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Visualising Chaos through Real-Time Heat Maps in a Flocking Simulation

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Abstract

Simulations of chaotic real-world scenarios are useful for attempting to understand, predict and even manage these complex systems. This study simulates flocking birds and a predator which disrupts the flock, causing the birds to scatter and reform. Three measures of chaos: polarisation, kinetic energy and Shannon entropy are visualised using a real-time heat map. Our results find the heat map to be effective in highlighting the subtle differences in measurements, offering potential use-cases in future studies.

CCS Concepts

• ***Computing methodologies*** → ***Scientific visualisation;*** • ***Information systems*** → *Information systems applications;*

1. Introduction

Complex, chaotic systems have been extensively studied and measured, using metrics such as directional variance and probability [PVG02] [PMO19]. Combining a simulation with a visualisation of chaos would provide useful insights into the understanding of complex systems.

This paper presents early work which considers a chaotic simulation of flocking birds (boids) [Rey87] accompanied with a visualisation of varying measures of chaos. Despite the seeming randomness of chaotic systems [Tso12], they can also be predictable and repeatable, through emergent behaviour, and this offers the opportunity to understand why systems behave in such a way, and how they can be managed or manipulated [GG94]. There are many accepted methods for measuring chaos [AAPR13], for example, combination methods [HMA15]. In this study we consider directional variance, velocity, and entropy which are calculated using measures of polarisation, kinetic temperature, and Shannon entropy, respectively. Whilst there is existing research on measuring chaotic systems, to the authors' knowledge, this study is novel in the simulation chosen, the use of a system disruptor, the specific measures of chaos, and use of a real-time heat map. We discuss the effectiveness of our approach and consider future work.

2. Background

Chaos theory was discovered by Edward Lorenz in 1963 while trying to predict weather patterns [Oes07]. Counter-intuitively, chaotic behaviour can emerge from deterministic laws. For example, the behaviour of turbulent fluids is based on the deterministic motion of particle elements. The complexity of these systems results in unpredictablility at the micro level [Jen87], yet at the macro level can often result in predictable emergent behaviour [Joh02].

Therefore, it seems intuitive that our understanding of chaotic systems could be improved by narrowing the gap between the macro and micro [Kad00], by measuring chaos [HMA15], and visualising this in real time [Ouy24]. The result of such work could have a variety of use cases, for example in crowd management [SP24].

The Lyapunov exponent (λ) [Bro97] is a common measure of chaos [Sto24]. The exponent measures the average rate by which the distance between close points changes after one iteration. Other measures include the estimation of fractal dimensionality [DKBP23], and Kolmogrov-Sinai entropy [Fri04] which measures the rate of information production.

When visualising chaos, several common methods are in use. Phase space portraits are used for the Lorenz Attractor [Lor17]. Bifurcation diagrams [MW24] illustrate the transitions between order and chaos. Heat maps are useful in demonstrating the evolving chaos within a system, with the temperature component acting as a signifier of probability densities [dMBJBSRR*22].

3. Methodology

Our methodology considers a simulation with chaos and emergence, distinct measures of chaos, and a heat map visualisation.

The boids model by Reynolds [Rey87] was chosen as a simple but effective demonstration of both chaos and emergent order [PVG02]. An analogy of bird movement, the model simulates flocking with simple rules for each bird. To reliably disrupt the flock, an extension to the model by Oboshi [OKMI03] and later White [Whi06] includes the addition of a predator. The predator, in this instance, does not predate birds, but causes the flock to disperse and reform.

The simulation was written in C++ and DirectX 11. Any graphical method or programming language would have been suitable; in this case the authors have a particular expertise in this choice. The simulation is in 3D space but movement only occurs on the XY plane and the flock consists of 1000 birds.

Three measures of chaos are chosen for their distinctness and applicability to the model. For each bird, at each simulation cycle, a sample of neighbouring birds, and their respective movement vectors are taken to calculate an individual chaos value per bird. First, to measure the degree of collective alignment, polarisation is calculated from the normalised average velocity of the agents [PVG02] using the following formula, where N is the number of birds and u_i is the direction of a bird:

$$Chaos = 1 - |\frac{1}{N}\sum_{i=1}^{N} u_i| \tag{1}$$

Second, kinetic temperature (speed variance) measures how agent speed, isolated from direction, is affected by movement [PVG02], where v_i is a bird's speed:

$$Chaos = \frac{\frac{1}{N}\sum_{i=1}^{N} |v_i - (\frac{1}{N}\sum_{i=1}^{N} v_i)|^2}{v_{\max}^2} \tag{2}$$

The third measure is Shannon entropy [PMO19], which calculates a probability for agent direction, providing a general measure of uncertainty within the group. This is given by:

$$Chaos := -K \sum_{x\epsilon\chi} p_i(x) log p_i(x) \tag{3}$$

This expression considers a discrete random variable $x\epsilon\chi$ characterised by the probability distribution $p(x)$. The parameter K is a positive constant, set to 1, and is used to express H (from Shannon's formula) in a given unit of measure.

A heat map visualisation is rendered in real time by adapting techniques from particle systems and volumetric rendering [JA17]. The first pass accumulates bird chaos values in world-space, *splatting* [BHZK05] these onto an off-screen accumulation buffer. The second pass samples the accumulation buffer and performs a texture lookup on a 1-D colour map, reassigning chaos (from 0 to 1) to a colour gradient (from blue to red) onto the screen buffer.

4. Results

Figure 1 demonstrates the three measures during predator response (*affected*, or *unaffected*), and *recovering*. See work by Ballerini [BCC*08] for examples of this in real-world populations. The heat map colours equate to blue and green where chaos is between 0 and 0.3 (ordered), yellow in the region of 0.3 and 0.6, and red above 0.6 (chaotic).

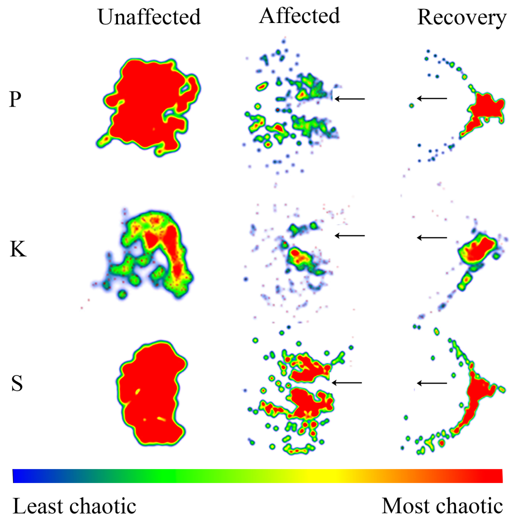

Figure 1: *Comparing the three measures; Polarisation (P), Kinetic (K), and Shannon (S). The arrow signifies the direction of the predator.*

Each measure has a unique visual difference, with kinetic demonstrating the least chaos, and Shannon entropy the most; a relationship also observed by [PVG02]. Kinetic energy has pools, or hot spots, within the flock that show fidelity in speed variance, this mirrors Ballerini's [BCC*08] observation that flocks of starlings expand, split and regroup. The polarisation measurement demonstrates this when recovering, but in the flock state it is uniformly chaotic. Shannon entropy is uniformly chaotic in each state, especially in the flock state, but shows medium / high chaos in the recovery state, too. This aligns with Garnier [GGT07] who discusses how collective behaviour relies on stochastic structures.

5. Conclusion

The behaviour of the birds in each scenario is predictable, we consistently observe a pattern of flocking, disruption and recovery. Our measurements demonstrate different perspectives of the chaos within.

The use of a heat map has been effective in showing subtle differences between measurements, underlining its common adoption [WF09]. A real-time heat map has been especially effective in observing similarity for measures across several simulation runs.

When comparing chaos measurements, the study has demonstrated how each might have different use-cases. For example, if a similar approach were used to simulate crowds to design safe public spaces, polarisation may be useful for comparing Brownian motion and ordered crowds [War18], kinetic energy to identify or predict when crowds might panic [BKS*02], and Shannon entropy to observe the general predictability of crowds [ZYSC15].

This study has a few limitations. First, the boid population is constant. Parrish [PVG02] discusses how flocks differ based on population size. The study could also compare chaos measurements with different flocking algorithms. Future work could also consider whether the different measurements are effective in classifying different types of agents, building on work by Nabeel [NM22].

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