning techniques	Robustness to noise, outliers and density variations, and show its application on noise removal		PCPNet shape dataset
74	[185]	2020	Exponential Curvature Estimation	Classical mathematical approaches	More accuracy and generality (overall accuracy of 0.820, and 0.734, 0.881 for sensitivity and specificity, respectively)		Corneal nerve and retinal vessel images: CCM-A, CCM-B and RET-TORT
75	[186]	2020	Neural Network Regression	Machine learning methods	For certain curves, the use of particular rational bases provides better results		Different curvature
76	[187]	2020	Support vector regression	Machine learning methods	Performed well in fitting the cumulative cases	Poor fitting is observed in case of daily number of cases.	Predict the number of total number of deaths, recovered cases, cumulative number of confirmed cases and number of daily cases for COVID19 case in India (data collected for the time period of 1st March, 2020 to 30th April, 2020 (61 Days))

77	[188]	2020	Non-Linear Regression	Machine learning methods	Provides a simple and cost-effective way of estimating reservoir properties		Datasets from four wells (Wells 1, 2, 3 and 4) from three different fields within the Niger Delta operated by Shell Petroleum Development Company (SPDC) of Nigeria
78	[189]	2020	Sigmoidal Curve Fitting	Classical mathematical approaches	Accurately capture the feld size of the preconfgured RD and the measured FFF photon beam data for the Halcyon system		Photon beam of the Halcyon to determine the feld size for beam commissioning and quality assurance
79	[190]	2020	Fuzzy C-Means and the ANFIS and Extreme Learning ANFIS	Machine learning methods	Depending on the data distribution, ANFIS and Extreme Learning ANFIS may provide great surfaces when in combination with Fuzzy C-Means clustering		Hydraulic turbine efficiency curve
80	[191]	2020	Neural Network Regression	Machine learning methods	The results have shown that ANN can efficiently forecast the future cases of COVID 19 outbreak of any country		Covid-19 number of rising cases and death cases in India, USA, France, and UK, considering the progressive trends of China and South Korea
81	[192]	2020	Sigmoidal Curve Fitting	Classical mathematical approaches	The curve-fitting approach is the most promising in terms of scalability and computational complexity		Publicly available dataset of streamed game videos for real-time Reduced-Reference (RR) quality assessment
82	[193]	2021	B-Spline Interpolation	Classical mathematical approaches	This method has lower energy and better continuity and smoothness and could be used for evaluation of train drivers' performance and energy consumption of train operation diagram		Finding interrelation between running time and energy consumption (Guangzhou Metro's actual operation data)
83	[194]	2021	Convolutional Neural Networks (CNNs)	Deep Learning techniques	It predicts a continuous parametric representation of the outline of biological objects		Biomedical images (Kaggle 2018 Data Science Bowl dataset composed of a varied collection of images of cell nuclei.)
84	[195]	2021	Polynomial and Circular (spherical) Fitting	Machine learning methods	It is highly associated with its profile		Young's modulus determines how easily a material can stretch and deform. It's defined as the ratio of tensile stress (σ) to tensile strain (ε)
85	[196]	2021	Neural Network Regression	Machine learning methods	It is suitable for both explicit polynomial fitting and implicit polynomial fitting. The algorithm is relatively simple, practical, easy to calculate, and can efficiently achieve the fitting goal. At the same time, the computational complexity is relatively low, which has certain application value		Polynomials (a variety of nonlinear functions)
86	[197]	2021	Weingarten map	Machine learning methods	It is general for point clouds in any dimension, and is efficient to implement due to low complexity and yields better results than the quadratic fitting in both MSE and robustness		
87	[198]	2021	Ratio of Parallelogram Diagonals (RPD)	Classical mathematical approaches	The main advantage of RPD detector is that only once square root operation is required to calculate the curvature value at each point on a contour while maintaining good noise robustness		Corner detection
88	[199]	2021	Geometric Least Square Curve Fitting Method for Localization (GLSCFL)	Classical mathematical approaches	It provides better localization accuracy than other geometric schemes		Localization of wireless sensor network

89	[200]	2021	Quadratic Polynomials and Least Squares	Machine learning methods	Proposed method can complete the identification of mobile specific emitter sources in the unsupervised state with more than 95% identification rate		Specific emitter identification (SEI)
90	[201]	2022	Neural Network Regression	Machine learning methods	Capable of creating a function that can even be used to predict curvatures of complex interfaces arising from fluid simulations		The interface curvature in the context of a Front-Tracking framework
91	[202]	2022	Linear Regression	Machine learning methods	The best algorithm is Cfit K-means as it has a maximum IoU score value of 95.75		Inferring Agronomical Insights for Wheat Canopy
92	[203]	2022	Logarithmic, Polynomial and Exponential curve fitting	Machine learning methods	High accuracy in prediction of magnitude and depth		Experimental analysis of earthquake prediction in India
93	[204]	2022	Linear Regression (Least square regression)	Machine learning methods	Compared with the existing methods, this method can still perform data cleaning well when the historical wind turbine data contains many abnormal data, and the method is insensitive to parameters, simple in the calculation, and easy to automate		Abnormal Data Cleaning Method for Wind Turbines
94	[205]	2022	Linear regression (CurFi)	Machine learning methods	A great resource for the users having limited technical knowledge who will also be able to find the best fit regression model for a dataset using the developed "CurFi" system		An automated system to find best regression analysis
95	[206]	2022	LSTM	Deep Learning techniques	Accurate and economical at the same time		Hydraulic Turbines
96	[207]	2022	Levenberg-Marquardt	Machine learning methods	It provides a new solution for filling the bathymetric gap in very shallow water, which is very essential for topobathymetry mapping		Decompose Airborne LiDAR Bathymetric Waveform in Very Shallow Waters Combining
97	[208]	2022	Linear regression	Machine learning methods	The correlation coefficient of the ANN model and curve-fitting model were 0.9992 and 0.9557, respectively. It shows the ANN model's higher accuracy than the curve-fitting model in R-Event prediction		Prediction of SARS-CoV-2 in Office Environment
98	[209]	2022	Hybrid	Hybrid	Both approaches yielded useful results, and although the machine learning application outcomes had a wider range, they typically presented around 10% better error metrics overall		Wind Power Curve Modelling
99	[210]	2022	Convolutional Neural Networks (CNNs)	Deep Learning techniques	With a worst-case mean absolute percentage error of 4.0%, 4.2%, and 3.7% on the training (108 cells), validation (35 cells), and test (35 cells) datasets, respectively	The presented modelling approach provides a foundation for early-stage battery degradation characterization	Predict the entire battery capacity fade curve - a critical indicator of battery performance degradation
100	[211]	2022	Hybrid	Hybrid	The accuracy of proposed method in fitting short-term trajectories has increased by 49.16% and 29.89% on average compared with the LSTM and BiGRU. The average fitting accuracy of method is 96 m, and the minimum fitting error is 64 m		Marine vessel Automatic Identification System

101	[212]	2023	MLP	Deep Learning techniques	It shows superior performance compared to its standard counterpart and has similar accuracy and convergence properties with the state [1] of-the-art conventional method despite using smaller stencil	Estimation of interface curvature in surface-tension dominated flows
102	[213]	2023	Hybrid	Hybrid	It obtains the most accurate WTPCs on four wind datasets, showing the superiority of the proposed DL approach	Wind turbine power curve modelling
103	[214]	2023	Hybrid	Hybrid	Much small mean square error (MSE), mean absolute error (MAE), and mean absolute percentage error (MAPE) than other methods	Curvature Prediction Method of Profile Roll Bending
104	[215]	2023	Discrete Gaussian Curvature (the whole method is Curvature Weighted Decimation (CWD))	Classical mathematical approaches	CWD reduces introduced error values over Random Decimation when 15 to 50% of the points are retained	Improve Lidar Point Decimation of Terrain Surfaces
105	[216]	2023	Gaussian Curvature	Classical mathematical approaches	It opens up a geometric analysis perspective on model bias and pay attention to model bias on non-long-tailed and even sample balanced datasets	Long-Tailed Classification

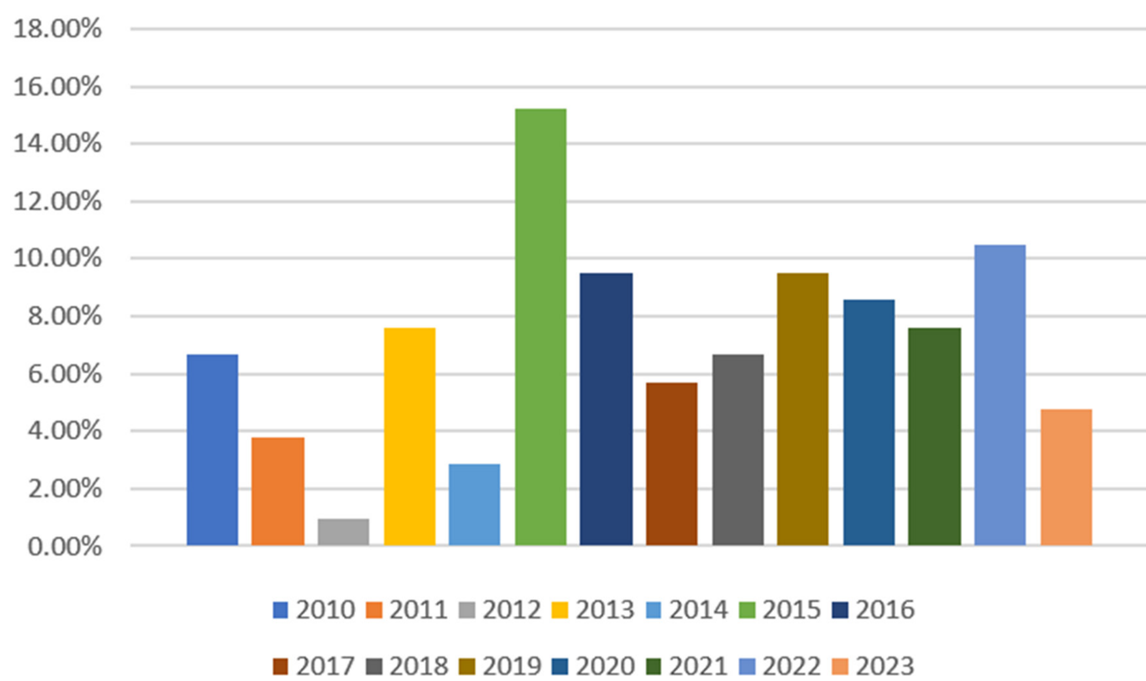

Figure 2. Curvature estimation research percentage in different years.

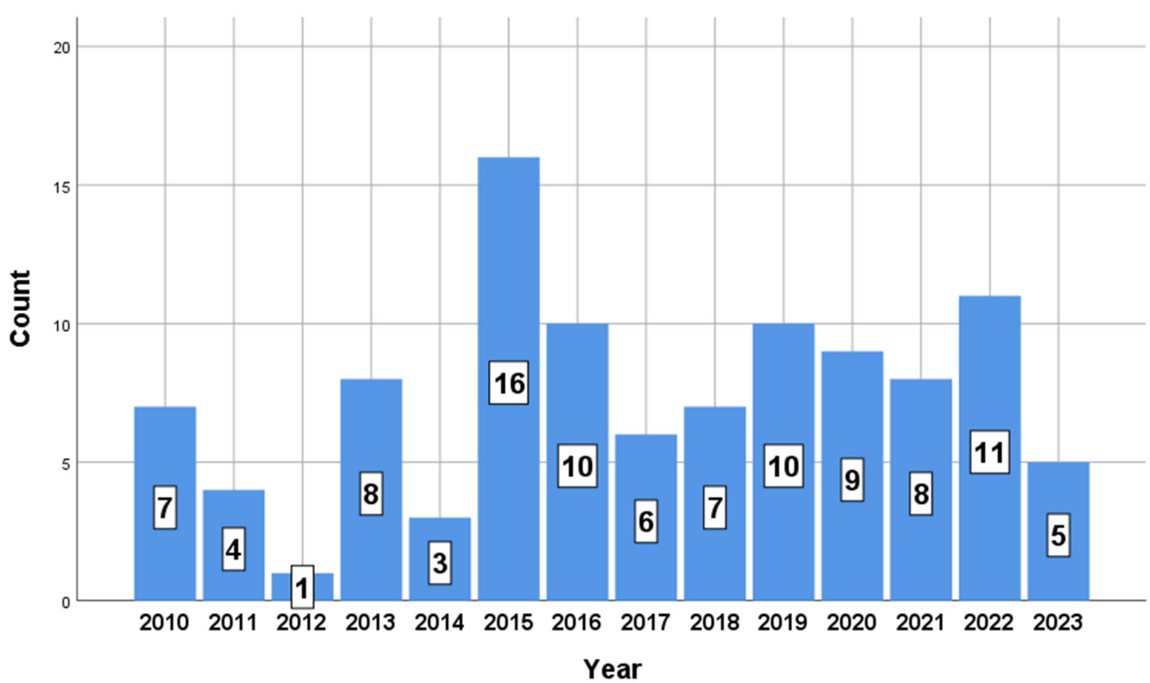

Figure 3. A histogram depicting the research on curvature estimation across various years.

relied on machine learning methods, five used deep learning techniques, and the remaining 13 applied a combination of various categories.

Table 2 displays the research techniques utilized in 105 papers. As per the data presented in **Table 1**, it can be observed that the majority of the 105 research papers (equivalent to 15

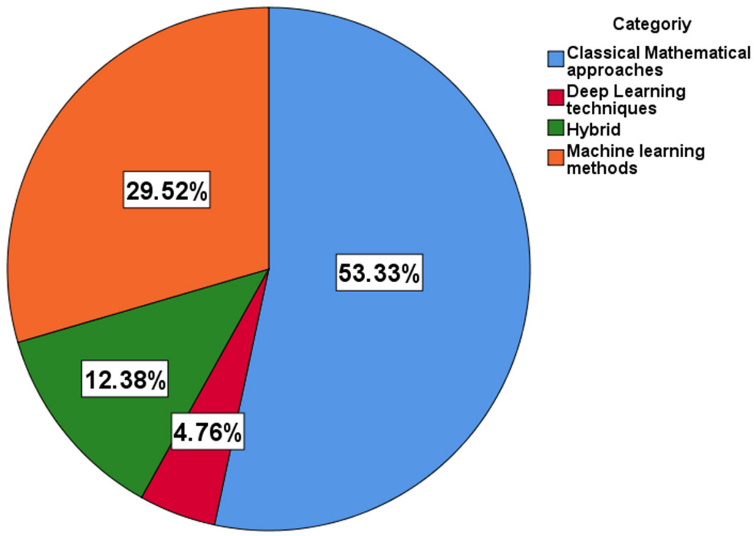

Figure 4. Classification of percentage of curvature estimation research categories.

papers, accounting for 13.6% of the total) used hybrid methods to estimate curvature. The second most commonly used method for estimating curvature was the Algebraic Equation (9 papers, equivalent to 8.2%). Additionally, the Principal Component Analysis method was applied in seven papers (equivalent to 6.4%). The Neural Network Regression method was utilized by the authors of six papers (equivalent to 5.5%) for estimating curvature. Curvature Scale Space, Multiscale Analysis, and Ridge Regression methods were used in five papers each, accounting for 4.5% of the total each. The remaining methods were used in four papers or less.

Table 2 utilizes the colours blue, red, green, and black to denote the classical mathematical approaches, machine learning methods, deep learning techniques, and hybrid methods, respectively.

Discussion

The application of curvature estimation in neurodegenerative diseases underscores its value not only as a tool for understanding complex structural changes but also as a means of enhancing diagnostic precision and monitoring disease progression. While this systematic review provides a comprehensive analysis of the state-of-the-art algorithms and methods for curvature estimation, there are some limitations that need to be acknowledged. First, the scope of the review was limited to papers published between 2010 and 2023. Consequently, earlier works that may have influenced the development of curvature estimation methods are not included in the analysis. Second, the classification of methods into four categories (classical mathematical approaches, machine learning methods, deep learning techniques, and hybrid methods) may not capture the full diversity of approaches in the field. Some algorithms may not fit neatly into these categories, and there may be overlaps between them. Third, this review focused on the number of papers published and the methods employed, but it did not include an in-depth analysis of the performance of these methods on various datasets and tasks. As a result, this review cannot provide a definitive ranking of the most effective methods for specific applications. The paper offers valuable insights into curvature estimation methods and provides recommendations for future research. To further guide upcoming studies, it is beneficial to expand on these recommendations with potential research questions and challenges in the field. Here are the revised suggestions:

1. Exploring the potential of machine learning and deep learning: The increasing adoption of machine learning and deep learning techniques in curvature estimation is evident. However, future research should delve deeper into this domain by addressing questions such as:

• What specific machine learning architectures and algorithms are most suitable for different curvature estimation tasks?

• How can we effectively train deep learning models with limited labeled data for curvature estimation?

• Are there transfer learning techniques that can enhance the generalization of curvature estimation models across different domains and datasets?

Table 2. Curvature estimation research methods

	Frequency	Percent
Valid	2	1.9
Algebraic Equation	9	8.6
B-spline (Kappa)	1	1.0
B-Spline Interpolation	4	3.8
Circle Fitting Method	1	1.0
Convolutional Neural Networks (CNNs)	2	1.9
Curvature Scale Space	5	4.8
Differential Geometry-based Geometric Learning	1	1.0
Discrete Gaussian Curvature (the whole method is Curvature Weighted Decimation (CWD))	1	1.0
Discrete Volume Fraction	1	1.0
Environmental Kuznets curve (EKC)	1	1.0
Exponential Curvature Estimation	1	1.0
Fitting a parabolic Surface	1	1.0
Fourier spectral method	1	1.0
Fuzzy C-Means and the ANFIS and Extreme Learning ANFIS	1	1.0
Gaussian Curvature	1	1.0
Geometric Least Square Curve Fitting Method for Localization (GLSCFL)	1	1.0
Height Function	1	1.0
Hybrid	13	12.4
K-NN	1	1.0
KD Technique	1	1.0
Least Square Method	1	1.0
Levenberg-Marquardt	1	1.0
Linear Regression	4	3.8
Local Quadric Surface Fitting	1	1.0
Local Regression (LOESS)	1	1.0
Logarithmic, Polynomial and Exponential Curve Fitting	1	1.0
LSTM	1	1.0
Mean curvature	2	2.0
MLP	2	1.9
Monte Carlo Simulation	1	1.0
Multiscale Analysis	4	3.8
Neural Network Regression	6	5.7
non-Linear Regression	2	1.9
PC-MSDM	1	1.0
Polynomial and Circular (spherical) Fitting	1	1.0
Polynomial Regression	3	2.9
Principal Component Analysis	7	6.7
Principal Curvature	2	1.9
Quadratic Polynomials and Least Squares	1	1.0
Ratio of Parallelogram Diagonals (RPD)	1	1.0
Ridge Regression	4	3.8
Sigmoidal Curve Fitting	2	1.9
Spline Interpolation	1	1.0
Support Vector Regression	1	1.0
The Color-Gradient Two-Phase Lattice Boltzmann Method	1	1.0
Total curvature	2	2.0
Weingarten map	1	1.0
Total	105	100.0

2. Revisiting the role of classical mathematical approaches: While classical mathematical approaches have seen a decline in usage, it's essential to reevaluate their relevance in contemporary research. Researchers can explore:

• Are there specific scenarios or data types where classical mathematical methods still outperform modern techniques in curvature estimation?

• Can advancements in computational resources and algorithms enhance the efficiency and accuracy of classical approaches?

• How can classical and modern methods be combined to harness their respective strengths?

3. Innovative integration of methods: The interest in hybrid methods suggests potential benefits in combining various algorithms. Future research should address challenges and questions like:

• What are the most effective strategies for combining classical, machine learning, and deep learning techniques in curvature estimation?

• How can these hybrid approaches be adapted to handle noisy or incomplete data?

• Are there theoretical frameworks for understanding the interactions between different algorithms in hybrid methods?

4. Benchmark dataset development and evaluation metrics: To facilitate meaningful comparisons and assessments in the field, the creation of standardized benchmark datasets and evaluation metrics is crucial. Researchers should consider:

• How can benchmark datasets be designed to represent diverse real-world scenarios and challenges in curvature estimation?

• What performance metrics are most appropriate for quantifying the accuracy, robustness, and efficiency of curvature estimation methods?

• Can collaborative efforts within the research community lead to the establishment of universally accepted benchmarks and evaluation protocols?

By addressing these research questions and challenges, future studies can contribute to advancing the field of curvature estimation and provide a clearer roadmap for researchers in this domain. The review has identified several popular methods within each category, such as Neural Network Regression for machine learning and Deep Multilayer Perceptron (MLP) and Convolutional Neural Networks (CNNs) for deep learning. Future research could investigate the strengths and weaknesses of these methods on a variety of datasets and tasks to provide a more comprehensive understanding of their performance and potential applications.

In addition to summarizing the methods and their respective applications, this review underscores the growing necessity of integrating machine learning and deep learning techniques into clinical neurodegenerative disease research. These methods, particularly hybrid approaches, address the complexity of brain morphology and its subtle changes over time, which are often challenging to capture using classical methods alone. For instance, the use of deep neural networks has demonstrated significant promise in handling high-dimensional neuroimaging data, enabling more precise and earlier detection of pathological changes. The shift toward hybrid methods indicates a paradigm where leveraging complementary strengths of classical and advanced techniques can pave the way for innovative diagnostic tools.

Furthermore, this analysis highlights the need for developing standardized benchmarks to evaluate these methods effectively. The authors advocate for future research to focus on creating large-scale, diverse datasets and robust evaluation protocols that bridge the gap between algorithmic advancements and their practical applicability in clinical settings. By fostering collaborations among computational and medical researchers, these advancements can be seamlessly translated into tools that directly benefit patient care.

This systematic review has provided a comprehensive overview of the state-of-the-art in dataset curvature estimation, highlighting the diverse range of methods and their evolution over the past 14 years. Based on the analysis, we can draw several recommendations for researchers, practitioners, and students in the field:

1. Consider the specific dataset and application requirements when selecting a curvature estimation method. Factors such as dataset complexity, computational resources, and the desired level of accuracy should be taken into account.

2. Stay updated on the latest developments in machine learning and deep learning techniques, as these methods continue to evolve and offer new possibilities for curvature estimation.

3. Researchers should continue to explore novel machine learning and deep learning techniques for curvature estimation, as these methods have shown significant promise and growth in recent years.

4. Classical mathematical approaches should not be disregarded, as they can still offer valuable insights in specific contexts and may contribute to the development of hybrid methods.

5. The development of hybrid methods should be a focus of future research, as they have the potential to combine the strengths of different techniques and offer more accurate and efficient curvature estimation.

6. Researchers should conduct performance analyses of the various methods on different datasets and tasks to provide a more comprehensive understanding of their effectiveness and applicability.

7. Developing benchmark datasets and performance metrics: To facilitate the comparison and evaluation of different curvature estimation methods, it is crucial to develop standardized benchmark datasets and performance metrics. This would enable researchers to objectively assess the strengths and weaknesses of various methods and promote the development of more effective curvature estimation techniques.

By following these recommendations, researchers, practitioners, and students can contribute to the continued development and refinement of curvature estimation methods, paving the way for improved computer graphics, computer vision, and pattern recognition applications.

Disclosure of conflict of interest

None.

Address correspondence to: Seyed-Ali Sadegh-Zadeh, Department of Computing, School of Digital, Technologies and Arts, Staffordshire University, Leek Road, Stoke-on-Trent ST4 2DF, United Kingdom. E-mail: ali.sadegh-zadeh@staffs.ac.uk

References

[1] Mitra NJ, Gelfand N, Pottmann H and Guibas L. Registration of point cloud data from a geometric optimization perspective. Second Eurographics Symposium on Geometry Processing 2004; 22-31.

[2] Cohen-Steiner D and Morvan JM. Restricted delaunay triangulations and normal cycle. Annual Symposium on Computational Geometry 2003; 312-321.

[3] Sadegh-Zadeh SA, Fakhri E, Bahrami M, Bagheri E, Khamsehashari R, Noroozian M and Hajiyavand AM. An approach toward artificial intelligence Alzheimer's disease diagnosis using brain signals. Diagnostics (Basel) 2023; 13: 477.

[4] Seyed-Ali SZ, Kambhampati C and Darryl D. Ionic imbalances and coupling in synchronization of responses in neurons. J (Basel) 2019; 2: 17-40.

[5] Sadegh Zadeh SA and Kambhampati C. All-or-none principle and weakness of Hodgkin-Huxley mathematical model. Int J Math Comput Sci 2017; 11: 453.

[6] Sadegh-Zadeh SA, Kambhampati C and Darryl D. Computational Investigation of amyloid peptide channels in Alzheimer's disease. J (Basel) 2018; 2: 1-14.

[7] Botsch M and Sorkine O. On linear variational surface deformation methods. IEEE Trans Vis Comput Graph 2008; 14: 213-30.

[8] Cazals F and Pouget M. Estimating differential quantities using polynomial fitting of osculating jets. Comput Aided Geom Des 2005; 22: 121-146.

[9] Sadegh-Zadeh SA, Rahmani Qeranqayeh A, Benkhalifa E, Dyke D, Taylor L and Bagheri M. Dental caries risk assessment in children 5 years old and under via machine learning. Dent J (Basel) 2002; 10: 164.

[10] Sadegh-Zadeh SA. Machine learning modelling for compressive strength prediction of superplasticizer-based concrete. Infrastructures (Basel) 2023; 8: 21.

[11] Patel HV, Panda A, Kuipers JAM and Peters E. Computing interface curvature from volume fractions: a machine learning approach. Comput Fluids 2019; 193: 104263.

[12] Hernández-Bautista M and Melero FJ. Deep learning of curvature features for shape completion. Comput Graph 2023; 115: 204-215.

[13] Sadegh-Zadeh SA, Bahrami M, Najafi A, Asgari-Ahi M, Campion R and Hajiyavand AM. Evaluation of COVID-19 pandemic on components of social and mental health using machine learning, analysing united states data in 2020. Front Psychiatry 2022; 13: 933439.

[14] Sadegh-Zadeh SA, Sakha H, Movahedi S, Fasihi Harandi A, Ghaffari S, Javanshir E, Ali SA, Hooshanginezhad Z and Hajizadeh R. Advancing prognostic precision in pulmonary embolism: a clinical and laboratory-based artificial intelligence approach for enhanced early mortality risk stratification. Comput Biol Med 2023; 167: 107696.

[15] Bober J, Monod A, Saucan E and Webster KN. Rewiring networks for graph neural network training using discrete geometry. arXiv preprint arXiv:2207.08026, 2022.

[16] Sadegh-Zadeh SA, Nazari MJ, Aljamaeen M, Yazdani FS, Mousavi SY and Vahabi Z. Predictive models for Alzheimer's disease diagnosis and MCI identification: the use of cognitive scores and artificial intelligence algorithms. NPG Neurologie-Psychiatrie-Gériatrie 2024.

[17] Garcia S and Herrera F. An extension on "statistical comparisons of classifiers over multiple data sets" for all pairwise comparisons. J Mach Learn Res 2008; 9.

[18] Hastie T, Tibshirani R, Friedman JH and Friedman JH. The elements of statistical learning: data mining, inference, and prediction. Springer 2009; 2.

[19] Batista GE and Monard MC. An analysis of four missing data treatment methods for supervised learning. Applied Artificial Intelligence 2003; 17: 519-533.

[20] Saar-Tsechansky M and Provost F. Handling missing values when applying classification models. J Mach Learn Res 2007.

[21] Torralba A and Efros AA. Unbiased look at dataset bias. CVPR 2011; 1521-1528.

[22] He H and Garcia EA. Learning from imbalanced data. IEEE Trans Knowl Data Eng 2009; 21: 1263-1284.

[23] Domingos P. A few useful things to know about machine learning. Commun ACM 2012; 55: 78-87.

[24] Jourahmad Z. Machine learning techniques for predicting the short-term outcome of resective surgery in lesional-drug resistance epilepsy. arXiv preprint arXiv:2302.10901, 2023.

[25] Zarringhalam A, Shiry Ghidary S, Mohades A and Sadegh-Zadeh SA. CUDA and OpenMp implementation of boolean matrix product with applications in visual SLAM. Algorithms 2023; 16: 74.

[26] Žliobaitė I. Learning under concept drift: an overview. arXiv preprint arXiv:1010.4784, 2010.

[27] Nazari MJ, Shalbafan M, Eissazade N, Khalilian E, Vahabi Z, Masjedi N, Ghidary SS, Saadat M and Sadegh-Zadeh SA. A machine learning approach for differentiating bipolar disorder type II and borderline personality disorder using electroencephalography and cognitive abnormalities. PLoS One 2024; 19: e0303699.

[28] Sadegh-Zadeh SA and Bagheri M. Harnessing the power of clinical data in dentistry: importance and guidelines for dentists in AI modelling for enhanced patient care. Open J Clin Med Images 2024; 4: 1188.

[29] Sadegh-Zadeh SA, Bagheri M and Saadat M. Decoding children dental health risks: a machine learning approach to identifying key influencing factors. Front Artif Intell 2024; 7: 1392597.

[30] Provost F and Domingos P. Tree induction for probability-based ranking. Mach Learn 2003; 52: 199-215.

[31] Sadegh-Zadeh SA, Soleimani Mamalo A, Kavianpour K, Atashbar H, Heidari E, Hajizadeh R, Roshani AS, Habibzadeh S, Saadat S, Behmanesh M, Saadat M and Gargari SS. Artificial intelligence approaches for tinnitus diagnosis: leveraging high-frequency audiometry data for enhanced clinical predictions. Front Artif Intell 2024; 7: 1381455.

[32] Hand DJ and Vinciotti V. Local versus global models for classification problems: fitting models where it matters. Am Stat 2003; 57: 124-131.

[33] Asuncion A. Uci machine learning repository, university of california, irvine, school of information and computer sciences. http://www.ics.uci.edu/mlearn/MLRepository.Html. 2007.

[34] Aboaoja FA, Zainal A, Ghaleb FA, Al-rimy BAS, Eisa TAE and Elnour AAH. Malware detection issues, challenges, and future directions: a survey. Applied Sciences 2022; 12: 8482.

[35] Sadegh-Zadeh SA, Movahhedi T, Hajiyavand AM and Dearn KD. Exploring undergraduates' perceptions of and engagement in an AI-enhanced online course. Frontiers in Education, Frontiers Media SA 2023; 1252543.

[36] Hanczar B, Hua J, Sima C, Weinstein J, Bittner M and Dougherty ER. Small-sample precision of ROC-related estimates. Bioinformatics 2010; 26: 822-830.

[37] Zarringhalam A, Shiry Ghidary S, Mohades A and Sadegh-Zadeh SA. Semisupervised vector quantization in visual SLAM using HGCN. Int J Intell Syst 2024; 2024: 9992159.

[38] García S, Luengo J and Herrera F. Tutorial on practical tips of the most influential data preprocessing algorithms in data mining. Knowl Based Syst 2016; 98: 1-29.

[39] Bishop CM and Nasrabadi NM. Pattern recognition and machine learning. Springer 2006; 4.

[40] Sohrabi MA, Zare-Mirakabad F, Ghidary SS, Saadat M and Sadegh-Zadeh SA. A novel data augmentation approach for influenza A subtype prediction based on HA proteins. Comput Biol Med 2024; 172: 108316.

[41] Smola AJ and Schölkopf B. A tutorial on support vector regression. Stat Comput 2004; 14: 199-222.

[42] Sadegh-Zadeh SA, Khanjani S, Javanmardi S, Bayat B, Naderi Z and Hajiyavand AM. Catalyzing IVF outcome prediction: exploring advanced machine learning paradigms for enhanced success rate prognostication. Front Artif Intell 2024; 7: 1392611.

[43] Goodfellow I, Bengio Y and Courville A. Deep learning. MIT press, 2016.

[44] Sadegh-Zadeh SA, Khezerlouy-Aghdam N, Sakha H, Toufan M, Behravan M, Vahedi A, Rahimi M, Hosseini H, Khanjani S and Bayat B. Precision diagnostics in cardiac tumours: integrating echocardiography and pathology with advanced machine learning on limited data. Inform Med Unlocked 2024; 49: 101544.

[45] Simonyan K and Zisserman A. Very deep convolutional networks for large-scale image recognition. arXiv preprint arXiv:1409.1556, 2014.

[46] Sadegh-Zadeh SA, Sadeghzadeh N, Soleimani O, Shiry Ghidary S, Movahedi S and Mousavi SY. Comparative analysis of dimensionality reduction techniques for EEG-based emotional state classification. Am J Neurodegener Dis 2024; 13: 23-33.

[47] Brox T, Bruhn A and Weickert J. Variational motion segmentation with level sets. Computer Vision-ECCV 2006: 9th European Conference on Computer Vision, Graz, Austria, May 7-13, 2006. Proceedings, Part I 9. Springer; 2006 pp. 471-483.

[48] Do Carmo MP. Differential geometry of curves and surfaces: revised and updated second edition. Courier Dover Publications 2016.

[49] Gray A, Abbena E and Salamon S. Modern differential geometry of curves and surfaces with Mathematica. Chapman & Hall/CRC; 2006.

[50] O'neill B. Elementary differential geometry. Elsevier; 2006.

[51] Milnor JW. On the total curvature of knots. Ann Math 1950; 248-257.

[52] Mokhtarian F and Mackworth AK. A theory of multiscale, curvature-based shape representation for planar curves. IEEE Trans Pattern Anal Mach Intell 1992; 14: 789-805.

[53] Rasmussen C and Williams C. Gaussian processes for machine learning. Cambridge, MA: MIT press; 2006.

[54] Caglar H, Caglar N, Özer M, Valarıstos A and Anagnostopoulos AN. B-spline method for solving Bratu's problem. Int J Comput Math 2010; 87: 1885-1891.

[55] Zahn CT and Roskies RZ. Fourier descriptors for plane closed curves. IEEE Transactions on Computers 1972; 100: 269-281.

[56] Mallat S. A wavelet tour of signal processing-the sparse way. Academic. Burlington, MA: Elsevier MA EEUU; 2009.

[57] Silverman BW. Density estimation for statistics and data analysis. CRC Press; 1986; 26.

[58] Meyer Y. Wavelets: algorithms and applications. Translated by Ryan RD. Society for Industrial and Applied Mathematics 1993.

[59] Unser M. Splines: a perfect fit for signal and image processing. IEEE Signal Process Mag 1999; 16: 22-38.

[60] Buhmann MD. Radial basis functions: theory and implementations. Cambridge University Press; 2003; 12.

[61] LeCun Y, Bengio Y and Hinton G. Deep learning. Nature 2015; 521: 436-444.

[62] Kalman RE. A new approach to linear filtering and prediction problems. J Basic Eng 1960; 82: 35-45.

[63] Metropolis N and Ulam S. The monte carlo method. J Am Stat Assoc 1949; 44: 335-341.

[64] Mallat S. A wavelet tour of signal processing. Elsevier; 1999.

[65] Pearson K. LIII. On lines and planes of closest fit to systems of points in space. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 1901; 2: 559-572.

[66] Heras J and Martín M. Multiscale analysis for power-to-gas-to-power facilities based on energy storage. Comput Chem Eng 2021; 144: 107147.

[67] Wang B, Wang L, Liang X, Sheng F, Zhang J, Hong Y and Wang L. 3D multiscale dynamic analysis of offshore wind turbine blade under fully coupled loads. Renew Energy 2024; 223: 119985.

[68] Galán G, Martín M and Grossmann IE. Multiscale analysis for the utilization of CO2 towards the production of chemicals at the country level: case study of Spain. J Clean Prod 2024; 437: 140551.

[69] Deng H, Zhang W, Mortensen E, Dietterich T and Shapiro L. Principal curvature-based region detector for object recognition. 2007 IEEE Conference on Computer Vision and Pattern Recognition, IEEE 2007; 1-8.

[70] Zhang W, Deng H, Dietterich TG and Mortensen EN. A hierarchical object recognition system based on multi-scale principal curvature regions. 18th International Conference on Pattern Recognition (ICPR'06), IEEE 2006; 778-782.

[71] Shi J, Bai Y, Zhou J and Zhang B. Multi-crop navigation line extraction based on improved YOLO-v8 and threshold-DBSCAN under complex agricultural environments. Agriculture 2023; 14: 45.
[72] Wang RB, Wang WF, Geng FG, Pan JS, Chu SC and Xu L. UAV path planning in mountain areas based on a hybrid parallel compact arithmetic optimization algorithm. Neural Comput Appl 2023; 1-16.
[73] James G, Witten D, Hastie T and Tibshirani R. An introduction to statistical learning. Springer; 2013; 112.
[74] Cleveland WS. Robust locally weighted regression and smoothing scatterplots. J Am Stat Assoc 1979; 74: 829-836.
[75] De Boor C and De Boor C. A practical guide to splines. Springer-verlag New York 1978; 27.
[76] Rasmussen CE and Williams CKI. Gaussian processes for machine learning. Springer 2006; 1.
[77] Drucker H, Burges CJ, Kaufman L, Smola A and Vapnik V. Support vector regression machines. Adv Neural Inf Process Syst 1996; 9.
[78] Bishop CM. Neural networks for pattern recognition. Oxford University Press; 1995.
[79] Breiman L. Random forests. Mach Learn 2001; 45: 5-32.
[80] Breiman L. Classification and regression trees. Routledge 2017.
[81] Friedman JH. Greedy function approximation: a gradient boosting machine. Ann Stat 2001; 1189-1232.
[82] Altman NS. An introduction to kernel and nearest-neighbor nonparametric regression. Am Stat 1992; 46: 175-185.
[83] Hoerl AE and Kennard RW. Ridge regression: biased estimation for nonorthogonal problems. Technometrics 1970; 12: 55-67.
[84] Tibshirani R. Regression shrinkage and selection via the lasso. J R Stat Soc Series B Stat Methodol 1996; 58: 267-288.
[85] Zou H and Hastie T. Regularization and variable selection via the elastic net. J R Stat Soc Series B Stat Methodol 2005; 67: 301-320.
[86] Hosmer DW Jr, Lemeshow S and Sturdivant RX. Applied logistic regression. John Wiley & Sons; 2013; 398.
[87] Domingos P and Pazzani M. On the optimality of the simple Bayesian classifier under zero-one loss. Mach Learn 1997; 29: 103-130.
[88] Shi M, Lin F, Jing X, Li B, Shi Y and Hu Y. Ecological environment quality assessment of arid areas based on improved remote sensing ecological index-a case study of the loess plateau. Sustainability 2023; 15: 13881.
[89] Hu C, Song M and Zhang A. Dynamics of the eco-environmental quality in response to land use changes in rapidly urbanizing areas: a case study of Wuhan, China from 2000 to 2018. Journal of Geographical Sciences 2023; 33: 245-265.
[90] Ullah A, Kilic M, Habib G, Sahin M, Khalid RZ and Sanaullah K. Reliable prediction of thermophysical properties of nanofluids for enhanced heat transfer in process industry: a perspective on bridging the gap between experiments, CFD and machine learning. J Therm Anal Calorim 2023; 148: 5859-5881.
[91] Barkalov K, Lebedev I, Usova M, Romanova D, Ryazanov D and Strijhak S. Optimization of turbulence model parameters using the global search method combined with machine learning. Mathematics 2022; 10: 2708.
[92] LeCun Y, Bengio Y and Hinton G. Deep learning. Nature 2015; 521: 436-444.
[93] Hochreiter S and Schmidhuber J. Long short-term memory. Neural Comput 1997; 9: 1735-1780.
[94] Goodfellow I. Advances in neural information processing systems. Curran Associates 2014; 27: 2672-2680.
[95] Hinton GE and Salakhutdinov RR. Reducing the dimensionality of data with neural networks. Science 2006; 313: 504-507.
[96] Hinton GE, Osindero S and Teh YW. A fast learning algorithm for deep belief nets. Neural Comput 2006; 18: 1527-1554.
[97] Bengio Y, Lamblin P, Popovici D and Larochelle H. Greedy layer-wise training of deep networks. Adv Neural Inf Process Syst 2006; 19.
[98] Masci J, Meier U, Cireşan D and Schmidhuber J. Stacked convolutional auto-encoders for hierarchical feature extraction. Artificial Neural Networks and Machine Learning-ICANN 2011: 21st International Conference on Artificial Neural Networks, Espoo, Finland, June 14-17, 2011, Proceedings, Part I 21. Springer 2011; 52-59.
[99] Krizhevsky A, Sutskever I and Hinton GE. Imagenet classification with deep convolutional neural networks. Commun ACM 2017; 60: 84-90.
[100] Makhzani A, Shlens J, Jaitly N, Goodfellow I and Frey B. Adversarial autoencoders. arXiv preprint arXiv:1511.05644, 2015.
[101] Socher R, Lin CC, Manning C and Ng AY. Parsing natural scenes and natural language with recursive neural networks. Proceedings of the 28th international conference on machine learning (ICML-11) 2011; 129136.
[102] Davison AJ, Reid ID, Molton ND and Stasse O. MonoSLAM: real-time single camera SLAM. IEEE Trans Pattern Anal Mach Intell 2007; 29: 1052-1067.
[103] Yosinski J, Clune J, Bengio Y and Lipson H. How transferable are features in deep neural networks? Adv Neural Inf Process Syst 2014; 27.

[104] Logothetis NK. The ins and outs of fMRI signals. Nat Neurosci 2007; 10: 1230-1232.
[105] Hochreiter S and Schmidhuber J. Long short-term memory. Neural Comput 1997; 9: 1735-1780.
[106] Hinton GE and Salakhutdinov RR. Reducing the dimensionality of data with neural networks. Science 2006; 313: 504-507.
[107] Shi X, Chen Z, Wang H, Yeung DY, Wong WK and Woo W. Convolutional LSTM network: a machine learning approach for precipitation nowcasting. Adv Neural Inf Process Syst 2015; 28.
[108] Smola AJ and Schölkopf B. A tutorial on support vector regression. Stat Comput 2004; 14: 199-222.
[109] Nazari AA, Janabi-Sharifi F and Zareinia K. Image-based force estimation in medical applications: a review. IEEE Sens J 2021; 22: 8805-8830.
[110] Li J, Fu X, Sun Q, Ji C, Tan J, Wu J and Peng H. Curvature graph generative adversarial networks. Proceedings of the ACM Web Conference 2022; 1528-1537.
[111] Kossaifi J, Tran L, Panagakis Y and Pantic M. Gagan: geometry-aware generative adversarial networks. Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition 2018; 878-887.
[112] Terleev VV, Mirschel W, Schindler U and Wenkel KO. Estimation of soil water retention curve using some agrophysical characteristics and Voronin's empirical dependence. Int Agrophys 2010; 24: 381-387.
[113] Awrangjeb M, Lu G and Fraser CS. A comparative study on contour-based corner detectors. 2010 International Conference on Digital Image Computing: Techniques and Applications, IEEE 2010; 92-99.
[114] Poorter H, Niinemets U, Walter A, Fiorani F and Schurr U. A method to construct dose-response curves for a wide range of environmental factors and plant traits by means of a meta-analysis of phenotypic data. J Exp Bot 2010; 61: 2043-2055.
[115] Carson RT. The environmental Kuznets curve: seeking empirical regularity and theoretical structure. Rev Environ Econ Policy 2010.
[116] Landa E, Keydar S and Moser TJ. Multifocusing revisited-inhomogeneous media and curved interfaces. Geophys Prospect 2010; 58: 925-938.
[117] Kahm M, Hasenbrink G, Lichtenberg-Fraté H, Ludwig J and Kschischo M. grofit: Fitting Biological Growth Curves with R. Journal of Statistical Software 2010; 33: 1-21.
[118] He J and Richard P. Environmental Kuznets curve for CO2 in Canada. Ecol Econ 2010; 69: 1083-1093.
[119] Anzanello MJ and Fogliatto FS. International Journal of Industrial Ergonomics Learning curve models and applications: literature review and research directions. Int J Ind Ergon 2011; 41: 573-583.
[120] Kumar R and Indrayan A. Receiver operating characteristic (roc) curve for medical researchers. Indian Pediatr 2011; 48: 277-287.
[121] Stanimirova I, Tauler R and Walczak B. A comparison of positive matrix factorization and the weighted multivariate curve resolution method. Application to environmental data. Environ Sci Technol 2011; 45: 10102-10110.
[122] Monroy A, Eigenstetter A and Ommer B. Beyond straight lines-object detection using curvature. 2011 18th IEEE International Conference on Image Processing, IEEE 2011; 3561-3564.
[123] Vaas LA, Sikorski J, Michael V, Göker M and Klenk HP. Visualization and curve-parameter estimation strategies for efficient exploration of phenotype microarray kinetics. PLoS One 2012; 7: e34846.
[124] Boyd K, Eng KH and Page CD. Area under the precision-recall curve: point estimates and confidence intervals. Machine Learning and Knowledge Discovery in Databases: European Conference, ECML PKDD 2013, Prague, Czech Republic, September 23-27, 2013, Proceedings, Part III 13. Springer 2013; 451-466.
[125] Lydia M, Selvakumar AI, Kumar AA and Kumar GEP. Advanced algorithms for wind turbine power curve modeling. IEEE Trans Sustain Energy 2013; 4: 827-835.
[126] He B, Lin Z and Li YF. An automatic registration algorithm for the scattered point clouds based on the curvature feature. Opt Laser Technol 2013; 46: 53-60.
[127] Ruijter JM, Pfaffl MW, Zhao S, Spiess AN, Boggy G, Blom J, Rutledge RG, Sisti D, Lievens A, De Preter K, Derveaux S, Hellemans J and Vandesompele J. Evaluation of qPCR curve analysis methods for reliable biomarker discovery: bias, resolution, precision, and implications. Methods 2013; 59: 32-46.
[128] Sofia G, Pirotti F and Tarolli P. Variations in multiscale curvature distribution and signatures of LiDAR DTM errors. Earth Surf Process Landf 2013; 38: 1116-1134.
[129] Annaert J, Claes AGP, De Ceuster MJK and Zhang H. Estimating the spot rate curve using the nelson-siegel model: a ridge regression approach. Int Rev Econ 2013; 27: 482-496.
[130] Saha-Chaudhuri P and Heagerty PJ. Non-parametric estimation of a time-dependent predictive accuracy curve. Biostatistics 2013; 14: 42-59.
[131] Papalazarou C, De With PHN and Rongen P. Sparse-plus-dense-RANSAC for estimation of

multiple complex curvilinear models in 2D and 3D. Pattern Recognit 2013; 46: 925-935.
[132] Lydia M, Kumar SS, Selvakumar AI and Kumar GEP. A comprehensive review on wind turbine power curve modeling techniques. Renew Sustain Energy Rev 2014; 30: 452-460.
[133] Park JY, Lee JK, Oh KY and Lee JS. Development of a novel power curve monitoring method for wind turbines and its field tests. IEEE Transactions on Energy Conversion 2014; 29: 119-128.
[134] Hauenstein JD and Newman TS. On reliable estimation of curvatures of implicit surfaces. 2014 2nd International Conference on 3D Vision, IEEE 2014; 697-704.
[135] Nurunnabi A, West G and Belton D. Outlier detection and robust normal-curvature estimation in mobile laser scanning 3D point cloud data. Pattern Recognit 2015; 48: 1404-1419.
[136] Moreira P and Misra S. Biomechanics-based curvature estimation for ultrasound-guided flexible needle steering in biological tissues. Ann Biomed Eng 2015; 43: 1716-1726.
[137] Dahmani H, Chadli M, Rabhi A and El Hajjaji A. Vehicle dynamics and road curvature estimation for lane departure warning system using robust fuzzy observers: experimental validation. Vehicle System Dynamics 2015; 53: 1135-1149.
[138] Ivey CB and Moin P. Accurate interface normal and curvature estimates on three-dimensional unstructured non-convex polyhedral meshes. J Comput Phys 2015; 300: 365-386.
[139] Fernandez C, Llorca DF, Stiller C and Sotelo MA. Curvature-based curb detection method in urban environments using stereo and laser. IEEE Intell Veh Symp 2015; 2015: 579-584.
[140] Marin D, Zhong Y, Drangova M and Boykov Y. Thin structure estimation with curvature regularization. Proc IEEE Int Conf Comput Vis 2015; 2015: 397-405.
[141] Zhang S, Yang D, Huang S, Tu L and Zhang X. Corner detection using Chebyshev fittingbased continuous curvature estimation. Electron Lett 2015; 51: 1988-1990.
[142] Yacine Z, Ichalal D, Ait-Oufroukh N, Mammar S and Djennoune S. Takagi-sugeno observers: experimental application for vehicle lateral dynamics estimation. IEEE Trans Control Syst Technol 2015; 23: 754-761.
[143] Zhou Y, Zhou M, Wu Z, Shui W, Deng Q and Wang W. Aging simulation of 3D models based on curvature analysis and color transition. International Conference on Virtual Reality and Visualization 2015; 2-6.
[144] Echeagaray-Patron BA, Miramontes-Jaramillo D and Kober V. Conformal parameterization and curvature analysis for 3D facial recognition. International Conference on Computational Science and Computational Intelligence 2015; 1-2.
[145] Lin YH, Chen YJ and Huang KC. Estimation of radius of curvature using reflected-spot method. Conference Record - IEEE Instrumentation and Measurement Technology Conference 2015; 2015: 1382-1386.
[146] Murphy RJ and Armand M. Estimating the configuration of a continuum dexterous manipulator with variable curvature bending using partial shape-sensing. Annu Int Conf IEEE Eng Med Biol Soc 2015; 2015: 5268-5271.
[147] Huang Q and Wischgoll T. The influence of gradient estimation on the extraction of boundary point cloud. Proceedings of the IEEE National Aerospace Electronics Conference 2016; 2016: 1-8.
[148] Nasiri AA and Fathy M. An effective algorithm for fingerprint reference point detection based on filed flow curves. Proceedings of the International Symposium on Artificial Intelligence and Signal Processing 2015; 79-83.
[149] Qi R, Zhang W, Tian Y and Xiao L. High order curvature and torsion continuous trajectory planning method for space flight robot. Proceedings - 2015 7th International Conference on Intelligent Human-Machine Systems and Cybernetics 2015; 2: 60-63.
[150] On the role of non-local menger curvature in image processing technion - Israel institute of technology. International Conference on Image Processing 2015; 1: 4437-4341.
[151] Bogner S, Rüde U and Harting J. Curvature estimation from a volume-of-fluid indicator function for the simulation of surface tension and wetting with a free-surface lattice Boltzmann method. Phys Rev E 2016; 93: 043302.
[152] Chen S, Meng H, Zhang C and Liu C. A KD curvature based corner detector. Neurocomputing 2016; 173: 434-441.
[153] Kawakami Y, Hattori T, Imai Y, Horikawa Y and Janaka Rajapakse RPC. Automated multiple-brightness peak image processing method using curvature and variance estimation. Proceedings of International Conference on Artificial Life and Robotics 2016; 21: 32-35.
[154] Meng J and Wang S. The recognition of overlapping apple fruits based on boundary curvature estimation. Proceedings - 2015 6th International Conference on Intelligent Systems Design and Engineering Applications, ISDEA 2015, Institute of Electrical and Electronics Engineers Inc. 2016; 874-877.
[155] Váša L, Vaněček P, Prantl M, SkorkovskáV, Martínek P and Kolingerová I. Mesh statistics for robust curvature estimation. Eurographics Symposium on Geometry Processing 2016; 35: 271-280.

[156] Yang Z, Radzienski M, Kudela P and Ostachowicz W. Two-dimensional modal curvature estimation via fourier spectral method for damage detection. Composite Structure 2016.
[157] Váša L, Kühnert T and Brunnett G. Multivariate analysis of curvature estimators. Comput Aided Des Appl 2017; 14: 58-69.
[158] Li Z, Wang GJ, Liao K and Zhu ZH. Model-independent estimations for the curvature from standard candles and clocks. Astrophys J 2016; 833: 240.
[159] Coquerelle M and Glockner S. A fourth-order accurate curvature computation in a level set framework for two-phase flows subjected to surface tension forces. J Comput Phys 2016; 305: 838-876.
[160] Prantl M, Váša L and Kolingerová I. Fast screen space curvature estimation on GPU. VISIGRAPP 2016 - Proceedings of the 11th Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications 2016; 151-160.
[161] Dharmasiri T, Spek A and Drummond T. Joint prediction of depths, normals and surface curvature from RGB images using CNNs. IEEE International Conference on Intelligent Robots and Systems 2017; 2017: 1505-1512.
[162] Spek A, Li WH and Drummond T. A fast method for computing principal curvatures from range images. Australasian Conference on Robotics and Automation 2015.
[163] Evrard F, Denner F and van Wachem B. Estimation of curvature from volume fractions using parabolic reconstruction on two-dimensional unstructured meshes. J Comput Phys 2017; 351: 271-294.
[164] Duan N, Wu SJ, Dai SL, Miao TT, Chen J and Wang ZP. Simultaneous detection of pathogenic bacteria using an aptamer based biosensor and dual fluorescence resonance energy transfer from quantum dots to carbon nanoparticles. Microchimica Acta 2015; 182: 917-923.
[165] Lin N and Shi S. Road grade estimation based on the curvature of the vertical profile of a road. Proceedings of the Institution of Mechanical Engineers Part D: Journal of Automobile Engineering 2017; 231: 476-486.
[166] Tang Y, Li H, Sun X, Morvan JM and Chen L. Principal curvature measures estimation and application to 3D face recognition. J Math Imaging Vis 2017; 59: 211-233.
[167] Doroz R, Wrobel K and Porwik P. An accurate fingerprint reference point determination method based on curvature estimation of separated ridges. International Journal of Applied Mathematics and Computer Science 2018; 28: 209-255.
[168] Lee TY, Skvortsov V, Kim MS, Han SH and Ka MH. Application of W-Band FMCW radar for road curvature estimation in poor visibility conditions. IEEE Sens J 2018; 18: 5300-5312.
[169] Denissenya M, Linder EV and Shafieloo A. Cosmic curvature tested directly from observations. J Cosmol Astropart Phys 2018; 2018.
[170] Kimia BB, Li X, Guo Y and Tamrakar A. Differential geometry in edge detection: accurate estimation of position, orientation and curvature. IEEE Trans Pattern Anal Mach Intell 2019; 41: 1573-1586.
[171] Taheri Andani M, Peterson A, Munoz J and Ahmadian M. Railway track irregularity and curvature estimation using doppler LIDAR fiber optics. Proc Inst Mech Eng F J Rail Rapid Transit 2018; 232: 63-72.
[172] Liu J, Fu H, Qin J and Jin H. Sliding look-ahead window-based real-time feedrate planning for non-uniform rational B-splines curves. Advances in Mechanical Engineering 2018; 10: 1-19.
[173] Kronenberger M, Wirjadi O and Hagen H. Empirical comparison of curvature estimators on volume images and triangle meshes. IEEE Trans Vis Comput Graph 2019; 25: 3032-3041.
[174] Khameneifar F and Ghorbani H. On the curvature estimation for noisy point cloud data via local quadric surface fitting. Comput Aided Des Appl 2019; 16: 140-149.
[175] Cao Y, Li D, Sun H, Assadi AH and Zhang S. Efficient curvature estimation for oriented point clouds. Stat 2019; 1050: 26.
[176] Lyu H, Sha N, Qin S, Yan M, Xie Y and Wang R. Manifold denoising by nonlinear robust principal component analysis. Adv Neural Inf Process Syst 2019; 32: 1-11.
[177] Qi Y, Lu J, Scardovelli R, Zaleski S and Tryggvason G. Computing curvature for volume of fluid methods using machine learning. J Comput Phys 2019; 377: 155-161.
[178] Akai T, Lin Q, Alhosani A, Bijeljic B and Blunt MJ. Quantification of uncertainty and best practice in computing interfacial curvature from complex pore space images. Materials (Basel) 2019; 12: 2138.
[179] Nguyen DD and Wei GW. DG-GL: differential geometry-based geometric learning of molecular datasets. Int J Numer Method Biomed Eng 2019; 35: e3179.
[180] Meynet G, Digne J and Lavoue G. PC-MSDM: a quality metric for 3D point clouds. 2019 11th International Conference on Quality of Multimedia Experience 2019; 1-3.
[181] Catanuto G, Taher W, Rocco N, Catalano F, Allegra D, Milotta FLM, Stanco F, Gallo G and Nava MB. Breast shape analysis with curvature estimates and principal component analysis for cosmetic and reconstructive breast surgery. Aesthet Surg J 2019; 39: 164-173.

[182] Mizoguchi T, Ishii A and Nakamura H. Individual tree species classification based on terrestrial laser scanning using curvature estimation and convolutional neural network. International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences - ISPRS Archives 2019; 42: 1077-1082.
[183] Mary H and Brouhard GJ. Kappa (κ): analysis of curvature in biological image data using B-splines. bioRxiv 2019; 1-14.
[184] Ben-Shabat Y and Gould S. DeepFit: 3D surface fitting via neural network weighted least squares BT - computer vision - ECCV 2020. In: Vedaldi A, Bischof H, Brox T and Frahm JM, eds. Cham: Springer International Publishing; 2020; pp. 20-34.
[185] Zhao Y, Zhang J, Pereira E, Zheng Y, Su P, Xie J, Zhao Y, Shi Y, Qi H, Liu J and Liu Y. Automated tortuosity analysis of nerve fibers in corneal confocal microscopy. IEEE Trans Med Imaging 2020; 39: 2725-2737.
[186] Gonzalez-Diaz R, Mainar E, Paluzo-Hidalgo E and Rubio B. Neural-network-based curve fitting using totally positive rational bases. Mathematics 2020; 8: 1-19.
[187] Parbat D and Chakraborty M. A python based support vector regression model for prediction of COVID19 cases in India. Chaos Solitons Fractals 2020; 138: 109942.
[188] Urang JG, Ebong ED, Akpan AE and Akaerue EI. A new approach for porosity and permeability prediction from well logs using artificial neural network and curve fitting techniques: a case study of Niger Delta, Nigeria. J Appl Geophy 2020; 183: 104207.
[189] Choi MG, Law M, Yoon DK, Tamura M, Matsumoto K, Otsuka M, Kim MS, Djeng SK, Monzen H and Suh TS. Simplified sigmoidal curve fitting for a 6 MV FFF photon beam of the Halcyon to determine the field size for beam commissioning and quality assurance. Radiat Oncol 2020; 15: 273.
[190] Abritta R, Panoeiro FF, de Aguiar EP, de M. Honório L, Marcato ALM and da Silva Junior IC. Fuzzy system applied to a hydraulic turbine efficiency curve fitting. Electrical Engineering 2020; 102: 1361-1370.
[191] Tamang SK, Singh PD and Datta B. Forecasting of Covid-19 cases based on prediction using artificial neural network curve fitting technique. Global Journal of Environmental Science and Management 2020; 6: 53-64.
[192] Van Damme S, Torres Vega M, Heyse J, De Backere F and De Turck F. A low-complexity psychometric curve-fitting approach for the objective quality assessment of streamed game videos. Signal Process Image Commun 2020; 88: 115954.
[193] Deng L, Mei H, Zhou W and Jing E. Fitting method of optimal energy-running time curve based on train operation data of an urban rail section. J Adv Transp 2021; 2021.
[194] Mandal S and Uhlmann V. A learning-based formulation of parametric curve fitting for bioimage analysis. Lecture Notes in Computational Science and Engineering 2021; 139: 1031-1038.
[195] Wang LG, Li Z, Zhang L, Zhou R and Chen X. On the measurement of particle contact curvature and young's modulus using X-ray μCT. Appl Sci 2021; 11: 1752.
[196] Tong Y, Yu L, Li S, Liu J, Qin H and Li W. Polynomial fitting algorithm based on neural network. ASP Transactions on Pattern Recognition and Intelligent Systems 2021; 1: 32-39.
[197] Cao Y, Li D, Sun H, Assadi AH and Zhang S. Efficient weingarten map and curvature estimation on manifolds. Mach Learn 2021; 110: 1319-1344.
[198] Zhang S, Liu S, Wang Q, Rong L, Li B and Zheng Q. An efficient corner detector based on ratio of parallelogram diagonal. The Journal of Engineering 2021; 2021: 726-775.
[199] Singh M, Bhoi SK and Panda SK. Geometric least square curve fitting method for localization of wireless sensor network. Ad Hoc Networks 2021; 116.
[200] Zhang Q, Guo Y and Song Z. Dynamic curve fitting and BP neural network with feature extraction for mobile specific emitter identification. IEEE Access 2021; 9: 33897-33910.
[201] França HL and Oishi CM. A machine learning strategy for computing interface curvature in front-tracking methods. J Comput Phys 2022; 450: 110860.
[202] Gupta A, Kaur L and Kaur G. Inferring agronomical insights for wheat canopy using image-based curve Fit K-means segmentation algorithm and statistical analysis. Int J Genomics 2022; 2022: 1875013.
[203] Chittora P, Chakrabarti T, Debnath P, Gupta A, Chakrabarti P, Praveen SP, Margala M and Elngar AA. Experimental analysis of earthquake prediction using machine learning classifiers, curve fitting, and neural modeling. Research Square 2022.
[204] Yin X, Liu Y, Yang L and Gao W. Abnormal data cleaning method for wind turbines based on constrained curve fitting. Energies (Basel) 2022; 15: 6373.
[205] Roy A, Al Zubayer T, Tabassum N, Islam MN and Sattar MA. CurFi: an automated tool to find the best regression analysis model using curve fitting. Engineering Reports 2022; 4: 1-19.
[206] Pan H, Hang C, Feng F, Zheng Y and Li F. Improved neural network algorithm based flow characteristic curve fitting for hydraulic tur-

bines. Sustainability (Switzerland) 2022; 14: 10757.
[207] Qi C, Ma Y, Su D, Yang F, Liu J and Wang XH. A method to decompose airborne LiDAR bathymetric waveform in very shallow waters combining deconvolution with curve fitting. IEEE Geosci Remote Sens Lett 2022; 19: 1-5.
[208] Kapoor NR, Kumar A, Kumar A, Kumar A and Kumar K. Transmission probability of SARS-CoV-2 in office environment using artificial neural network. IEEE Access 2022; 10: 121204-121229.
[209] Filho DPC, da S Ramos DN, Filho MDC, Medrado RC, Weyll ALC and Maritan TR. Wind power curve modeling through data-driven approaches: evaluating piecewise linear fitting and machine learning applications in a real-unit case. 2022 Workshop on Communication Networks and Power Systems (WCNPS) 2022; 1-6.
[210] Saxena S, Ward L, Kubal J, Lu W, Babinec S and Paulson N. A convolutional neural network model for battery capacity fade curve prediction using early life data. J Power Sources 2022; 542: 231736.
[211] Chen J, Chen H, Zhao Y and Li X. FB-BiGRU: a deep learning model for AIS-based vessel trajectory curve fitting and analysis. Ocean Engineering 2022; 266: 112898.
[212] Önder A and Liu PLF. Deep learning of interfacial curvature: a symmetry-preserving approach for the volume of fluid method. J Comput Phys 2023; 485: 112110.
[213] Wang Y, Duan X, Zou R, Zhang F, Li Y and Hu Q. A novel data-driven deep learning approach for wind turbine power curve modeling. Energy 2023; 270: 126908.
[214] Cao H, Yu G, Liu T, Fu P, Huang G and Zhao J. Research on the curvature prediction method of profile roll bending based on machine learning. Metals 2023; 13: 143.
[215] Schrum PT, Jameson CD, Tateosian LG, Blank GB, Wegmann KW and Nelson SAC. Curvature weighted decimation: a novel, curvature-based approach to improved lidar point decimation of terrain surfaces. Geomatics 2023; 3: 266-289.
[216] Ma Y, Jiao L, Liu F, Yang S, Liu X and Li L. Curvature-balanced feature manifold learning for long-tailed classification. arXiv preprint arXiv:2303.12307, 2023.